Core-vortex pinning contribution to mutual friction force

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in collaboration with

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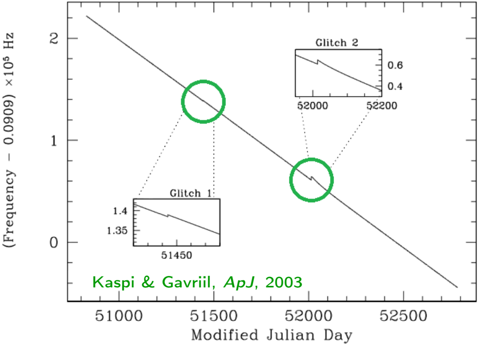
Pulsar glitches



- short **rise time:**

  $\boxed{\tau_r < 30 \text{ s}}$ ←-- Vela

- exponential **relaxation** on several days or months.

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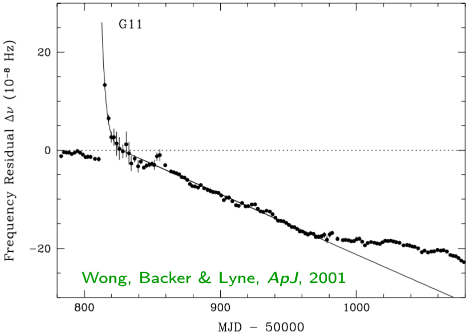

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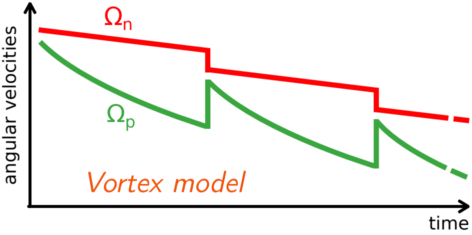

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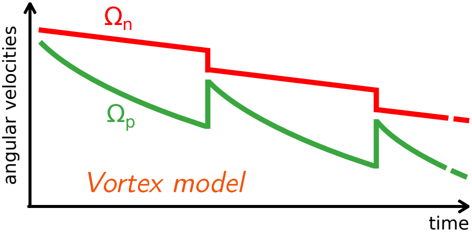

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--→ **Vortex pinning** shall take place somewhere in the star!

Vela glitch puzzle

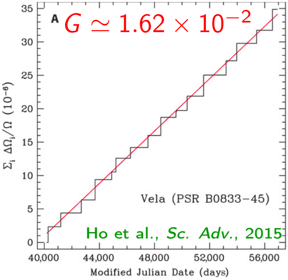
Andersson et al., *PRL*, 2012 & Chamel, *PRL*, 2013

Glitches have been thought to originate from the **crust**.

- the *core* superfluid is expected to be *strongly coupled* to the crust, Alpar+, 1984
- analysis of glitch data:

$$\frac{I_n}{I} \gtrsim G \sim 0.02$$

Rk: $I_n^{crust}/I \sim 0.02 - 0.05$.



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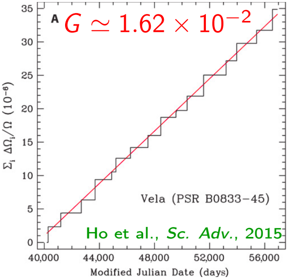

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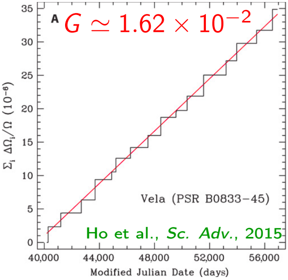

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⇢ possible role of the outer core...

The outer core of neutron stars

The **outer core** is expected to contain:

- a *neutron* superfluid,
- a type II *proton* superconductor,
- a degenerate gas of *electrons* (and possibly muons).

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Two kinds of quantised lines:

- superfluid **vortex lines**,
- proton **flux tubes** (or **fluxoids**).

Vortex-fluxoid interpinning

Different scenarios can lead to pinning:

- A moving vortex line is likely to *intersect* several flux tubes.

The vortex-fluxoid junction is **energetically favorable**

⇢ vortices may **pin** to fluxoids in the outer core.

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Objectives:

- Study the impact of *perfect* vortex-fluxoid pinning on the **mutual friction force**,
- Explore possible implications for **glitches**.

Hydrodynamical approach

Intervortex spacing length:

$$d_{\rm n} \simeq 1.10^{-3} \times \left(\frac{P}{10 \text{ ms}} \right)^{1/2} \text{ cm.}$$

⇢ on scales L verifying $d_{\rm n} \ll L \ll R$, a smooth-averaged *hydrodynamical* approach is relevant.

Two-fluid model

The **outer core** of NSs can be described as a mixture of two fluids:

- a neutron superfluid moving at $v_{\rm n}^i$,
- a charge-neutral fluid made of protons and electrons moving at a common velocity, $v_{\rm p}^i$.

Origin of the mutual friction force

Euler equations:

$$\left(\partial_t + v_{\rm n}^j \nabla_j\right)\left(v_{\rm n}^i + \varepsilon_{\rm n} w_{\rm pn}^i\right) + \nabla^i\left(\tilde{\mu}_{\rm n} + \Phi\right) + \varepsilon_{\rm n} w_j^{\rm pn} \nabla^i v_{\rm n}^j = f_{\rm v\to n}^i / \rho_{\rm n},$$

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where $w_{\rm pn}^i = v_{\rm p}^i - v_{\rm n}^i$ and ε_X stands for **entrainment** effects.

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⇢ **microscopic interactions** between the vortices and the fluids

Model assumptions

We assume that N_p flux tubes are pinned to each vortex line:

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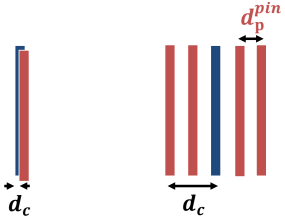

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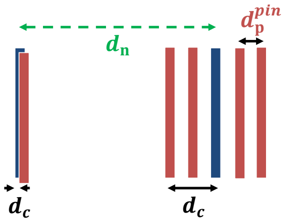

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$d_c \ll d_\mathrm{n}$ ⇢ neglect any *possible interactions* between bundles.

Forces acting on a single bundle

A bundle undergoes the following *averaged forces per unit length*:

- **non-dissipative Magnus forces**:

$$\mathcal{F}^i_{\rm J} = \mathcal{F}^i_{\rm M\,n} + \mathcal{F}^i_{\rm M\,p},$$

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- a **dissipative drag force** associated with the electrons:

$$\mathcal{F}^i_{\mathrm{d}} = -\mathcal{R}\left(v^i_{\mathrm{L}} - v^i_{\mathrm{p}}\right),$$

where $\mathcal{R} \geq 0$ is called the *resistivity coefficient* and v^i_{L} is the mean velocity of the bundles in the fluid element.

Generalized Joukowski lift formula

Carter, Langlois & Prix, *Springer*, 2002

- **Neutron** Magnus force:

$$\mathcal{F}^i_{\mathrm{M\,n}} = -\varepsilon^{ijk}\rho_{\mathrm{n}}\kappa\hat{\kappa}_j\left(v_{\mathrm{n}\,k} - v_{\mathrm{L}k}\right),$$

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where $\mathcal{C}_{\mathrm{p}}$ is the *momentum circulation* of the proton fluid around the bundle: $\mathcal{C}_{\mathrm{p}} = \oint_{\mathrm{b}} \pi^{\mathrm{p}}_i \,\mathrm{d}x^i$.

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- If pinning takes place $\dashrightarrow$ $\mathcal{C}_{\mathrm{p}} = N_{\mathrm{p}} \times m \times \kappa$.

Resistivity coefficient

Main source of drag:

⇢ electrons scattering off the magnetic fields carried by the N_p fluxoids present in each bundle.

Drag-to-lift ratio for a **single** flux tube:

$$\xi_p = \frac{\mathcal{R}}{\rho_n \kappa} \simeq 4 \times 10^{-4} \quad (1 - \varepsilon_p)^{-1/2} \left(\frac{x_p}{0.05}\right)^{7/6} \left(\frac{\rho}{10^{14}\ \mathrm{g.cm^{-3}}}\right)^{1/6}$$

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- **Contact pinning** ⇢ $\xi_c \simeq \xi_p \times {N_p}^2$,
- **Dynamical pinning** ⇢ $\xi_c \simeq \xi_p \times N_p$.

Mutual friction force

The **stationary** motion of a bundle is governed by

$$\boxed{\mathcal{F}^i_{\mathrm{d}} + \mathcal{F}^i_{\mathrm{M\,n}} + \mathcal{F}^i_{\mathrm{M\,p}} = 0} \dashrightarrow \quad v^i_{\mathrm{L}}.$$

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where

$$\beta = \frac{\xi_c}{(1+X)^2 + \xi_c{}^2} \quad \& \quad \beta' = \frac{X(1+X) + \xi_c{}^2}{(1+X)^2 + \xi_c{}^2} \quad \text{with} \quad \boxed{X \simeq x_{\mathrm{p}} N_{\mathrm{p}}}.$$

Coupling time scale

The *dynamical evolution* of the relative velocity w^i_{pn} is governed by

$$\frac{\partial w^i_{\mathrm{pn}}}{\partial t} + \ldots = -\frac{\mathcal{N}_{\mathrm{n}}\kappa}{x_{\mathrm{p}}\left(1-\varepsilon_{\mathrm{n}}-\varepsilon_{\mathrm{p}}\right)} \times \beta \times w^i_{\mathrm{pn}} = -\frac{w^i_{\mathrm{pn}}}{\tau_c}.$$

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Coupling time scale between the fluids through MF ($\mathcal{N}_{\rm n}\kappa \simeq 2\Omega$):

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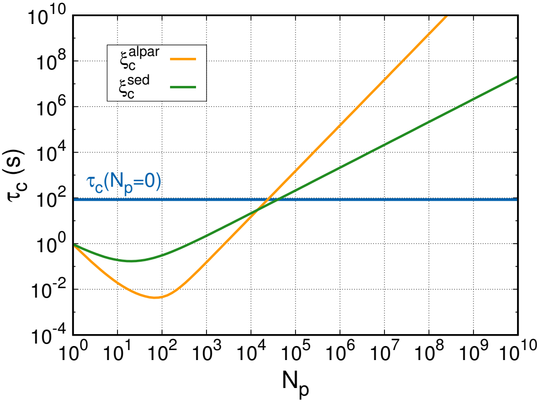
Without pinning:

$$\tau_c^0 \simeq 10\, P(\mathrm{s})\ (1-\varepsilon_{\rm p})^{3/2}\ \varepsilon_{\rm p}^{-2}\ \left(\frac{x_{\rm p}}{0.05}\right)^{-1/6} \left(\frac{\rho}{10^{14}\ \mathrm{g.cm^{-3}}}\right)^{-1/6}\ \mathrm{s}.$$

Coupling time scale with core vortex pinning

$\rho = 10^{14}$ g.cm^{-3}, $x_p = 0.05$, $\varepsilon_p = 0.1$ & $P = 89$ ms.



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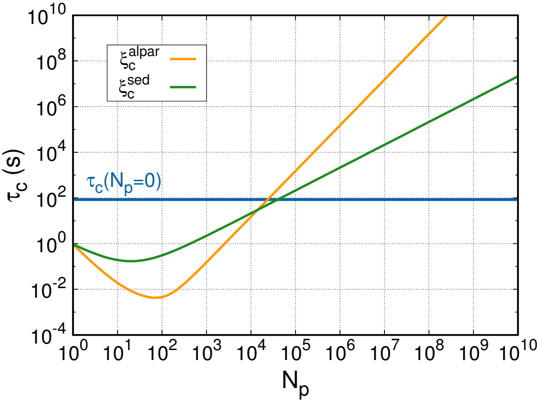

⇢ The **core** may take part to the **glitch** process!

Conclusion & perspectives

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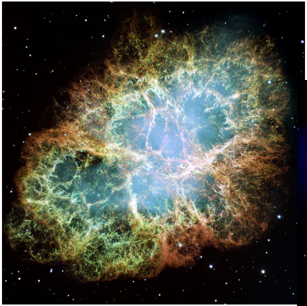
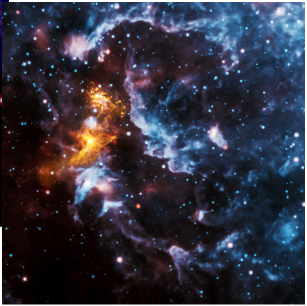
Perspectives:

- explore other astrophysical implications: precession, r-modes, ...





Thank you!

Appendices

Vortex lines & flux tubes

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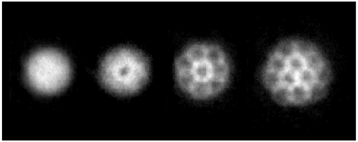

Madison et al., *PRL*, 2000

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$$\kappa = h/(2m_{\mathrm{n}}) \simeq 2.10^{-3}\ \mathrm{cm}^2.\mathrm{s}^{-1}$$

Mean surface density of vortex lines:

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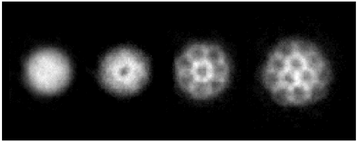

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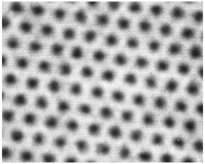
Mean surface density of vortex lines:

$$\mathcal{N}_{\rm n} = \frac{4m_{\rm n}\Omega_{\rm n}}{h} \simeq 6.10^5 \left(\frac{P}{10\ {\rm ms}}\right)^{-1}\ {\rm cm}^{-2}$$

Likewise, a type II superconductor is threaded by *flux tubes* (or *fluxoids*).



Hess et al., *PRL*, 1989

Quantum magnetic flux:

$$\phi_0 = hc/(2e) \simeq 2.10^{-7}\ {\rm G.cm}^2$$

Mean surface density of fluxoids:

$$\mathcal{N}_{\rm p} = \frac{B}{\phi_0} \simeq 5.10^{18} \left(\frac{B}{10^{12}\ {\rm G}}\right)\ {\rm cm}^{-2}$$

Velocity of the vortex array

Velocity of the vortex array:

$$v^i_{\mathrm{L}} = \dot{r}\delta^i_r + r\Omega_{\mathrm{v}}\delta^i_\varphi$$

- radial velocity:

$$\dot{r} = -\frac{\xi}{(1+X)^2+\xi^2} r\left(\Omega_{\mathrm{p}} - \Omega_{\mathrm{n}}\right),$$

- angular velocity:

$$\Omega_{\mathrm{v}} = \frac{\Omega_{\mathrm{n}}+(X+\tilde{\xi}^2+X\tilde{\xi}^2)\Omega_{\mathrm{p}}}{1+X+\tilde{\xi}^2+X\tilde{\xi}^2}.$$

If $X = 0$:

$$\dot{r} = -\frac{\xi}{1+\xi^2} r\left(\Omega_{\mathrm{p}} - \Omega_{\mathrm{n}}\right) \quad \& \quad \Omega_{\mathrm{v}} = \frac{\Omega_{\mathrm{n}}+\xi^2\Omega_{\mathrm{p}}}{1+\xi^2}$$

Magnus force

Size of a vortex core:

$$\Lambda_* \sim 130 \text{ fm}.$$

On *microscopic* scales $\Lambda_* \ll L' \ll d_{\rm n}$, a single vortex line undergoes a **Magnus force** from the neutron superfluid.

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On *macroscopic* scales $d_\mathrm{n} \ll L \ll R$, the action of the neutron superfluid on each vortex line is represented by an *averaged* Magnus force *per unit length*:

$$\boxed{\mathcal{F}^i_{\mathrm{M\,n}} = -\varepsilon^{ijk}\rho_\mathrm{n}\kappa_j\left(v_{\mathrm{n}\,k} - v_{\mathrm{L}k}\right)},$$

where $\kappa^i = \kappa\hat{\kappa}^i$ and v^i_L is the mean velocity of the vortex lines in the fluid element of size L.

Mutual friction & model assumptions

Mutual friction force:

$$\boxed{f^i_{\mathsf{mf}} = -f^i_{\mathsf{n}\to\mathsf{v}} = -n_{\mathsf{v}}\mathcal{F}^i_{\mathrm{M}\,\mathsf{n}}}.$$

Mutual friction & model assumptions

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- n_{v}: *local* surface density of vortex lines. Assuming straight vortices parallel to the rotation axis, we get

$$n_{\mathrm{v}} \simeq \mathcal{N}_{\mathrm{n}}.$$

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We shall consider *stationary*, *axisymmetric* and *circular* configurations:

$$v^i_{\mathrm{n}} = r\Omega_{\mathrm{n}}u^i_{\varphi} \quad \text{and} \quad v^i_{\mathrm{p}} = r\Omega_{\mathrm{p}}u^i_{\varphi}, \quad \text{with} \quad \Omega_{\mathrm{p}} = \Omega.$$

Force balance

The **stationary** motion of a bundle is governed by

$$\boxed{\mathcal{F}^i_{\rm d} + \mathcal{F}^i_{\rm M\,n} + \mathcal{F}^i_{\rm M\,p} = 0}.$$

Solving this equation, the bundle velocity is given by

$$v^i_{\rm L} = v^i_{\rm p} + \frac{\rho_{\rm n}\kappa}{\mathcal{R}}\left(\mathcal{B}\varepsilon^{ijk}\hat{\kappa}_j\varepsilon_{klm}\hat{\kappa}^l w^m_{\rm pn} + \mathcal{B}'\varepsilon^{ijk}\hat{\kappa}_j w^{\rm pn}_k\right)$$

where we have introduced the following **mutual friction coefficients**

$$\mathcal{B} = \frac{\xi_c(1+X)}{(1+X)^2+\xi_c^{\,2}}, \quad \mathcal{B}' = \frac{\xi_c^{\,2}}{(1+X)^2+\xi_c^{\,2}},$$

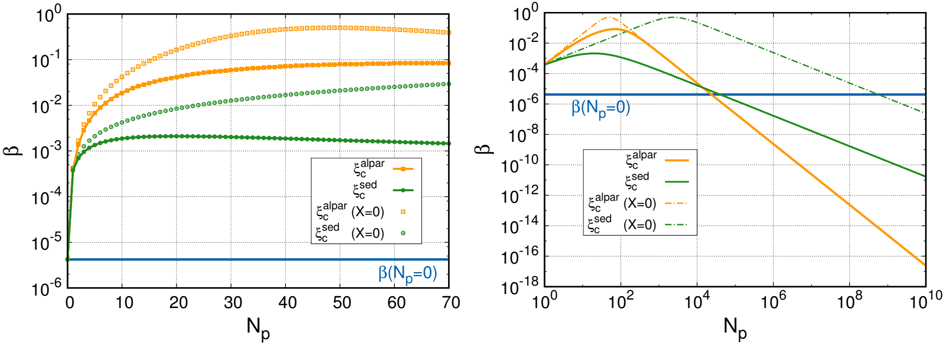
with

$$X = \frac{n_{\rm p}\mathcal{C}_{\rm p}}{\rho_{\rm n}\kappa} \simeq x_{\rm p} N_{\rm p}.$$

Rk: we confirm results obtained by Glampedakis & Andersson (2011).

β coefficient VS N_p

$$\rho = 10^{14}\ \mathrm{g.cm^{-3}},\ x_p = 0.05\ \&\ \varepsilon_p = 0.1$$



$$\beta(N_p = 0) = \frac{\xi_n}{1 + \xi_n^2}, \quad \text{where} \quad \xi_n = \xi_p \times \varepsilon_p^2$$

β' coefficient VS N_p

$$\rho = 10^{14}\ \mathrm{g.cm^{-3}},\ x_p = 0.05\ \&\ \varepsilon_p = 0.1$$