Analysis of the frequency response of a dynamically modulated, high Q, optical fiber ring resonator

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ABSTRACT:

In this work we analyze the optical transmittance of a high-Q fiber ring resonator in both the stationary case, and when the phase of the cavity is rapidly modulated. The frequency of the modulation signal plays a role in the transmittance of the device, as it is compared with the cavity lifetime.

First, we present the experimental characterization of a stationary fiber ring resonator setup with a 99:1 singlemode coupler and 1 m cavity length. The bandwidth of the resonances is 8.5 fm (Q = 1.83×10^{8}), and the finesse is 110. Second, we analyze the response of the output signal when the cavity phase is dynamically modulated. The frequency behavior of the transmittance is alike that of a low-bandpass filter: the higher the Q-value of the cavity, the lower the cutoff frequency of the filter. This feature has an effect on the selection of the optimal resonator for sensing applications.

Key words: optical fiber, ring resonator, mechanical modulator, dynamic response, optomechanics.

1.- Introduction

Photonic resonators are a technology of high interest, particularly in integrated optics. High Q-value resonators have been demonstrated in different shapes, specially microspheres or toroids for Whispering Gallery Modes (WGM). The tiny bandwidth of these resonances have opened the path to their use in a variety of applications, such as sensing and biosensing with extremely low detection limits, quantum light sources, optomechanics and materials characterization.

Optical fibers can be exploited to perform photonic resonators, being the most usual ones those used for laser cavities. Different configurations are possible, such as ring cavities, Fabry-Pérot cavities when combined with FBGs, or DFB gratings. Moreover, the cylindrical shape of a section of fiber can also support WGM, and Q-factors in the order of 10^7 have been demonstrated.

In this work we present an optical fiber ring analogous to those exploited in integrated photonics, but in a macroscopic version. We employed common fiber optics components (basically, a 99:1 single mode fiber coupler and a polarization controller) to obtain a ring resonator with a Q-factor of 10^8. The use of low-cost components together with the high Q-value, make them attractive for sensing and optomechanical applications.

In our case, we are interested in the detection of periodic modulations of a parameter of the ring resonator (for example, a mechanical vibration of the fiber). Because of the high Q-value of our ring resonator, we cannot use the conventional stationary solutions for the electromagnetic fields, since the period of modulation can be shorter than the cavity lifetime. We present here the analysis of the dynamic behavior of a high Q ring resonator when a periodic modulation is applied to it, and how its optical response is modified, compared to the stationary case.

2.- Stationary high Q ring resonator.

We setup a fiber ring resonator as the one depicted in Fig. 1. The 1% output arm of a 99:1 SM fiber coupler was spliced to the free input arm. The fiber was standard telecom fiber SMF28 and the total length of fiber within the ring cavity was 1.71 m. A polarization controller was placed outside the cavity. To measure the resonances of such fiber ring, a fine linewidth tunable laser (< 30 kHz) was used, and the output was registered by means of a photodiode and an oscilloscope (350 MHz bandwidth).

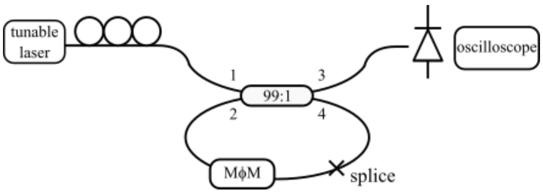

Fig. 1: Scheme of the setup.

Fig. 1 includes a mechanical phase modulator (*MϕM*) whose role will be described in latter sections. In this section we will analyze the response of the fiber ring without considering this element, that is, its stationary optical response.

The output optical fields of the coupler, E_3 and E_4 can be written in terms of the input fields, E_1 and E_2, and the parameters of the coupler: γ accounts for its loss, and α and β as the coupling and cross-coupling coefficients, which satisfy the condition $\alpha^2 + \beta^2 = 1$ to ensure conservation of energy, as follows [1, 2]:

$$\begin{pmatrix} E_3 \\ E_4 \end{pmatrix} = \gamma \begin{pmatrix} \alpha & j\beta \\ j\beta & \alpha \end{pmatrix} \begin{pmatrix} E_1 \\ E_2 \end{pmatrix} \qquad (1)$$

As the optical signal exits the coupler (E_4), it travels round and goes back in the loop as E_2; both fields are related through the expression

$$E_2 = f e^{-j\phi} E_4 \qquad (2)$$

where f is the loss of the elements within the cavity and ϕ the phase delay. From these equations, one can calculate the transmittance of the ring.

The optical response of the system is, thus, a series of equidistant attenuation notches that will be characterized by means of a few parameters: the visibility, v, the bandwidth, $\Delta\phi_{3dB}$, and the finesse, $F = (2\pi)/\Delta\phi_{3dB}$. They can be calculated easily from Eq. (3), which gives the transmittance T of the resonator.

$$T = \frac{P_3}{P_1} = |\gamma|^2 \left(1 - \frac{(1-\alpha^2)(1-(ft)^2)}{1+\alpha^2(ft)^2 - 2\alpha(ft)\cos(\phi)}\right) \qquad (3)$$

Moreover, the Q factor of the cavity will be calculated as the ratio between the resonant frequency or wavelength, and the correspondent bandwidth of the notch expressed in the proper unities.

The critical coupling condition for the resonances, that is, the parameters that will lead to null transmittance at the resonant wavelength, will be achieved when $\alpha = f\gamma$. Thus, for a coupler as ours, with a ratio 99:1, to reduce as much as possible the cavity loss is important to optimize the visibility of the resonances, and this is the reason why we placed the polarization controller outside the cavity, as well as to reduce the bandwidth of the notches.

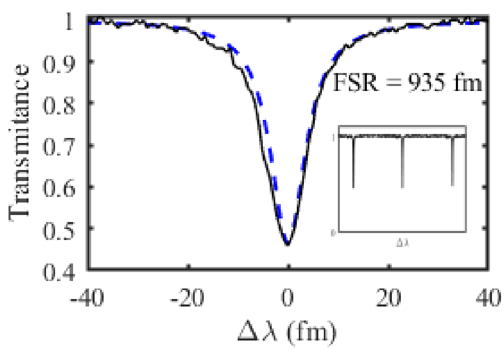

Fig. 2: Resonance centered at 1.55 μm, experimental (blue, continuous line) and calculated from the model (black, dashed line). Inset: consecutive resonances showing the FSR.

The experimental measurement of the resonances, see Fig. 2, was performed. The bandwidth in terms of wavelength, the Free Spectral Range (FSR), the finesse and the visibility of the resonances are compiled in Table I.

Table I. Experimental characterization of the resonances

$\Delta\lambda_{3dB}$ (fm)	FSR (fm)	F	v (%)
8.5±0.3	935±18	110±2	36.5±1.3

The central wavelength of the resonance, λ_R, is 1.55 μm. From these data, we can calculate the Q-factor as

$$Q = \frac{\lambda_R}{\Delta\lambda_{3dB}} = (1.83 \pm 0.07) \times 10^8$$

which is clearly in the range of high Q-factors for fiber resonators as ours. From this value, the cavity lifetime, τ_{lf}, can be estimated as,

$$\tau_{lf} = \frac{Q}{\omega} = 215\ ns$$

where ω is the frequency of the optical signal.

Table II. Parameters of the ring resonator.

α	β	(fγ)
0.9955 $\pm$0.0004	(9$\pm$0.8) $\times$ 10^{-3}	0.9764 $\pm$ 0.0007

Using Eq. (3) we can calculate the parameters of the resonator from our experimental measurements, see Table II. The dashed line in Fig. 2 depicts the transmittance according to this model and shows the high degree of concordance between experiment and theory.

3.- Dynamic modulation of a high Q ring resonator.

The cavity lifetime for high-Q factors can be relatively long: in our case, τ_{lf} is of hundreds of ns If we introduce a modulation in one of the parameters of the cavity (for example, its length or its refractive index) with a period much longer than this value, we may consider that the stationary optical response described in the previous case is valid. However, it could occur that the period of the modulation is much shorter than τ_{lf}. In this case, the previous solution must be modified to reproduce the effects on the optical response of the resonator induced by such modulation. From an experimental point of view, for our fiber ring resonator, we can introduce high frequency modulations by means of standing acoustic waves. In the experiments we are currently carrying out, the element that we called MϕM in Fig. 1 is a ~2 cm long section of bare fiber where we excite standing acoustic waves using a piezoelectric transducer. In such way, two resonators, an optical (the ring resonator) and a mechanical one, will be nested one within the other. In this section, we will analyze theoretically the effect of the modulation frequency on the optical response of this device.

In this case, the phase delay introduced in the optical field as it travels in the cavity, that is, from port 4 to 2, can be written as:

$$\Delta\phi(t) = \Delta\phi_{DC} + \Delta\phi_{AC}\cos(\Omega t) \qquad (4)$$

where $\Delta\phi_{DC}$ corresponds to the detuning between the frequency of the optical signal, ω, and the resonant frequency, ω_R. Ω is the frequency of the mechanical modulation introduced in the cavity and $\Delta\phi_{AC}$ corresponds to the amplitude of the variation in the phase due to the mechanical modulation.

Taking into account this, considering that we are in the case of a high Q resonator (that is, γ=1), and that E_1 remains constant over time, Eq. (1) and (2) can be rewritten as

$$\begin{pmatrix} E_3(t) \\ E_4(t) \end{pmatrix} = \begin{pmatrix} \alpha & j\beta \\ j\beta & \alpha \end{pmatrix} \begin{pmatrix} E_1 \\ E_2(t) \end{pmatrix} \qquad (5)$$

$$E_2(\mathrm{t}) = f e^{-j\phi(t)} E_4(t-\tau) \qquad (6)$$

where τ is the roundtrip time of the cavity, which is considered to be short (~ 8 ns in our case). By algebraic manipulation

$$\left.\begin{aligned} &E_4(t-\tau) = j\beta E_1 + \alpha E_2(t-\tau) \approx \\ &\quad \approx j\beta E_1 + +\alpha\left[E_2(t) - \tau\frac{dE_2(\tau)}{d\tau}\bigg|_{\tau=t}\right] \\ &E_3(t) = \alpha E_1 + j\beta E_2(t) \end{aligned}\right\} \quad (7)$$

Combining (6) and (7) and, after solving the differential equations [3, 4], the expression for the transmittance of the device results to be:

$$T = \frac{P_3}{P_1} \approx \frac{1}{2}\left|1 + \frac{\tau\Delta\phi_{AC}}{\sqrt{\Omega^2 - 2j\left(\frac{\omega}{2Q}\right)^2}}\cos(\Omega t + \varphi)\right|^2 \quad (8)$$

where φ is the phase detuning between the mechanical and the optical amplitude modulations. We have considered small amplitude modulations (small $\Delta\phi_{AC}$), $v \approx 1$ (thus $\alpha \approx 1$ and $\beta << 1$) and an optical frequency that fulfills T=0.5 when there is not mechanical modulation. This expression can be written as

$$T = \frac{1}{2}(1 + \Delta T_{AC}\cos(\Omega t + \varphi) \quad (9)$$

where ΔT_{AC} stands for the transmission modulation amplitude:

$$\Delta T_{AC} = Re\left(\frac{\tau\Delta\phi_{AC}}{\sqrt{\Omega^2 - j\left(\frac{\omega}{2Q}\right)^2}}\right) \quad (10)$$

It is worth to note the presence of a pole in the denominator of ΔT_{AC}, see Eq. 8 and 10. The zero value of the pole divides the behavior of the transmittance in two frequency regimes. We will define the cutoff frequency, Ω_c, as the value of the frequency at which the denominator is null. For an optical signal at 1.55 μm and our Q-value,

$$\Omega_C \equiv \frac{\omega}{Q\sqrt{2}} = \frac{1}{\tau_{lf}\sqrt{2}} = 3.2 \times 10^6\ s^{-1}$$

which is a frequency modulation easily attainable exciting standing acoustic waves in a 2-cm long mechanical resonator. Thus, we will analyze the asymptotic behavior of the transmittance at low and high frequencies, and Eq. (8) is transformed into:

$$T_s \approx \frac{1}{2}\left(1 + \tau\Delta\phi_{AC}\frac{Q}{\omega}\cos(\Omega t + \varphi)\ \text{if}\right)\ \Omega \ll \Omega_C$$
$$T_r \approx \frac{1}{2}\left(1 + \tau\Delta\phi_{AC}\frac{1}{\Omega}\cos(\Omega t + \varphi)\ \text{if}\right)\ \Omega \gg \Omega_C$$

where s and r denote for *slow* and *rapid*, respectively. These expressions resemble the behavior of a low-pass frequency filter.

Fig. 3 shows the amplitude in Eq.8 as a function of Ω, which represents the frequency response of the optical resonator. We can identify the typical trend of the Bode diagram of a low-pass frequency filter. Three different curves for three Q-values are shown, including our own value.

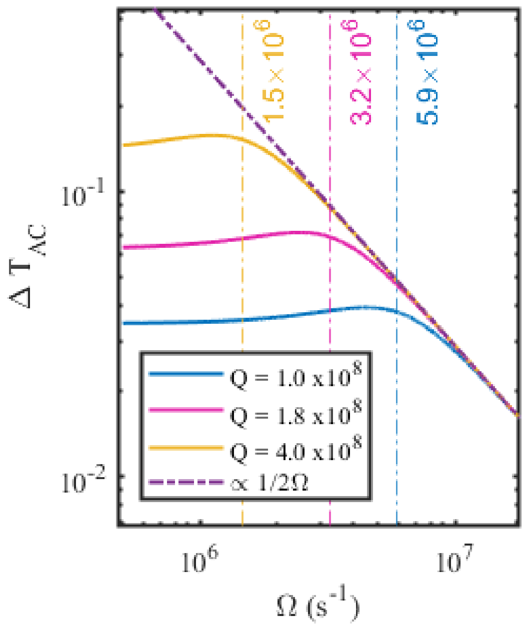

Fig. 3: Frequency response of the dynamically modulated optical ring resonator.

This feature has a direct effect on the selection of the optimal resonator for sensing applications. High Q values are usually associated with high sensitivities and, therefore, low detection limits. This is true in stationary and slowly modulated resonators but, when considering fast modulations of the cavity, the transmittance of a high Q resonator will not reproduce them, since the oscillatory term in Eq. (8) is damped as Ω increases. Consequently, the optimal election of an optical resonator as a sensing transducer must take into account not only this parameter but also the range of working frequencies: even when the resonance of a high Q cavity will present a narrower bandwidth (thus, a better sensitivity) than that of a low Q one, the damping effect of high modulation frequencies might result in a poorer performance. The measurements required to demonstrate this effect experimentally are in progress.

4.- Conclusion

In this work, we have analyzed the performance of a high Q factor ring resonator when the optical cavity is phase modulated. Such modulation might be introduced via a nested acoustic resonator in the optical cavity. We setup a simple, fiber ring resonator using a SM 99:1 coupler and a 1.71 m long section of fiber, and we experimentally characterized it, obtaining resonances of 8.5±0.3 fm bandwidth and a Q factor of $(1.83\pm0.07)\times10^8$. The comparison with the theoretical model is presented and it is used to extract the ring cavity parameters, α, β and $(f\gamma)$.

The analysis of the dynamic phase modulation of the resonator arises an interesting feature. When designing the optimum setup for using an optical resonator for sensing, intuitively one might think than the higher the Q value, the better the performance of the sensor, since the bandwidth of the resonances decrease with the Q value. However, it is important to take into account that for rapid modulations, the cavity lifetime might be comparable to the modulation period. In this case, the ring resonator behaves as a low-pass frequency filter and it damps high frequency modulations. Thus, the optical transmittance of the device will not reproduce the variations introduced in the cavity phase, which will result in an overall poorer performance. Experimental demonstration of such effect is work in progress.

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