Interaction of acoustic perturbations with a shock wave

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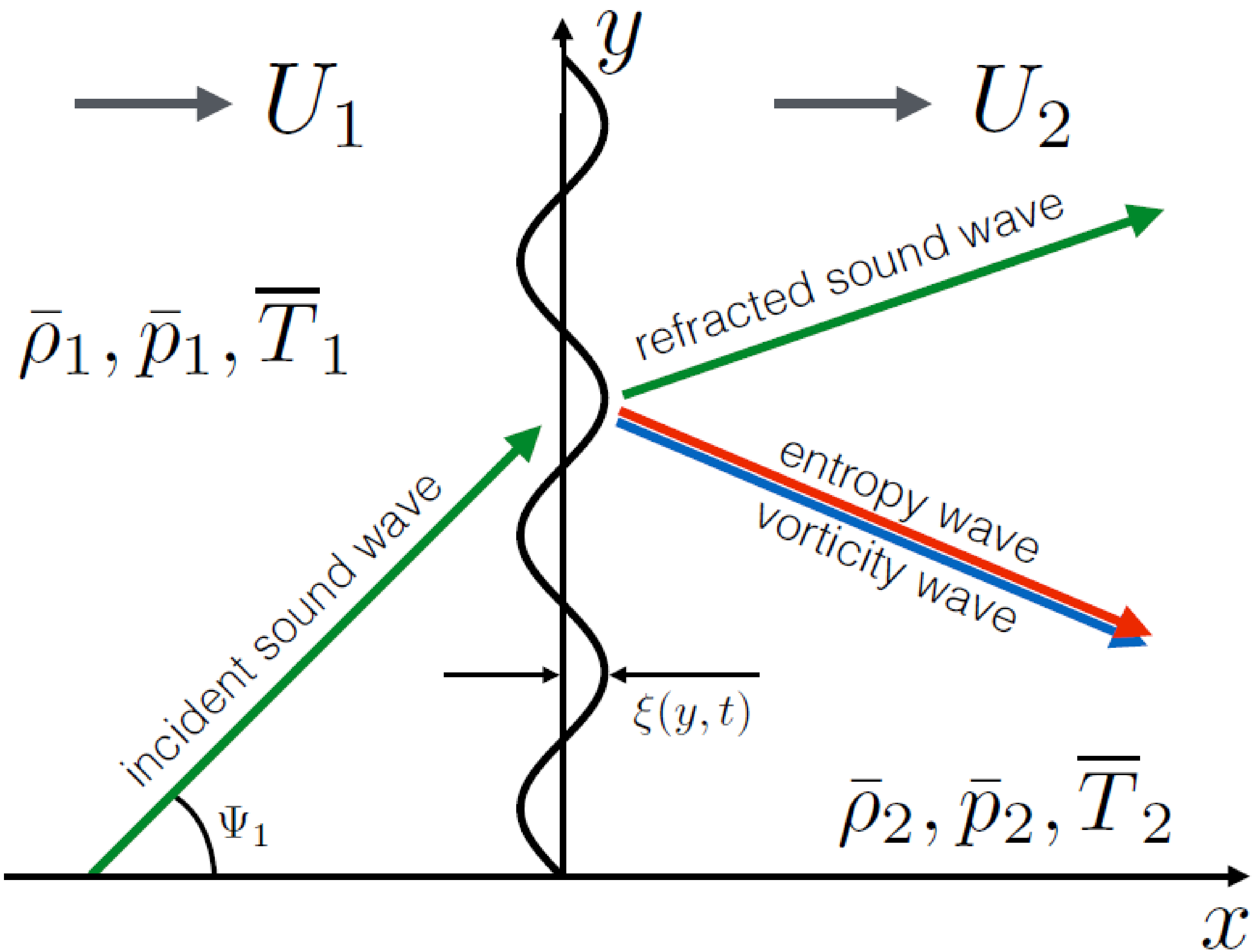
Outline

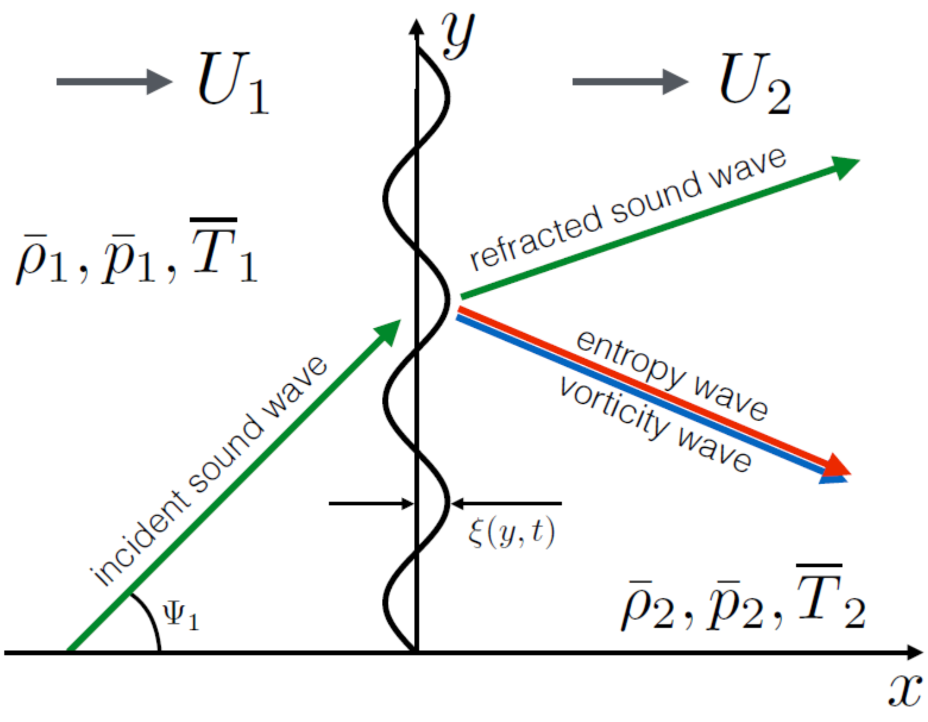
- Incident Acoustic Wave
- Formalism
- Geometrical Interpretation
- Results

Incident Acoustic Wave



Incident Acoustic Wave

$$\frac{p_1'}{\overline{p}_1} = A_p f[i\vec{k}_1 \cdot (\vec{r}_1 - \vec{a}_1 t)]$$



Jump Conditions

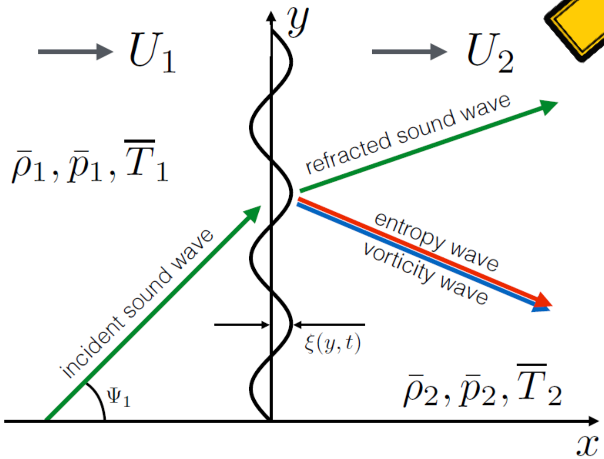
$$\rho_1 V_{1,\perp} = \rho_2 V_{2,\perp}$$

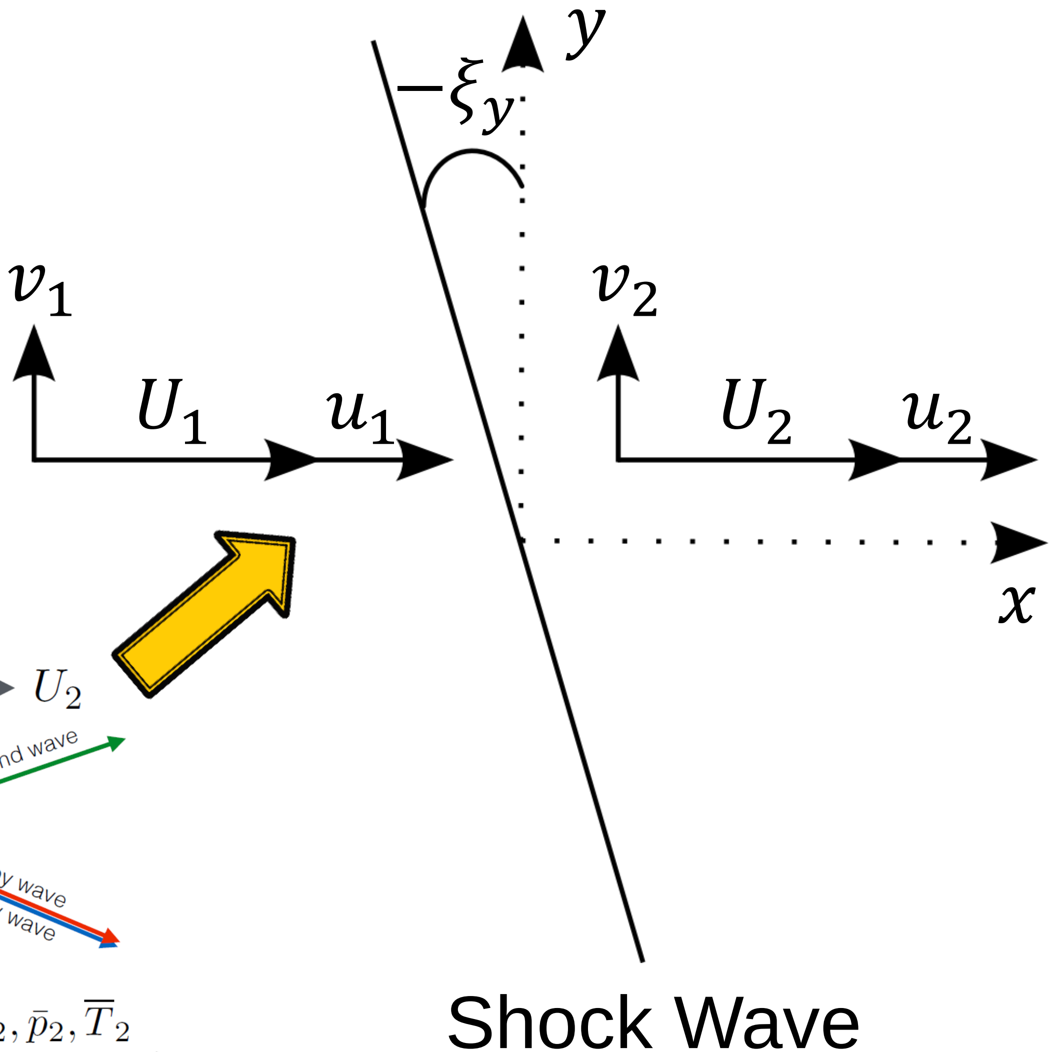
$$p_1 + \rho_1 V_{1,\perp}^2 = p_2 + \rho_2 V_{2,\perp}^2$$

$$\omega_1 + \frac{1}{2} V_{1,\perp}^2 = \omega_2 + \frac{1}{2} V_{2,\perp}^2$$

$$V_{1,\parallel} = V_{2,\parallel}$$

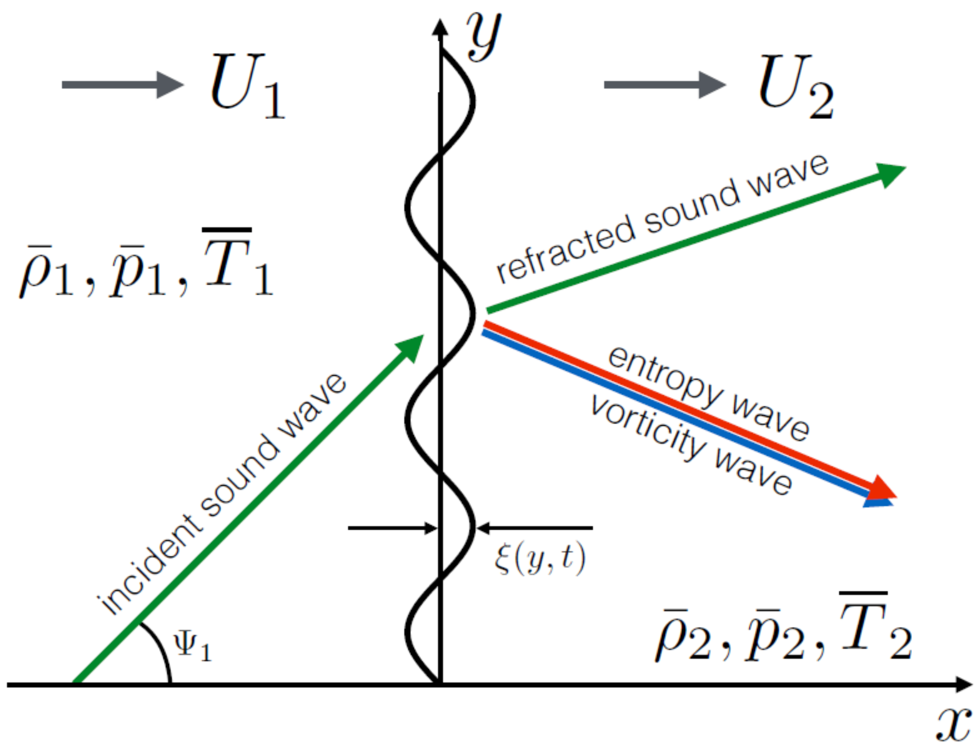
Jump Conditions



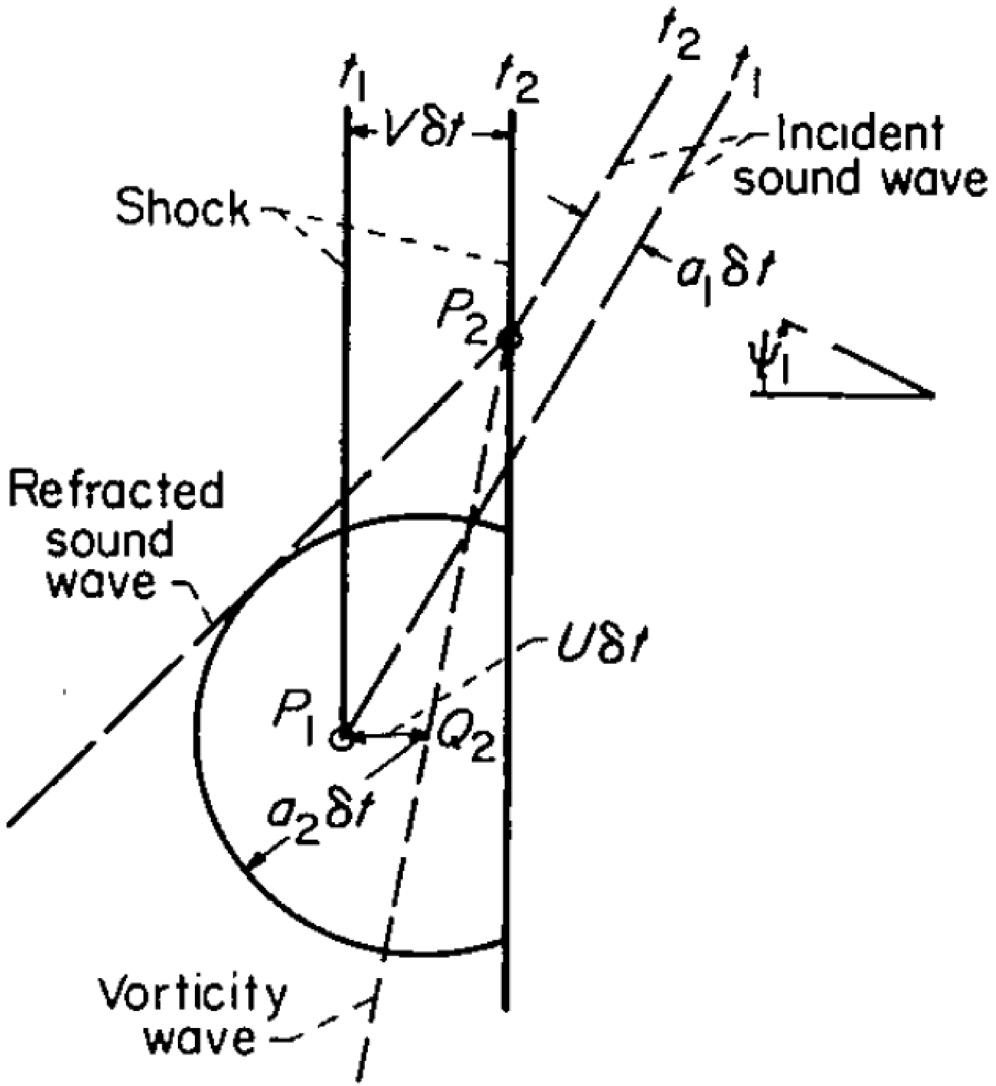


Post-Shock Region

- Acoustic mode: dependence on ψ_1

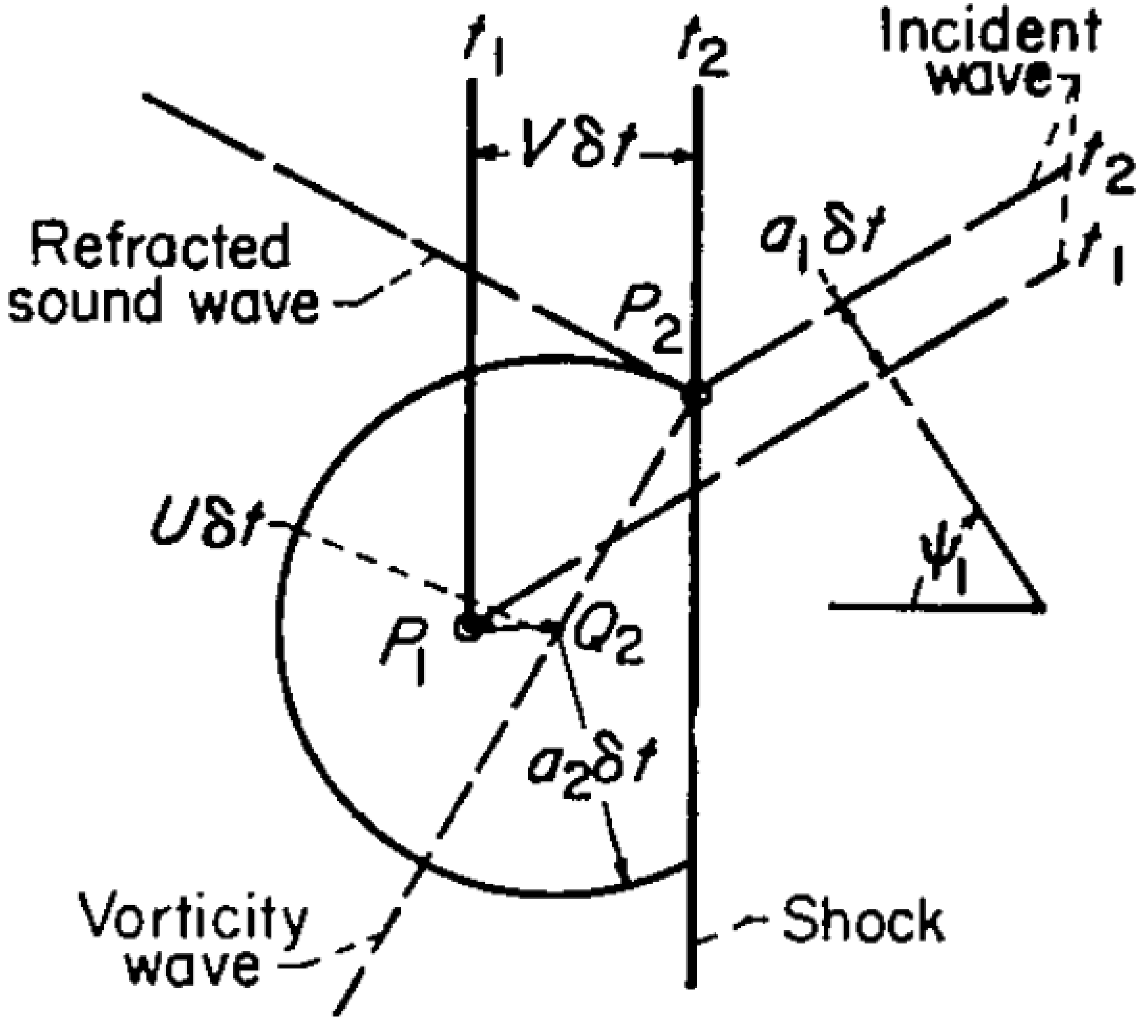


Geometrical Interpretation



[Moore, 1954]

Geometrical Interpretation



[Moore, 1954]

Geometrical Interpretation

$$\left(\frac{a_2}{U_1}\right)^2 - \left(\frac{U_2}{U_1}\right)^2 = \left(\cot(\psi_c) + \frac{a_1}{U_1}\csc(\psi_c)\right)^2$$

- Propagating regime:

$$0 \leq \psi_1 \leq \psi_{cl} \quad \text{or} \quad \psi_{cu} < \psi_1 \leq \pi$$

- Non-propagating regime:

$$\psi_{cl} < \psi_1 < \psi_{cu}$$

Solution for Propagating Regime

$$\frac{p_2'}{\overline{p}_2} = [A\,mode]$$

$$\frac{\rho_2'}{\overline{\rho}_2} = [A\,mode] + [E\,mode]$$

$$\frac{u_2}{U_1} = [A\,mode] + [V\,mode]$$

$$\frac{v_2}{U_1} = [A\,mode] + [V\,mode]$$

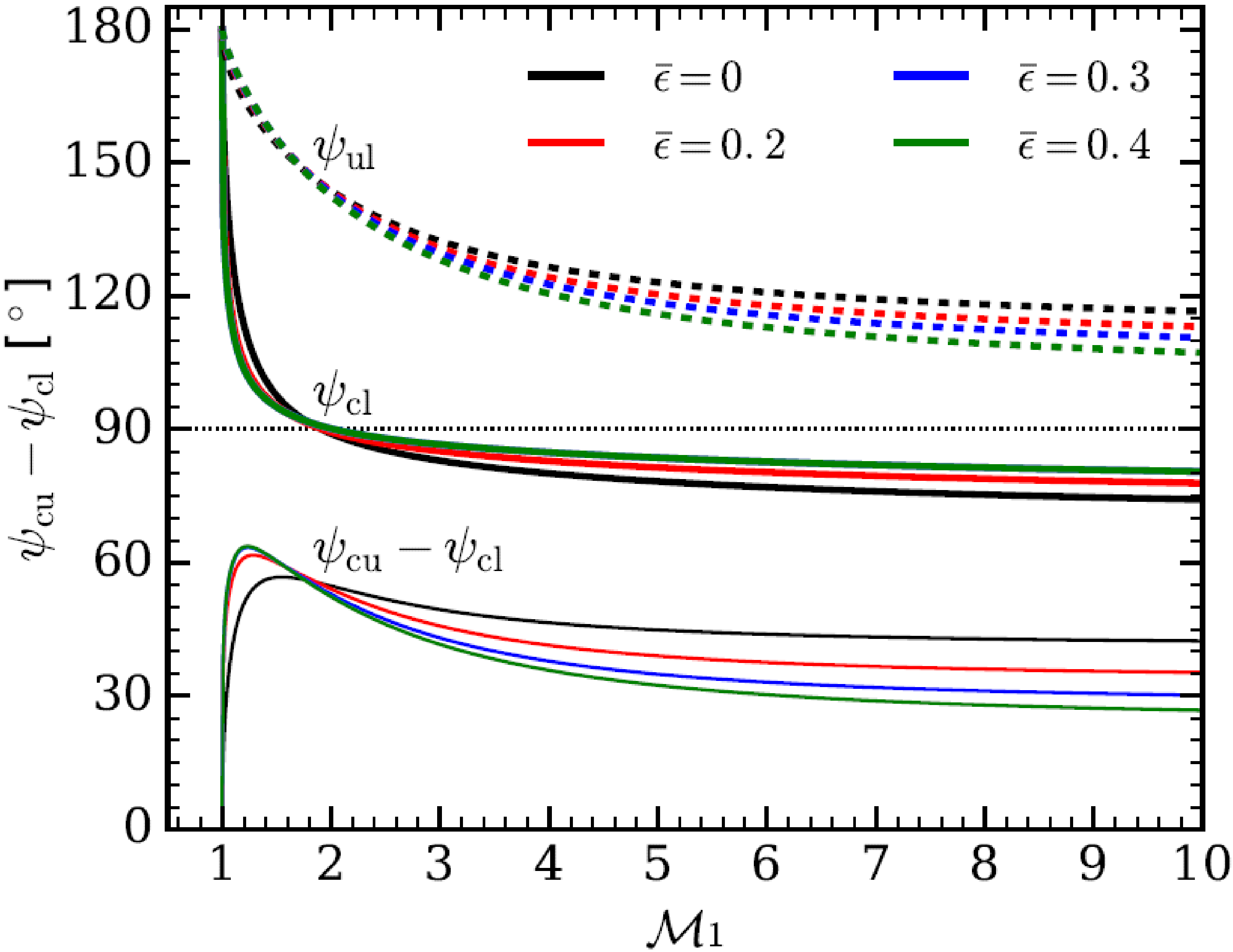
Addition of Nuclear Dissociation

$$C = \frac{\gamma + 1}{\gamma + \frac{1}{M_1^2} - \sqrt{\left(1 - \frac{1}{M_1^2}\right)^2 + (\gamma + 1)\frac{(\gamma - 1)M_1^2 + 2}{M_1^2}\bar{\epsilon}}}$$

- $\epsilon = \bar{\epsilon}\frac{1}{2}v_{ff}^2$
- $\bar{\epsilon} = 0$: no nuclear dissociation
- $\bar{\epsilon} \sim 0.4$: strong nuclear dissociation

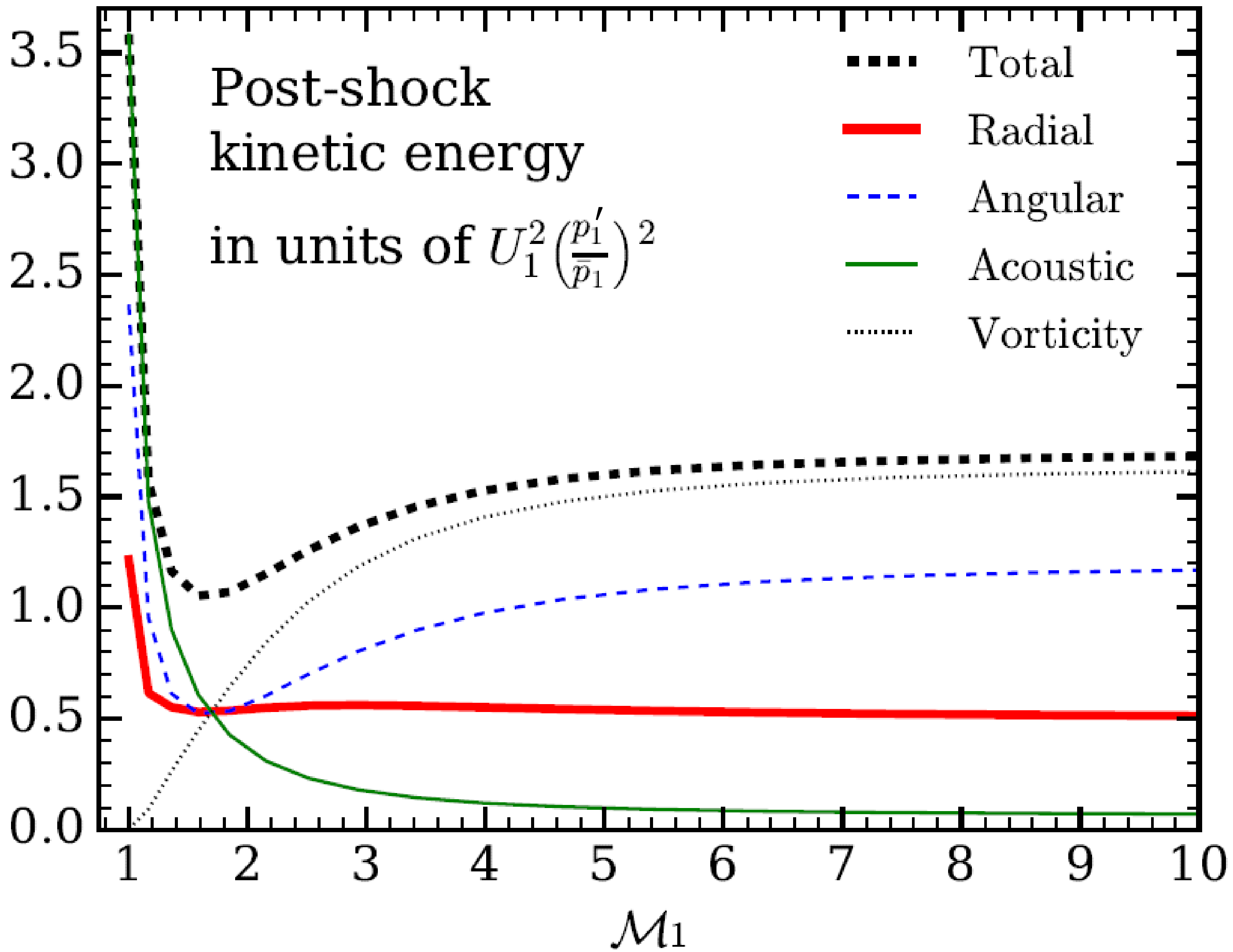
[Fernandez & Thompson, 2009]

Results



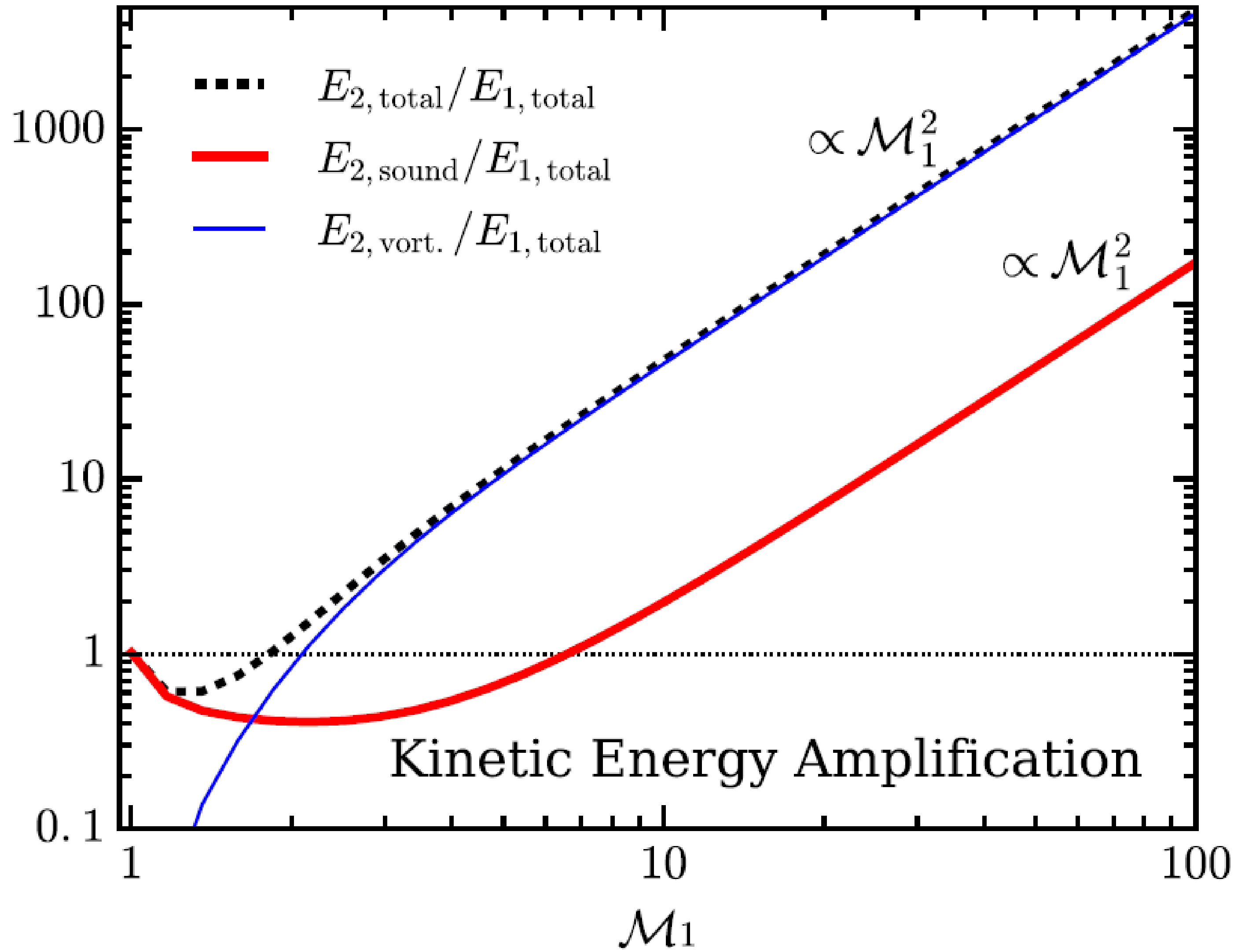
180
150
120
90
60
30
0
1
2
3
4
5
6
7
8
9
10
$\mathcal{M}_1$
$\psi_{cu} - \psi_{cl}$ [°]
$\bar{\epsilon} = 0$
$\bar{\epsilon} = 0.2$
$\bar{\epsilon} = 0.3$
$\bar{\epsilon} = 0.4$
ψ_{ul}
ψ_{cl}
$\psi_{cu} - \psi_{cl}$

Results



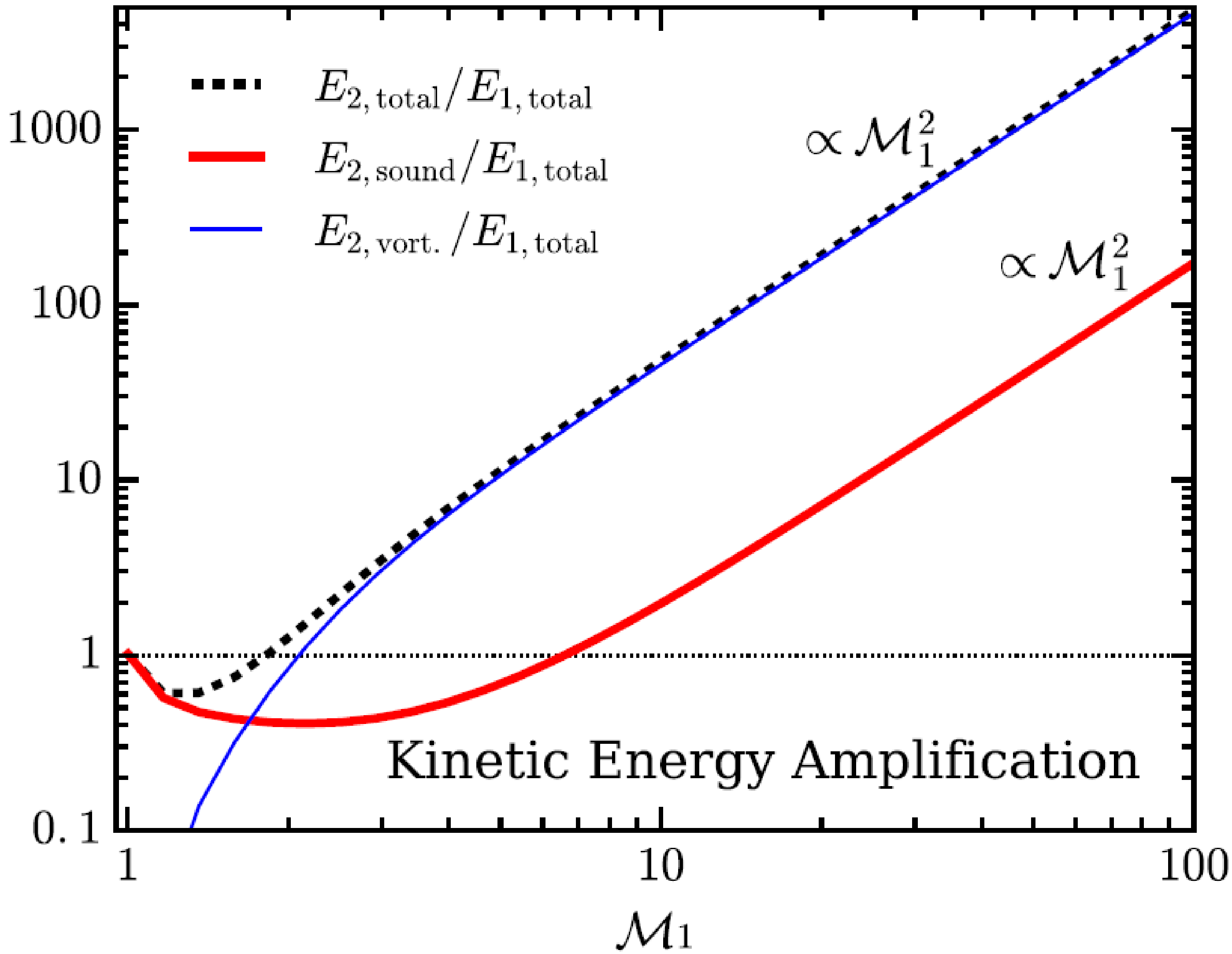
Post-shock
kinetic energy
in units of $U_1^2\left(\frac{p_1'}{\bar{p}_1}\right)^2$
Total
Radial
Angular
Acoustic
Vorticity
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
1
2
3
4
5
6
7
8
9
10
$\mathcal{M}_1$

Results



$E_{2,\,\mathrm{total}}/E_{1,\,\mathrm{total}}$
$E_{2,\,\mathrm{sound}}/E_{1,\,\mathrm{total}}$
$E_{2,\,\mathrm{vort.}}/E_{1,\,\mathrm{total}}$
$\propto \mathcal{M}_1^2$
$\propto \mathcal{M}_1^2$
1000
100
10
1
0.1
1
10
100
$\mathcal{M}_1$
Kinetic Energy Amplification

Results



$u_1 \propto M_1^{-1}$
$v_1 \propto M_1^{-1}$

Summary

- Incident acoustic wave creates vorticity and entropy waves in the post-shock region
- Acoustic wave in the post-shock region has propagating and non-propagating regimes
- $E_{2,total}$ is dominated by vorticity component
- $E_{2,total}/E_{1,total}$ is proportional to M_1^2
- Non-propagating regime shrinks as $\overline{\epsilon}$ is increased