A simple, robust and accurate a posteriori subcell finite volume limiter for the discontinuous Galerkin method

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ABSTRACT

In our talk we will present a novel *a posteriori* finite volume subcell limiter technique for the Discontinuous Galerkin finite element method for nonlinear systems of hyperbolic conservation laws in multiple space dimensions that works well for *arbitrary* high order of accuracy in space *and time* and that does *not* destroy the natural *subcell resolution* properties of the DG method. High order time discretization is achieved via a one-step ADER approach that uses a local space-time discontinuous Galerkin predictor method to evolve the data locally in time within each cell.

Our new limiting strategy is based on the so-called MOOD paradigm, which *a posteriori* verifies the validity of a discrete candidate solution against physical and numerical detection criteria after each time step. Here, we employ a relaxed discrete maximum principle in the sense of piecewise polynomials and the positivity of the numerical solution as detection criteria. Within the DG scheme on the main grid, the discrete solution is represented by piecewise polynomials of degree N. For those troubled cells that need limiting, our new limiter approach recomputes the discrete solution by scattering the DG polynomials at the previous time step onto a set of $N_s = 2N + 1$ finite volume subcells per space dimension. A robust but accurate ADER-WENO finite volume scheme then updates the subcell averages of the conservative variables within the detected troubled cells. The recomputed subcell averages are subsequently gathered back into high order cell-centered DG polynomials on the main grid via a subgrid reconstruction operator. The choice of $N_s = 2N + 1$ subcells is optimal since it allows to match the maximum admissible time step of the finite volume scheme on the subgrid with the maximum admissible time step of the DG scheme on the main grid, minimizing at the same time the local truncation error of the subcell finite volume scheme. It furthermore provides an excellent subcell resolution of discontinuities.

Our new approach is therefore *radically different* from classical DG limiters, where the limiter is using TVB or (H)WENO *reconstruction*, based on the discrete solution of the DG scheme on the *main grid* at the new time level. In our case, the discrete solution is instead *recomputed* within the troubled cells using a *different and more robust* numerical scheme on a *subgrid level*.

We illustrate the performance of the new *a posteriori* subcell ADER-WENO finite volume limiter approach for very high order DG methods via the simulation of numerous test cases run on Cartesian grids in two and three space dimensions, using DG schemes of up to *tenth* order of accuracy in space and time ($N = 9$). The method is also able to run on massively parallel large scale supercomputing infrastructure, which is shown via one 3D test problem that uses a tenth order scheme and **10 billion** space-time degrees of freedom per time step.

At the end of our talk, we will also present the extension of this novel *a posteriori* subcell limiter approach to space-time adaptive Cartesian grids (AMR) in two and three space dimensions, together with time-accurate local time stepping (LTS), as well as to unstructured triangular and tetrahedral meshes. The combination of the high order DG scheme and the sub-cell resolution of the new limiter with the advantages of AMR allows for an unprecedented ability in resolving even the finest flow details. The spectacular resolution properties of the new scheme have been shown through a wide number of test cases performed in two and in three space dimensions, both for the Euler equations of compressible gas dynamics and for the magnetohydrodynamics (MHD) equations.

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