MATHEMATICAL BIOGRAPHY OF FUENSANTA ANDREU VAILLO

Fuensanta Andreu Vaillo was born on August 18, 1955, in Almoradí (Alicante), Spain. She obtained her Ph. D. at Universitat de València in 1982, writing a thesis in abstract Functional Analysis, under the advice of Manuel Valdivia. She spent her entire academic career at the Universitat de València where she was promoted to full professor in 2006. She died last December 26, when she was 53 years old.

After her thesis in abstract Functional Analysis, more precisely in Echelon Spaces (see [1], [2], [3], [5]) Fuensanta started working with her husband José M. Mazón, also from Almoradí and student of Manuel Valdivia, with whom she collaborated during her whole mathematical life. They started working in differentiability in Banach spaces ([4]) and in Riesz spaces ([6], [7]) jointly with Sergio Segura de León, solving a problem proposed by Arnoud Van Rooij.

In 1987, Fuensanta spent 3 months in the Mathematisches Institut of the Eberhard-Karls Universität of Tübingen, where she started to work in Linear Semigroups of Positive Operators with the group of Rainer Nagel. Fuensanta's results concerning this subject are contained in the papers [8], [12], [10] and [11]. Some of these results are part of the Thesis of her first student Josep Martínez. The more relevant result from this period is the spectral mapping Theorem for perturbed semigroups given in [11], which solves a conjecture by Mokhtar-Karroubi in connection with the linear transport neutron equation. The above result is contained in the M. Mokhtar-Karroubi monograph *"Mathematical Topics in Neutron Transport Theory-New Aspects"*. Series on Advance in Math. for Applied Science **46** (1997). World-Scientific. It was in this period that takes place the first collaboration (of a long series) with Vicent Caselles.

In 1990, Fuensanta was at the Laboratoire de Mathématiques of the Université de Franche-Comté in Besançon for three months. There she began the study of nonlinear partial differential equations under the direction of Philippe Bénilan, who proposed her to study a nonlinear potential problem from the point of view of the accretivity. She solved this problem in the work [12] obtaining a nonlinear version of Hunt-Lions's Theorem from the point of view of T-accretivity. Since that time she has been working in different aspects of the theory of nonlinear partial differential equations. She obtained existence and uniqueness results for different non linear partial differential equations, using as a main tool the theory of evolution equations governed by accretive operators and also results about the behaviour of these solutions. The first nonlinear PDE she studied was a perturbation of the filtration equation that appears in some biological models, more precisely the problem

$$(1) \qquad \begin{cases} u_t = \Delta\phi(u) - \varphi(x,u) & \text{in} \quad \Omega \times (0,\infty) \\ \\ -\frac{\partial \phi(u)}{\partial \eta} \in \beta(u) & \text{on} \quad \partial\Omega \times (0,\infty) \\ \\ u(x,0) = u_0(x) & \qquad x \in \Omega \end{cases}$$

where β is a maximal monotone graph in $\mathbb{R} \times \mathbb{R}$ with $0 \in \beta(0)$ and $\phi : \mathbb{R} \to \mathbb{R}$ is a nondecreasing continuous function with $\phi(0) = 0$, and the absorbtion φ satisfies:

(a) for almost all $x \in \Omega$, $r \mapsto \varphi(x,r)$ is a nondecreasing continuous function,

(b) for any $r \in \mathbb{R}$, $x \mapsto \varphi(x,r)$ is in $L^1(\Omega)$.

In [13], jointly with J.M. Mazón and J. Toledo, she obtained existence and uniqueness of solutions for problem (1) and the stabilization of these solutions. These stabilization results generalized the one by N. D. Alikakos and R. Rostamian about the porous medium equation and also answered a question of H. Brezis. In 1996, the authors of the above paper started to work in collaboration with Fr. Simondon, from the University of Besançon, in some related problems. Fruit of this collaboration are the results, about qualitative behaviour of the solutions of certain nonlinear PDE's, contained in [16], [18] and [23]. In [16], after proving the existence and uniqueness of weak solutions (for initial data in $L^\infty(\Omega)$) for the problem

$$\begin{cases} u_t = \Delta\varphi(u) + f(u) & \text{in } \ ]0,\infty[\times\Omega \\ -\frac{\partial\varphi(u)}{\partial\eta} = g(u) & \text{on } \ ]0,\infty[\times\partial\Omega \\ u(0,x) = u_0(x) & x \in \Omega \end{cases} \tag{2}$$

where Ω is a bounded domain in $\mathbb{R}^n$, φ is a nondecreasing function satisfying some convexity property and the growth of the functions f and g are controlled by φ, it is proved the existence of a compact attractor in $L^\infty(\Omega)$ for the dynamical system generated by these weak solutions.

In [18] the following degenerated nonlinear diffusion problem with a source and a gradient term is studied,

$$\begin{cases} u_t = \Delta u^m - \|\nabla u^\alpha\|^q + u^p & \text{in } \ Q = \Omega \times (0,\infty) \\ u = 0 & \text{on } \ S = \partial\Omega \times (0,\infty) \\ u(x,0) = u_0(x) \geq 0 & x \in \Omega, \end{cases} \tag{3}$$

where Ω is a bounded domain in $\mathbb{R}^N$, $m \geq 1$, $\alpha > 0$, $p \geq 1$ and $q \geq 1$. Under the assumptions

$$m \geq 1, \quad \alpha > \frac{m}{2}, \quad 1 \leq q < 2 \ \text{ y } \ 1 \leq p < \alpha q, \tag{4}$$

it is proved the existence of global weak solution for initial datum $u_0 \in L^{m+1}(\Omega)^+$. The uniqueness of solution for problem (3) was open until E. Feireisl, H. Petzltová and F. Simondon introduced the concept of admissible solutions. In [23] the existence and uniqueness of global admissible solutions for problem (3) is proved under the assumptions (4), moreover the existence of blow up solutions in the complementary range of the assumptions (4) is also given.

In [25], jointly with S. Segura, L. Boccardo and L. Orsina she studied problem (3) in the case $m = 1$ and $q = 2$, obtaining the existence of global weak solutions for initial data in $L^1(\Omega)$. In this direction are the results of [29] where the problem

$$\begin{cases} u_t - \Delta u + u|u|^{\beta-2}|\nabla u|^q = u|u|^{\alpha-2}|\nabla u|^p & \text{in } Q := \Omega\times]0,+\infty[; \\ u(x,t) = 0 & \text{on } \ S := \partial\Omega\times]0,+\infty[; \\ u(x,0) = u_0(x) & x \in \Omega; \end{cases} \tag{5}$$

with Ω a bounded open subset of $\mathbb{R}^N$, $1 \leq q \leq 2$, $0 \leq p < q$, $\alpha, \beta > 1$ and $\alpha + p < \beta + q$, was studied. Also in the same line can be considered her first joint work with Julio Rossi [24], where the existence of global weak solutions or blow up solutions, depending of the range of the parameters, of the problem

$$\begin{cases} u_t = \Delta(|u|^{m-1}u) - \lambda|u|^{p-1}u & \text{in } \ \Omega \times (0,\infty) \\ \frac{\partial(|u|^{m-1}u)}{\partial\eta} = |u|^{q-1}u & \text{on } \ \partial\Omega \times (0,\infty) \\ u(x,0) = u_0(x) & x \in \Omega, \end{cases} \tag{6}$$

was studied, Ω being a bounded domain in $\mathbb{R}^N$, $m > 1$, $p > 0$, $q > 0$ and $\lambda > 0$, .

Following the techniques introduced in the seminal paper[Benilan, Ph., Boccardo, L., Gallouet, Th., Gariepy, R., Pierre, M., Vazquez, J.L., *An L^1-Theory of Existence and Uniqueness of Solutions of Nonlinear Elliptic Equations* Ann. Sc. Norm. Sup. Pisa **22** (1995), 241-273], in 1996, Fuensanta, jointly with the group of nonlinear PDE's at the University of Valencia, started to study elliptic and parabolic problems associated with operators in divergence form with growth of order $p > 1$. More precisely, let Ω a bounded domain in $\mathbb{R}^N$ with smooth boundary $\partial\Omega$ and $1 < p < \infty$. Let us consider a Carathéodory function $\mathbf{a} : \Omega \times \mathbb{R}^N \to \mathbb{R}^N$ satisfying

(H_1) there exists a $\lambda > 0$ such that

$$\langle \mathbf{a}(x, \xi), \xi \rangle \geq \lambda |\xi|^p$$

for each ξ and for almost all $x \in \Omega$, where $\langle , \rangle$ is the inner product in $\mathbb{R}^N$.

(H_2) For any ξ and $\eta \in \mathbb{R}^N$, $\xi \neq \eta$, and for almost all $x \in \Omega$ we have

$$\langle \mathbf{a}(x, \xi) - \mathbf{a}(x, \eta), \xi - \eta \rangle > 0.$$

(H_3) There exists $\Lambda \in \mathbb{R}$ such that

$$|\mathbf{a}(x, \xi)| \leq \Lambda(j(x) + |\xi|^{p-1})$$

for any $\xi \in \mathbb{R}^N$, with $j \in L^{p'}$, $p' = p/(p-1)$.
The model example of a function $\mathbf{a}$ satisfying the assumptions (H_1), (H_2) and (H_3), is $\mathbf{a}(x, \xi) = |\xi|^{p-2}\xi$ whose corresponding operator is the p-Laplacian $\Delta_p(u) = \operatorname{div}\big(|Du|^{p-2}\ Du\big)$. In [14] is studied the elliptic problem for data in $L^1(\Omega)$,

$$\left\{ \begin{array}{lll} u - \operatorname{div}\ \mathbf{a}(x, Du) = f & \text{in} & \Omega \\ \\ -\frac{\partial u}{\partial \eta_a} \in \beta(u) & \text{on} & \partial\Omega \end{array} \right. \tag{7}$$

where $\partial/\partial\eta_a$ is the Neumann boundary operator associated with $\mathbf{a}$, i.e.,

$$\frac{\partial u}{\partial \eta_a} := \langle \mathbf{a}(x, Du), \eta \rangle$$

and β is a maximal monotone graph in $\mathbb{R} \times \mathbb{R}$ with $0 \in \beta(0)$; proving existence and uniqueness of entropy solutions. These results permit to get existence and uniqueness of mild solution of the associated abstract Cauchy problem in $L^1(\Omega)$. The characterization of these mild solutions was done in [15], where existence and uniqueness of entropy solutions for problem

$$\left\{ \begin{array}{lll} u_t = \operatorname{div}\ \mathbf{a}(x, Du) & \text{in} & \Omega \times (0, \infty) \\ \\ -\frac{\partial u}{\partial \eta_a} \in \beta(u) & \text{on} & \partial\Omega \times (0, \infty) \\ \\ u(x, 0) = u_0(x) & & x \in \Omega \end{array} \right. \tag{8}$$

was established. In [17], using the Liapunov method for equations governed by accretive operators introduced by A. Pazy, it was studied the asymptotic behaviour of the entropy solutions of problem (8), proving the convergence of the solutions as time go to $+\infty$ to an stationary solution.

In collaboration with Mazón and M. Sofonea from the University of Perpignan, in [19], Fuensanta adapted the techniques developed in [15] to a problem from elasticity, proving existence an uniqueness of entropy solutions of a model for the antiplane shear deformations for elastic solids.

In 2005 Fuensanta, jointly with J.M. Mazón and J. Toledo, started a collaboration with Noureddine Igbida, from the University of Amiens, working in problems related with problem (8). Fruit of this collaboration are the results contained in the papers [38], [40], [45], [46], in which, for instance, existence and uniqueness of weak/renormalized solutions for a general degenerate elliptic-parabolic problem with nonlinear dynamical boundary conditions is proved. More precisely the problem

$$P_{\gamma,\beta}(f,g,z_0,w_0) \quad \begin{cases} z_t - \operatorname{div} \mathbf{a}(x,Du) = f, \ z \in \gamma(u) & \text{in } Q_T :=]0,T[\times\Omega \\ w_t + \mathbf{a}(x,Du)\cdot\eta = g, \ w \in \beta(u) & \text{on } S_T :=]0,T[\times\partial\Omega \\ z(0) = z_0 \ \text{ in } \Omega, \ w(0) = w_0 & \text{in } \partial\Omega, \end{cases}$$

where $T > 0$, Ω is a bounded domain in $\mathbb{R}^N$ with smooth boundary $\partial\Omega$, $z_0 \in L^1(\Omega)$, $w_0 \in L^1(\partial\Omega)$, $f \in L^1(0,T;L^1(\Omega))$, $g \in L^1(0,T;L^1(\partial\Omega))$ and η is the unit outward normal on $\partial\Omega$. Here the function $\mathbf{a} : \Omega \times \mathbb{R}^N \to \mathbb{R}^N$ is a Carathéodory function satisfying the assumptions (H_1), (H_2) and (H_3). The nonlinearities γ and β are maximal monotone graphs in $\mathbb{R}^2$ such that $0 \in \gamma(0)$, $\mathrm{Dom}(\gamma) = \mathbb{R}$, and $0 \in \beta(0)$. Particular instances of this problem appear in various phenomena with changes of phase like multiphase Stefan problem and in the weak formulation of the mathematical model of the so called Hele–Shaw problem. Moreover, the problem with non-homogeneous Neumann boundary condition is included.

In the case $D(\gamma) \neq \mathbb{R}$, we are dealing with obstacle problems. For this kind of problems the existence of a weak solution, in the usual sense, fails to be true for nonhomogeneous boundary conditions, so a new concept of solution has to be introduced, this was done in [46], where existence and uniqueness of this type of solution for problem

$$(S^{\gamma,\beta}_{\phi,\psi}) \quad \begin{cases} -\mathrm{div}\ \mathbf{a}(x,Du) + \gamma(u) \ni \phi & \text{in } \Omega \\ \mathbf{a}(x,Du)\cdot\eta + \beta(u) \ni \psi & \text{on } \partial\Omega, \end{cases}$$

was proved.

The last collaboration of Fuensanta with N. Igbida has been paper [49], in which existence and uniqueness of renormalized/entropy solutions is established for a degenerate elliptic problem with nonlinear boundary condition of the form

$$\begin{cases} -\mathrm{div}\ \mathbf{a}(x,Du) + \gamma(u) \ni \mu_1 & \text{in } \Omega \\ \mathbf{a}(x,Du)\cdot\eta + \beta(u) \ni \mu_2 & \text{on } \partial\Omega, \end{cases}$$

where $\mu_1,\ \mu_2$ are measures such that $\mu_1 = \mu_1 \llcorner \Omega,\ \mu_2 = \mu_2 \llcorner \partial\Omega$ and $\mu_1 + \mu_2$ is a diffuse measure (it does not charge sets of zero $p-$capacity).

In 1999, Fuensanta and Mazón started to work in colaboration with Coloma Ballester and Vicent Caselles, on problems coming from image processing theory of which Ballester and Caselles are specialists. Motivated by the L. Rudin, S. Osher and E. Fatemi models for image restoration based on total variation minimization, in [20], it is proved existence and uniqueness of solutions for the Minimizing Total Variation Flow with a Neumann boundary condition, namely

$$\begin{cases} \dfrac{\partial u}{\partial t} = \mathrm{div}\left(\dfrac{Du}{|Du|}\right) & \text{in } Q = (0,\infty)\times\Omega \\ \dfrac{\partial u}{\partial \eta} = 0 & \text{on } S = (0,\infty)\times\partial\Omega \\ u(0,x) = u_0(x) & \text{in } x \in \Omega, \end{cases} \tag{9}$$

where Ω is a bounded set in $\mathbb{R}^N$ with Lipschitz continuous boundary $\partial\Omega$ and $u_0 \in L^1(\Omega)$. Also some qualitative properties of the solutions of (9) were studied. Next, in [21], the Dirichlet problem

$$\begin{cases} \dfrac{\partial u}{\partial t} = \operatorname{div}\left(\dfrac{Du}{|Du|}\right) & \text{in} \quad Q = (0,\infty)\times\Omega \\ u(t,x) = \varphi(x) & \text{on} \quad S = (0,\infty)\times\partial\Omega \\ u(0,x) = u_0(x) & \text{in} \quad x \in \Omega, \end{cases} \tag{10}$$

where $u_0 \in L^1(\Omega)$ and $\varphi \in L^1(\partial\Omega)$, was studied. One of the motivations for studying this problem comes from a filling-in method introduced by C. Ballester, M. Bertamio, V. Caselles, G. Shapiro and J. Verdera.

Later, in a joint paper with J. I. Díaz [22] they studied the asymptotic behaviour of the solutions of problems (9) and (10).

Using some of the techniques developed to study the total variation flow, in the papers [26], [27] and [28], in collaboration with V. Caselles and J. M. Mazón, she studied the Dirichlet, Neumann and Cauchy problem for quasi-linear parabolic equations whose operator is in divergence form being the subdifferential of a Lagrangian which is convex and has linear growth in the magnitude of the gradient. More precisely, for the differential operator $-\operatorname{div}\mathbf{a}(x, Du)$, where $\mathbf{a}(x,\xi) = \nabla_\xi f(x,\xi)$, f being a convex function of ξ with linear growth as $\|\xi\| \to \infty$, which include many examples relevant in applications like the nonparametric area integrand, $f(\xi) = \sqrt{1+\|\xi\|^2}$, and Hencky plasticity. Existence follows by means of Crandall-Ligget's semigroup generation Theorem, while uniqueness is proved using Kruzhkov's method of doubling of variables.

In 2003 she was awarded the prize "Ferran Sunyer i Balaguer" for the monograph *Parabolic Quasilinear Problems Minimizing Linear Growth Functionals*, (with José M. Mazón and Vicent Caselles). This book mainly contain the results of the papers: [20], [21], [22], [26], [27] and [28].

Other papers of Fuensanta, with different collaborators, related with the total variation flow are the following: [35] in which existence and uniqueness for the total variation flow with nonlinear boundary conditions like the ones that appear in problem (8) was established; [30] in which the minimizing total variation flow with measures as initial conditions was studied and [32] dedicate to the study of the best constant for the Sobolev trace embedding from $W^{1,1}(\Omega)$ into $L^1(\partial\Omega)$.

In 2004 Fuensanta started a new project, again jointly with V. Caselles and J.M. Mazón, about the study of the equation

$$\frac{\partial u}{\partial t} = \operatorname{div}\,\mathbf{a}(u, Du)$$

where $u_0 \in L^1(\Omega)$, $\mathbf{a}(z,\xi) = \nabla_\xi f(z,\xi)$, f being a function with linear growth as $\|\xi\| \to \infty$. The first results ([33], [34]) in this direction were obtained assuming

$$C_0\|\xi\| - D_0 \le f(z,\xi) \le M(\|\xi\|+1), \tag{11}$$

for any $(z,\xi) \in \mathbb{R}\times\mathbb{R}^N$, $|z| \le R$ and some positive constants C_0, D_0, M depending on R. There are important equations, that appear for instance in models for heat and mass transfer in turbulent fluids or in the theory of phase transitions, where the corresponding free energy functional has a linear growth rate with respect to the gradient satisfying (11). On the other hand, there are some relevant cases like the *relativistic heat equation*

$$u_t = \nu \operatorname{div}\left(\frac{|u|Du}{\sqrt{u^2 + a^2|Du|^2}}\right) \tag{12}$$

for which the Lagrangian $f(z,\xi) = \frac{\nu}{a^2}|z|\sqrt{z^2 + a^2|\xi|^2}$ does not satisfy (11). Observe that, in this case, $f(z,\xi)$ satisfies the following condition:

$$C_0(z)\|\xi\| - D_0(z) \le f(z,\xi) \le M_0(z)(\|\xi\|+1) \tag{13}$$

for any $(z, \xi) \in \mathbb{R} \times \mathbb{R}^N$, and some positive and continuous functions C_0, D_0, M_0, such that $C_0(z) > 0$ for any $z \neq 0$. The equation (12) was introduced by Ph. Rosenau in 1992 to overcome the unphysical divergence of the flux with gradient predicted by the classical transport theory. The equation (12) was rediscovered by Y. Brenier in 2003, by means of Monge-Kantorovich's mass transport theory. As Brenier pointed out, this relativistic heat equation is one among the various *flux limited diffusion equations* used in the theory of radiation hydrodynamics. Indeed, a very similar equation,

$$u_t = \nu \operatorname{div} \left(\frac{uDu}{u + \frac{\nu}{c}|Du|} \right), \tag{14}$$

appears in the theory of radiation hydrodynamics. The study of this type of equations has been one of the main subjects studied by Fuensanta in her last years. One of the main difficulties in the study of this problem is to give sense to the differential operator, this was done in [36], where the elliptic problem is studied. Then, in [37], by means of the Crandall-Liggett Theorem and using Kruzhkov's method of doubling of variables to prove uniqueness, it is proved the existence and uniqueness of entropy solution for the Cauchy problem and for the Neumann problem. The Dirichlet problem, for the particular case of the relativistic heat equation, has been recently studied in [50] in collaboration with Salvador Moll, who was her second thesis student. The Dirichlet problem is however more delicate than the Cauchy problem in $\mathbb{R}^N$ or the homogeneous Neumann one. In particular, difficulties arise because of the fact that the boundary condition is in general not attained and this condition has to be weakened to become an obstacle condition on the boundary. Also jointly with S. Moll, in [39] it was proved that the support of solutions of the relativistic heat equation evolves at constant speed, identified as light's speed c, and in [41] it was studied the diffusion equation in transparent media with constant speed of propagation c

$$u_t = c \operatorname{div} \left(u \frac{Du}{|Du|} \right), \tag{15}$$

equation that is obtained formally when $\nu \to \infty$ in the relativistic heat equation. For the equation (15) it was possible to get explicit solutions which are expanding fronts at constant speed c.

Although the entropy solutions of the relativistic heat equation has discontinuity fronts moving at light's speed, in [44] it was proved some partial regularity results for these solutions. In particular, under some assumptions on the initial condition u_0, it was shown that $u_t(t)$ is a Radon measure in $\mathbb{R}^N$. Moreover, if u_0 is log-concave inside its support Ω, Ω being a convex set, then it is shown that the solution $u(t)$ is also log-concave in its support $\Omega(t)$. This implies its smoothness in $\Omega(t)$. In that case, a simpler characterization of the notion of entropy solution was given.

The use of the relativistic heat equation as model for diffusion problems instead of the classical linear diffusion equation opened a broad field of work which unfortunately her early death prevented her from engaging. Nevertheless, Fuensanta has left us two nice preprints ([51], [52]) in this direction, in which she was working until her last days. In the first one it is studied a Fisher-Kolmogorov type equation with a flux limited diffusion term, more precisely the problem

$$\begin{cases} u_t = \nu \operatorname{div} \left(\dfrac{uDu}{\sqrt{u^2 + \frac{\nu^2}{c^2}|Du|^2}} \right) + ku(1-u) & \text{in } \ Q_T = (0,T) \times \mathbb{R}^N \\ \\ u(0,x) = u_0(x) & \text{in } \ x \in \mathbb{R}^N. \end{cases} \tag{16}$$

The second one is devoted to an equation arising in the transport of morphogens. In this equation it is changed the linear diffusion by the relativistic heat equation arising to a more realistic model.

Another subject in which Fuensanta was working until her last days is about evolution problems with nonlocal diffusion. In this subject she obtained results in collaboration with J.M. Mazón, J. Rossi and J. Toledo, which have been published in the papers: [42], [43], [47] and [48]. The results of these papers, that correspond to nonlinear problems, jointly with the linear results obtained by

J. Rossi in collaborations with different authors, have been collected in the recent monograph [53]. The above papers deal with existence and uniqueness of solutions, and their asymptotic behaviour, of the nonlocal version of some classical nonlinear equations. Also some results concerning limits of solutions to nonlocal equations when a rescaling parameter goes to zero, are given, recovering in these limits some of the most frequently used diffusion models like the heat equation, the $p-$Laplacian evolution equation, the porous medium equation, the total variation flow. The type of nonlocal problems studied in the above papers is

$$\begin{cases} (u_p)_t(x,t) = \displaystyle\int_{\mathbb{R}^N} J(x-y)|u_p(y,t)-u_p(x,t)|^{p-2}(u_p(y,t)-u_p(x,t))dy \\ u_p(x,0) = u_0(x), \qquad\qquad\qquad\qquad x \in \mathbb{R}^N,\ t>0, \end{cases} \tag{17}$$

which correspond to the Cauchy problem for the p-Laplacian. Here $J : \mathbb{R}^N \to \mathbb{R}$ is a nonnegative continuous radial function with compact support, $J(0) > 0$ and $\int_{\mathbb{R}^N} J(x)dx = 1$. Again, the main tool used here to get existence and uniqueness of solutions is Crandall-Ligget's semigroup generation Theorem. Also a general result due to Brezis and Pazy, about convergence of the mild solutions of a sequence of problems, has been useful the obtain the convergence of the rescaled non local problems to the local one.

Taking limit as $p \to +\infty$ in problem (17), in [48] it was obtained a non local model for sandpile and it was proved that the solutions of rescaled problems converge to the clasical sandpile models by Arosson-Evans-Wu. Similar results for the Prigozhin model of sandpile were obtained in [47].

Publications of Fuensanta Andreu Vaillo

1. **ANDREU, F.** *On the Dvoretzky-Rogers Theorem.* Proceedings of the Edinburgh Mathematical Society **27** (1984), 105-113.
2. **ANDREU, F.** *Una clase de espacios de sucesiones vectoriales.* Collectanea Mathematica **35** (1984), 3-17.
3. **ANDREU, F.** *Espacios escalonados de orden (p,q)* Revista de la Real Academia de Ciencias Exactas, Físicas y Naturales **68** (1985), 119-129.
4. **ANDREU, F., MAZON, J.M.** *On the compact differentiability in function spaces.* Proceedings of the Royal Irish Academy **85** (1985), 109- 117.
5. **ANDREU, F., M. GARCIA-FALSET** *On certain echelon spaces.* Archiv der Mathematik **47** (1986), 70-78.
6. **ANDREU, F., MAZON, J. M., SEGURA, S.** *On the space of all regular operators from $C(K)$ into $C(K)$.* Indagationes Mathematicae **91** (1988), 225-230.
7. **ANDREU, F., MAZON, J. M., SEGURA, S.** *A Radon-Nikodym theorem for Riesz space valued measures.* Quaestiones Mathematicae **12** (1989), 149-167.
8. **ANDREU, MAZON, J. M.** *On the boundary spectrum of dominated C_0-semigroups,* Semigroup Forum **38**, (1989), 129-139.
9. **ANDREU, MAZON, J. M.** *Relation between the boundary point spectrum of a generator and of its adjoint,* Publ. Matemátiques **34**, (1990), 389-396.
10. **ANDREU, F., CASELLES, V., MARTINEZ, J., MAZON, J. M.** *The essential spectrum of AM-compact operators,* Indag. Math. **2**, (1991), 149-158.
11. **ANDREU, F., MARTINEZ, J., MAZON, J. M.** *A spectral mapping theorem for perturbed strongly continuos semigroups,* Math. Ann. **291**, (1991), 453-462.
12. **ANDREU, F., MAZON, J.M.** *A nonlinear version of Hunt-Lions's theorem from the point of view of T-accretivity.* Potential Analysis **1** (1992), 205-220.
13. **ANDREU, F., MAZON, J. M., TOLEDO, J.** *Stability of solutions of the filtration equation with absortion and non-linear flux.* NoDEA **2** (1995), 267-289.
14. **ANDREU, F., MAZON, J. M., SEGURA, S., TOLEDO, J.** *Quasi-linear elliptic and parabolic equations in L^1 with nonlinear boundary conditions.* Adv. in Math Sci. and Appl. **7**, (1997), 183-213.
15. **ANDREU, F., MAZON, J. M., SEGURA, S., TOLEDO, J.** *Existence and uniqueness of solutions for a degenerate parabolic equation with L^1-data.* Trans. Amer. Math. Soc. **351**(1999), 285-306.
16. **ANDREU, F., MAZON, J. M., SIMONDON, F., TOLEDO, J.** *Attractor for a degenerate nonlinear diffusion problem with nonlinear boundary condition.* J. of Dynamics and Diff. Eq. **10**, (1998), 347-377.

17. **ANDREU, F., MAZON, J. M., TOLEDO, J.** *Asymptotic behaviour of solutions of quasi-linear parabolic equations with non-linear flux.* Comput. Appl. Math. **17**(1998),203-217.
18. **ANDREU, F., MAZON, J. M., SIMONDON, F., TOLEDO, J.** *Global existence for a degenerate nonlinear diffusion problem with nonlinear gradient term and source.* Math. Ann. **314** (1999), 703-728.
19. **ANDREU, MAZON, J. M., SOFONEA, M.** *Entropy solutions in the study of antiplane shear deformations for elastic solids.* Mathematical Models and Methods in Applied Science.**10** (2000), 99-126.
20. **ANDREU, F., BALLESTER, C., CASELLES, V., MAZON, J. M.** *Minimizing Total Variation Flow.* Differential and Integral Equations **14** (2001), 321-360.
21. **ANDREU, F., BALLESTER, C., CASELLES, V., MAZON, J. M.** *The Dirichlet Problem for the Total Variation Flow* Journal of Functional Analysis. **180** (2001), 347-403.
22. **ANDREU, F., CASELLES, V., DIAZ, J. I., MAZON, J. M.** *Some qualitative properties for the total variation flow.* J. Funct. Anal.**188** (2002), 516-547.
23. **ANDREU, F., MAZON, J. M., SIMONDON, F., TOLEDO, J.** *Blow-up for a class of nonlinear parabolic problems.* Asymptotic Analysis **29** (2002), 143-155.
24. **ANDREU, F., MAZON, J. ROSSI J. M., TOLEDO, J.** *Porous medium equation with absorption and a nonlinear boundary condition.* Nonlinear Analysis TMA **49** (2002), 541-563.
25. **ANDREU, BOCCARDO, L., ORSINA, L., SEGURA, S.** *Existence results for some quasi-linear parabolic problems with a quadratic gradient term and source.* Mathematical Models and Methods in Applied Science **12** (2002), 1-16.
26. **ANDREU, CASELLES, V., MAZON, J. M.** *A Parabolic Quasilinear Problem for Linear Growth Functionals.* Rev. Mat. Iberoamericana **18** (2002), 135-185.
27. **ANDREU, F., CASELLES, V., MAZON, J. M.** *Existence and Uniqueness of Solution for a Parabolic Quasilinear Problem for Linear Growth Functionals.* Math. Ann. **322** (2002), 139-206.
28. **ANDREU, F., CASELLES, V., MAZON, J. M.** *The Cauchy Problem for Linear Growth Functionals.* J. Evol. Equ. **3** (2003), 39-65.
29. **ANDREU, F., SEGURA, S., TOLEDO, J.** *Quasilinear Diffusion Equations with Gradient Terms and L^1-Data.* Nonlinear Analysis TMA. **56** (2004), 1175–1209.
30. **ANDREU, F., CASELLES, V., MAZON, J. M., MOLL, S** *The Minimizing Total Variation Flow with Measures Initial Conditions.* Comm. in Contemporary Math. **6** (2004), 431-494.
31. **ANDREU, F., CASELLES, V., MAZON, J. M.** *Parabolic quasilinear equations minimizing linear growth functionals.* Progress in Mathematics, **223**. Birkhäuser Verlag, Basel, 2004.
32. **ANDREU, F., MAZON, J. ROSSI J. M.** *J. D. The best constant for the Sobolev trace embedding from $W^{1,1}(\Omega)$ into $L^1(\partial\Omega)$.* Nonlinear Anal. **59** (2004), 1125–1145.
33. **ANDREU, F., CASELLES, V., MAZON, J. M.** *A strongly degenerate quasilinear equation: the elliptic case.* Ann. Sc. Norm. Super. Pisa Cl. Sci. (5) 3 (2004), 555–587.
34. **ANDREU, F., CASELLES, V., MAZON, J. M.** *A strongly degenerate quasilinear equation: the parabolic case.* Arch. Ration. Mech. Anal. **176** (2005), 415–453.
35. **ANDREU, F., MAZON, J. M., MOLL, S.** *The Total Variation Flow with Nonlinear Boundary Conditions.* Asymptotic Analysis **43** (2005), 9- 46.
36. **ANDREU, F., CASELLES, V., MAZON, J. M.** *A strongly degenerate quasilinear elliptic equation.* Nonlinear Anal. **61** (2005), 637–669.
37. **ANDREU, F., CASELLES, V., MAZON, J. M.** *The Cauchy problem for a strongly degenerate quasilinear equation.* J. Eur. Math. Soc. **7** (2005), 361–393.
38. **ANDREU, F., IGBIDA, N., MAZON, J. M., TOLEDO, J.** *A degenerate elliptic-parabolic problem with nonlinear dynamical boundary conditions.* Interfaces Free Bound. **8** (2006), 447–479.
39. **ANDREU, F., CASELLES, V., MAZON, J. M., MOLL, S** *Finite Propagation Speed for Limited Flux Diffusion Equations.* Arch. Ration. Mech. Anal. **182** (2006), 269-297.
40. **ANDREU, F., IGBIDA, N., MAZON, J. M., TOLEDO, J.** *L^1 existence and uniqueness results for quasi-linear elliptic equations with nonlinear boundary conditions.* Ann. Inst. H. Poincaré Anal. Non Linéaire **24** (2007), 61–89.
41. **ANDREU, F., CASELLES, V., MAZON, J. M., MOLL, S** *A diffusion equation in transparent media.* J. Evol. Equ. **7** (2007), 113–143.
42. **ANDREU, F., MAZON, J. ROSSI J. M., TOLEDO, J.** *The Neumann problem for nonlocal nonlinear diffusion equations.* J. Evol. Equ. **8** (2008), 189–215.
43. **ANDREU, F., MAZON, J. ROSSI J. M., TOLEDO, J.** *A nonlocal p-Laplacian evolution equation with Neumann boundary conditions.* J. Math. Pures Appl. **90** (2008), 201–227.
44. **ANDREU, F., CASELLES, V., MAZON, J. M.** *Some regularity results on the 'relativistic' heat equation.* J. Differential Equations **245** (2008), 3639–3663.
45. **ANDREU, F., IGBIDA, N., MAZON, J. M., TOLEDO, J.** *Renormalized solutions for degenerate elliptic-parabolic problems with nonlinear dynamical boundary conditions and L^1-data.* J. Differential Equations **244** (2008), 2764–2803.

46. **ANDREU, F., IGBIDA, N., MAZON, J. M., TOLEDO, J.** *Obstacle problems for degenerate elliptic equations with nonhomogeneous nonlinear boundary conditions.* Math. Models Methods Appl. Sci. **18** (2008),1869–1893.
47. **ANDREU, F., MAZON, J. ROSSI J. M., TOLEDO, J.** *A nonlocal p-Laplacian evolution equation with nonhomogeneous Dirichlet boundary conditions.* SIAM J. Math. Anal. **40** (2009), 1815–1851.
48. **ANDREU, F., MAZON, J. ROSSI J. M., TOLEDO, J.** *The limit as $p \to \infty$ in a nonlocal $p-$Laplacian evolution equation. A nonlocal approximation of a model for sandpiles.* Calc. Var. Partial Differential Equations. **35**, (2009), 279–316.
49. **ANDREU, F., IGBIDA, N., MAZON, J. M., TOLEDO, J.** *Degenerate Elliptic Equations with Nonlinear Boundary Conditions and Measures Data.* To appear in Ann. Scuola Norm. Sup. Pisa.
50. **ANDREU, F., CASELLES, V., MAZON, J. M., MOLL, S** *The Dirichlet problem associated to the relativistic heat equation.* To appear in Math. Ann.
51. **ANDREU, F., CASELLES, V., MAZON, J. M.** *A Fisher-Kolmogorov Equation with Finite Speed of Propagation.* Preprint.
52. **ANDREU, F., CALVO, J., MAZON, J. M., SOLER, J.** *On a nonlinear flux–limited equation arising in the transport of morphogens.* Preprint.
53. **ANDREU, F., MAZON, J. ROSSI J. M., TOLEDO, J.** *Evolution Problem with nonlocal diffusion.* Forthcomming book.