

Hawking Radiation in Acoustic Black-Holes on an Ion Ring

Benni Reznik

In collaboration with, B. Horstman, *S. Fagnocchi*, J. I. Cirac

Towards the Observation of Hawking Radiation in Condensed
Matter Systems. IFIC, Valencia, February 1-7, 2009

Outline:

1. **A discrete analogue** model of a sonic hole toy model on a ion ring geometry.

- motivation, trapped ions.

- numerical classical simulation of a BH.

2. **Numerical evidence** of Hawking radiation:

- backward evolution and Bogolibouv particle creation.

- mode conversion (Unruh's sonic-hole)

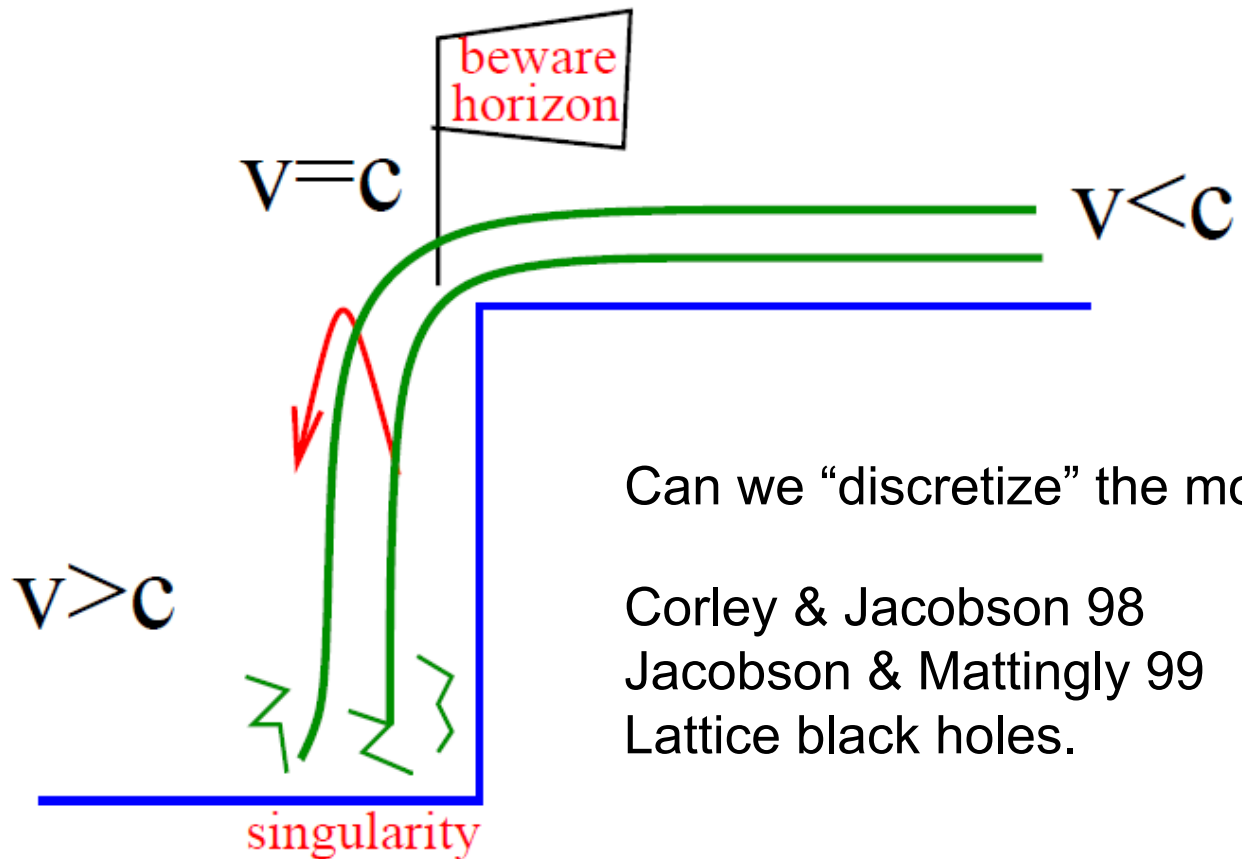
- Bloch oscillations (Corley and Jacobson, "falling lattice")

- Dynamical creation and point-to-point correlations.

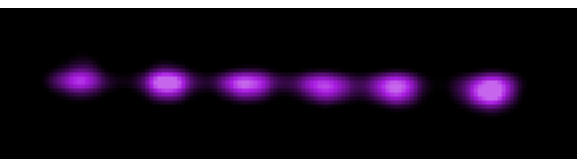
,

Sonic dumb-hole

Unruh 81, 95

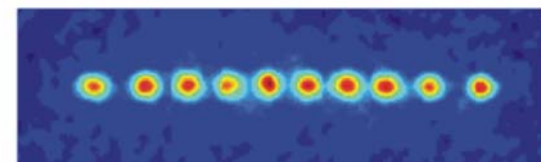
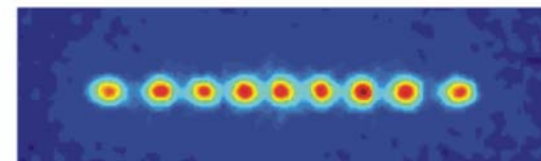
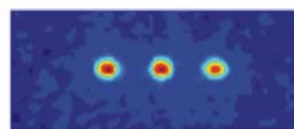
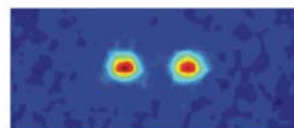
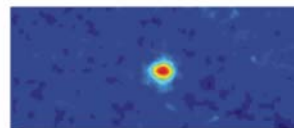
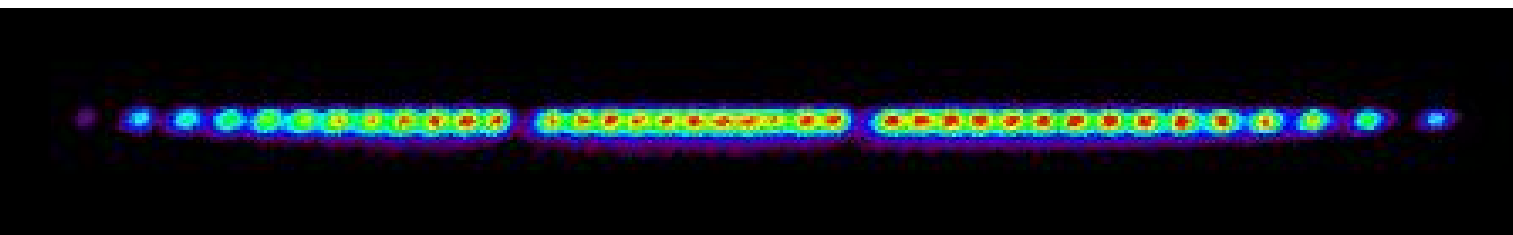


Ion lattices

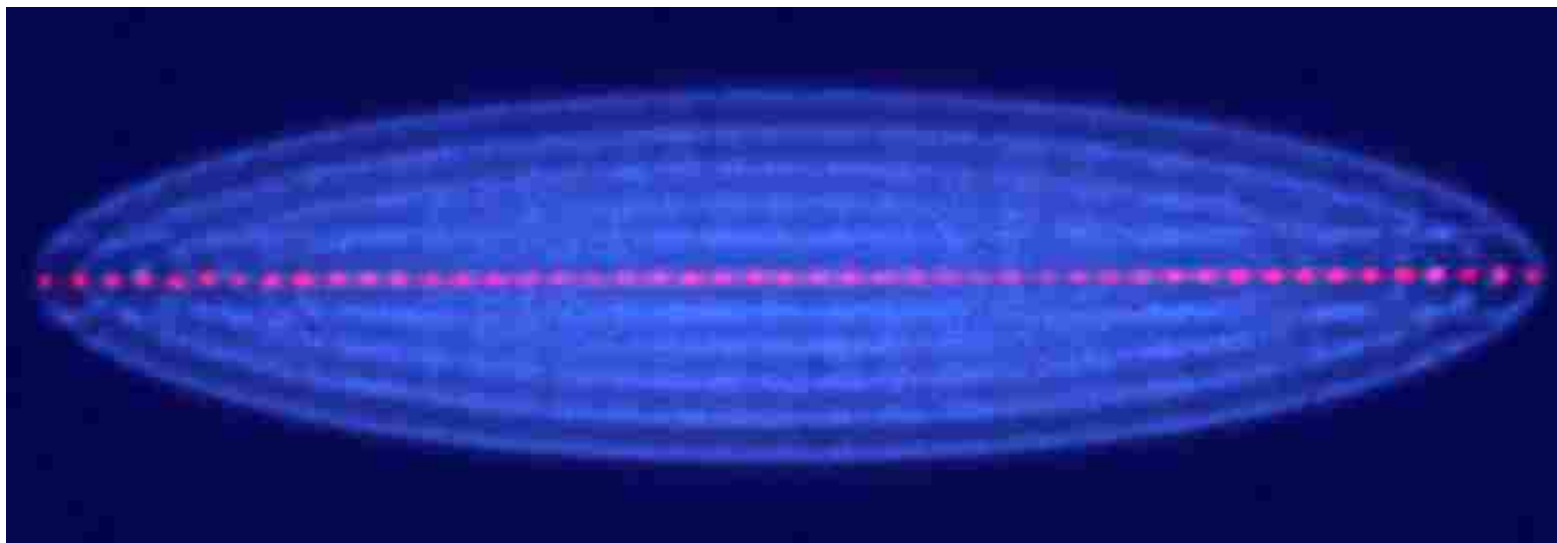


Oxford, England: $^{40}\text{Ca}^+$

Boulder, USA: Hg^+ (mercury)

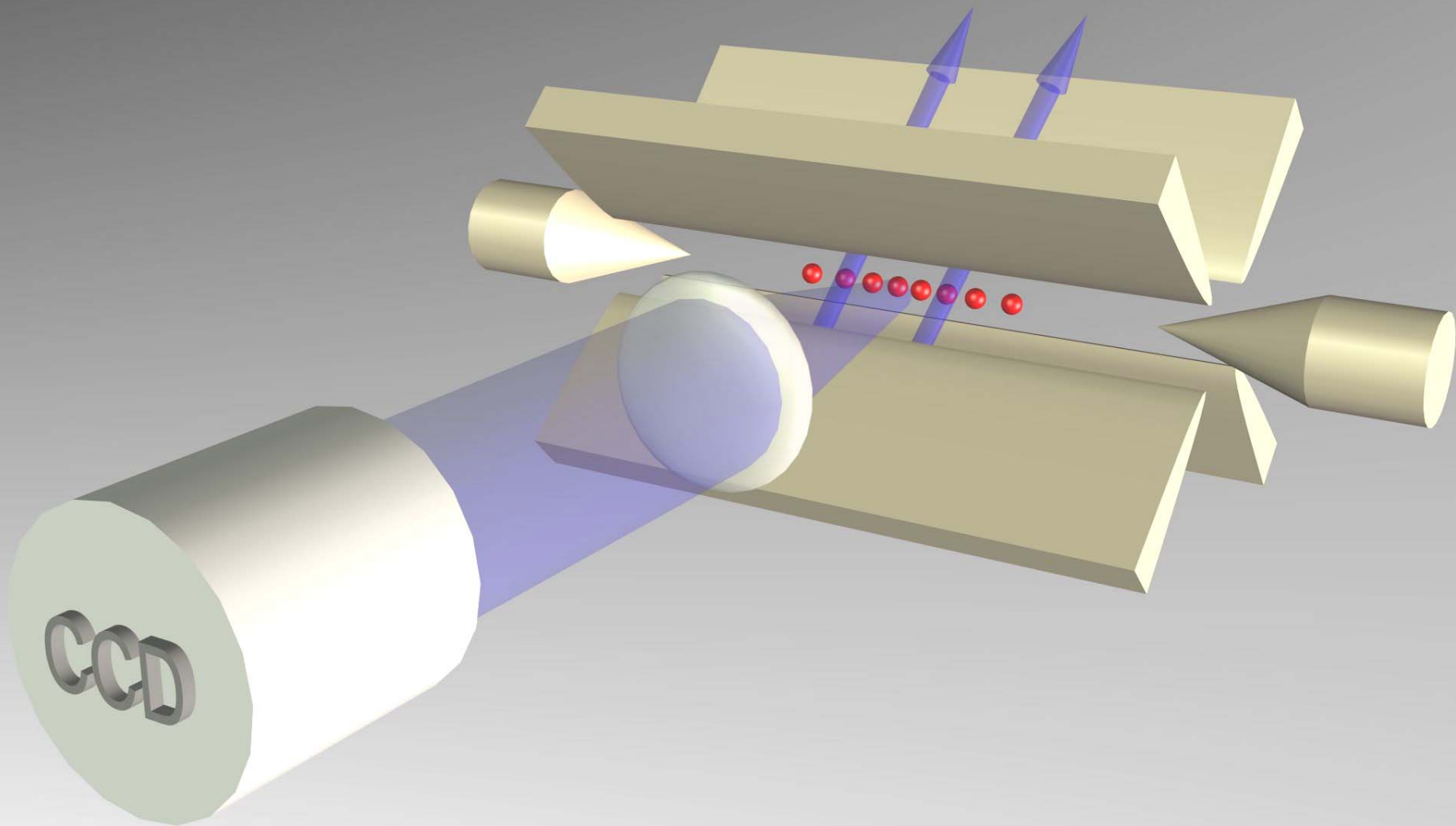


Innsbruck, Austria: $^{40}\text{Ca}^+$



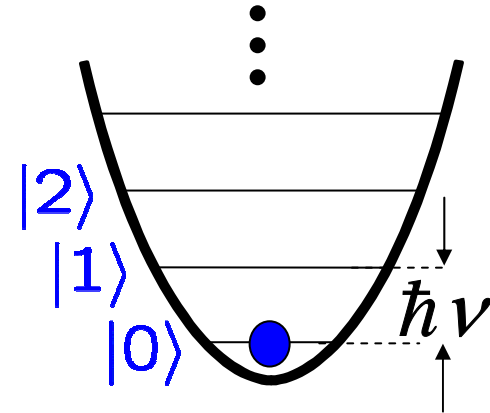
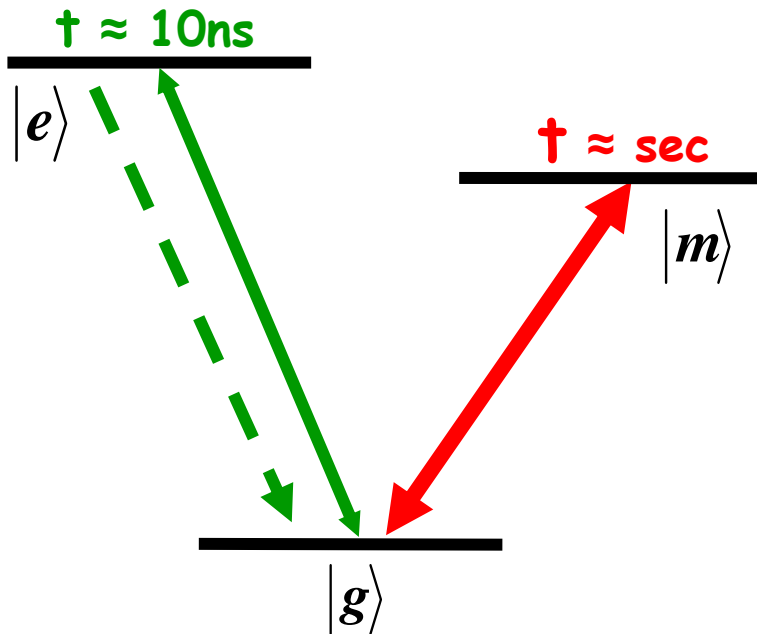
Aarhus, Denmark: $^{40}\text{Ca}^+$ (red) and $^{24}\text{Mg}^+$ (blue)

Trapped Ions



Courtesy of R. Blatt

Trapped ions



$$T = \frac{\hbar\nu}{k_B} \approx 50\mu\text{K}$$

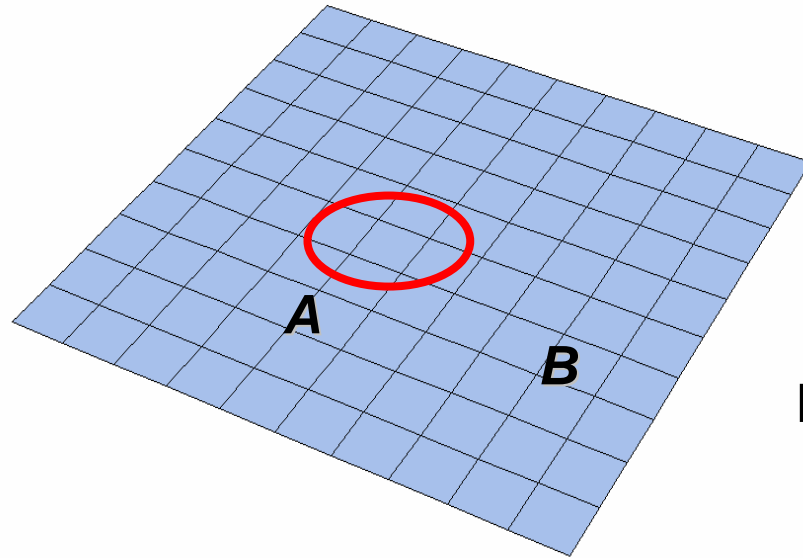
Separation between ions:

$$d \approx 5\mu\text{m}$$

$$\langle x^2 \rangle^{1/2} = \sqrt{\frac{\hbar}{2m\nu}} \approx 11\text{nm}$$

Size of the wave packet \ll wavelength of visible light.

Entanglement entropy in Field Theory



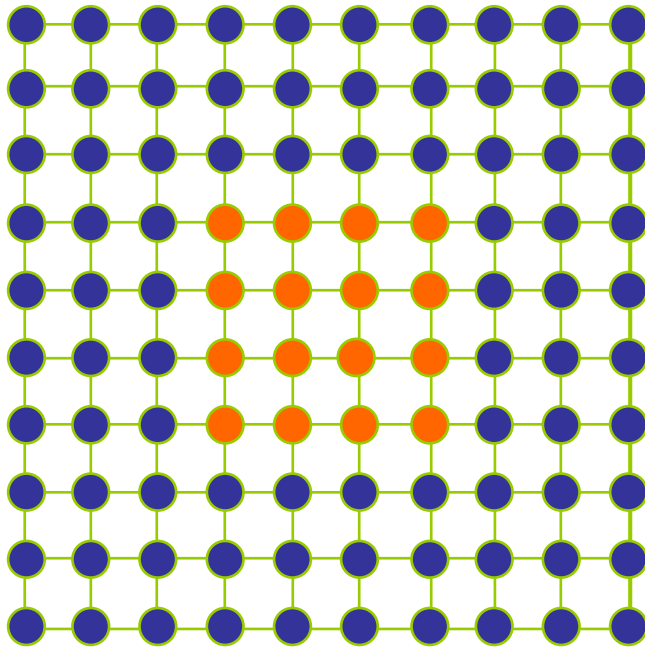
Entanglement \propto Area

Could it be that Entanglement is the quantum source of BH entropy?

Bombelli, Koul, Lee, Sorkin 86

Entanglement entropy

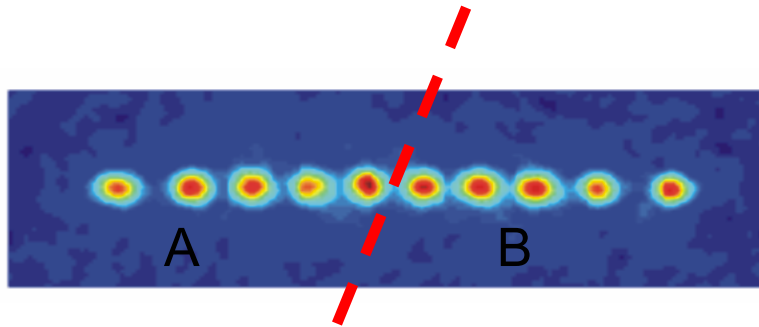
2-D Harmonic lattice



Entanglement \propto Area

$$S_{AB} = -tr(\rho_A \log \rho_A)$$

Simulating detection of vacuum entanglement



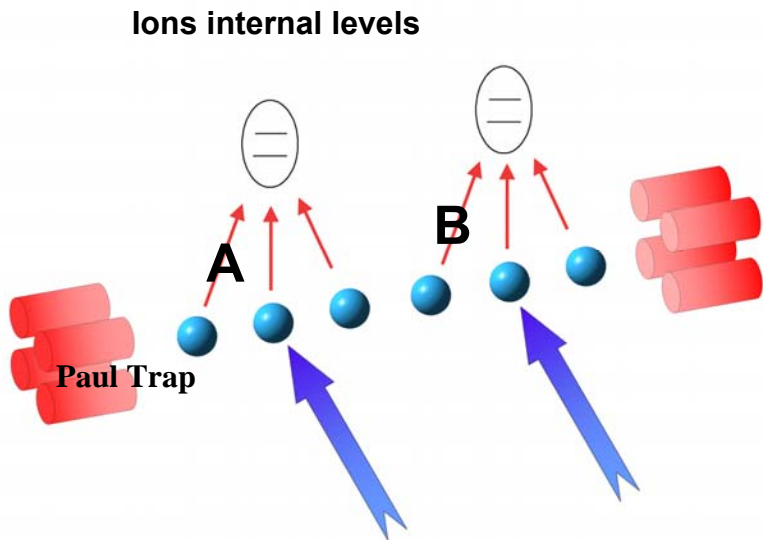
Entanglement Entropy

$$H=H_0+H_{\text{int}}$$

$$H_0=\omega_z(\sigma_z^A+\sigma_z^B)+\sum v_n a_n^\dagger a_n$$

$$H_{\text{int}}=\Omega(t)(e^{-i\phi}\sigma_+^{(k)}+e^{i\phi}\sigma_-^{(k)})x_k$$

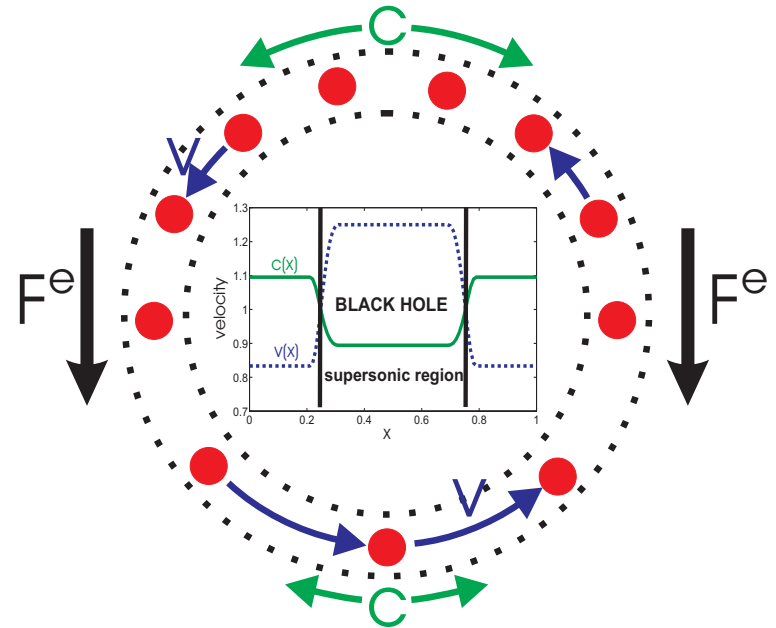
$$1/\omega_z \ll T \ll 1/v_0$$



Discrete BH analogue

Inhomogeneous, but stationary velocity profile $v(\theta)$.

The necessary (de-)acceleration of the ions is guaranteed by a force F^e on the ions.



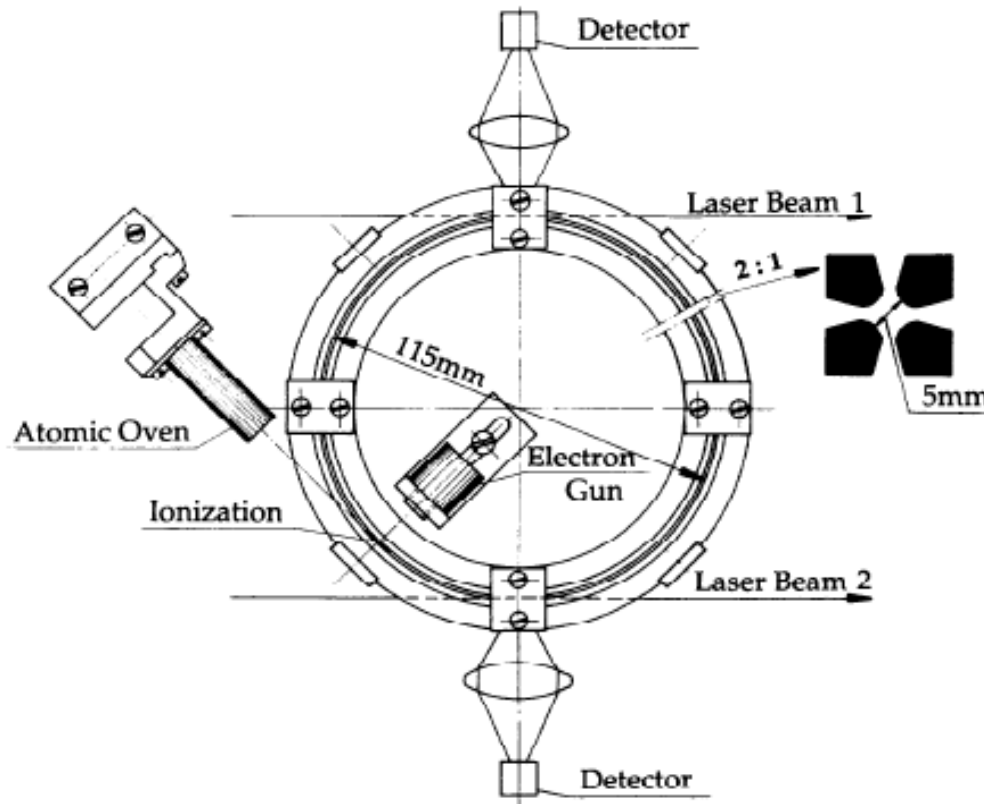
Harmonic oscillations around the equilibrium motion are phonons with velocities $c(\theta) \propto (v(\theta))^{-1/2}$.

When v increases the sound velocity decreases and a Black and White horizons can form.

Observation of Ordered Structures of Laser-Cooled Ions in a Quadrupole Storage Ring

I. Waki,^(a) S. Kassner, G. Birkel, and H. Walther

Nature (1992), PRL (1992)



Ions on a ring

N ions of mass m in a ring with radius L :

$$\mathcal{H} = - \sum_{i=1}^N \frac{4\pi^2 \hbar^2}{2mL^2} \frac{\partial^2}{\partial \theta_i^2} + \sum_{i=1}^N V^e(\theta_i) + V^c(\theta_1, \dots, \theta_N)$$

We treat small perturbations around the equilibrium motion

$$\theta_i(t) = \theta_i^0(t) + \delta\theta_i(t)$$

and expand the Hamiltonian to second order in $\delta\theta_i$

$$\mathcal{H} = - \sum_{i=1}^N \frac{4\pi^2 \hbar^2}{2mL^2} \frac{\partial^2}{\partial \delta\theta_i^2} + \frac{1}{2} \sum_{i \neq j} f_{ij}(t) \delta\theta_i \delta\theta_j \quad (1)$$

Large N effective field limit

For a slowly varying $v(\theta)$ the system Lagrangian for the scalar field $\Phi(\theta_i^0(t), t) = \delta\theta_i(t)$ becomes

$$\mathcal{L} = \frac{mL^2}{(2\pi)^2} \int d\theta \frac{n(\theta)}{2} \left[(\partial_t \Phi + v(\theta) \partial_\theta \Phi)^2 - (iD(\theta, -i\partial_\theta) \Phi)^2 \right]$$

with $D(\theta, k) = c(\theta)k + \mathcal{O}(k^3)$, the density $n(\theta) = 1/(v(\theta)T)$.
 $c(\theta) = \sqrt{2(2\pi)^3 n(\theta) e^2 / (4\pi\epsilon_0 \cdot mL^3)}$.

We have a position dependent dispersive fluid.

Creating a discrete BH

We want the classical dumb-hole equilibrium motion:

$$\theta_i(t) = g_v\left(\frac{i + t/T}{N}\right)$$

where g maps the normalized indices $i/N \in [0, 1]$ monotonically increasing onto the angles $\theta \in [0, 2\pi]$ and is periodically continued.

The velocity profile is then $v(\theta) = g'(g^{-1}(\theta)) / T$

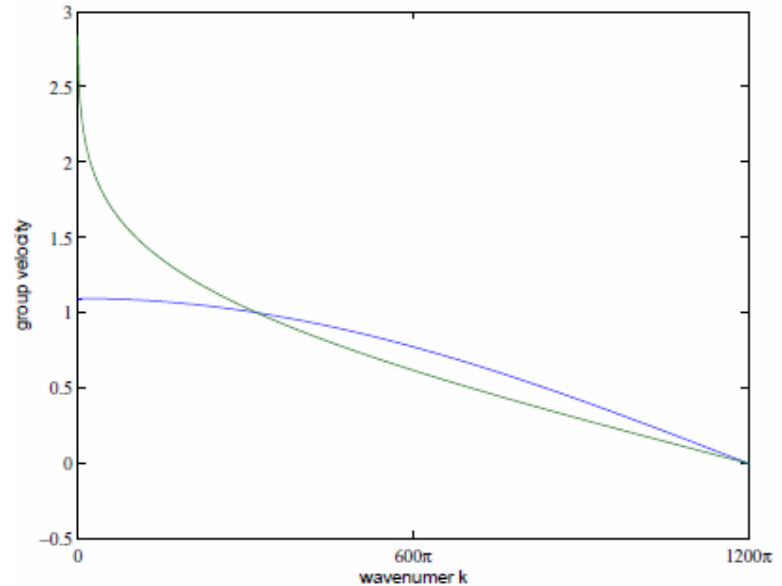
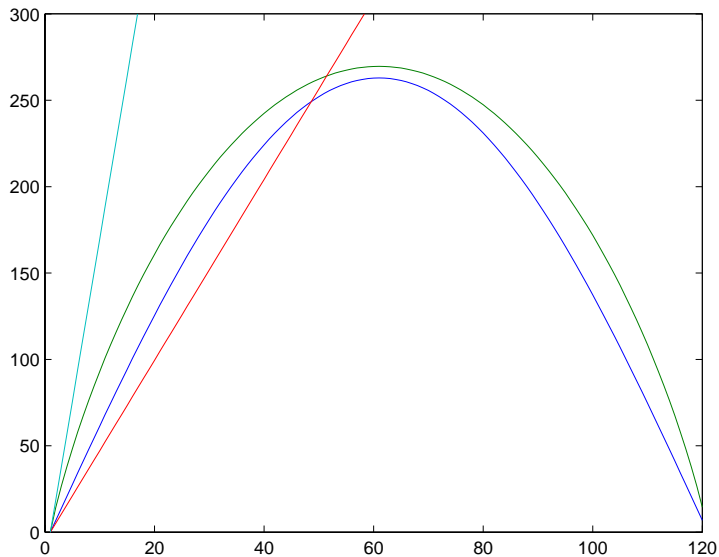
Dynamics

$$\frac{\partial}{\partial t} \langle \hat{\xi}_i \rangle_t = \sum_j G_{ij}(t) \langle \hat{\xi}_j \rangle_t$$

$$\frac{\partial}{\partial t} \Gamma(t) = G(t) \cdot \Gamma(t) + \Gamma(t) \cdot G(t)^T.$$

Note that since the initial state is Gaussian (either thermal or vacuum) The state will remain Gaussian for later times. Hence higher order Correlations can be computed via Wick's decomposition theorem.

Group velocity



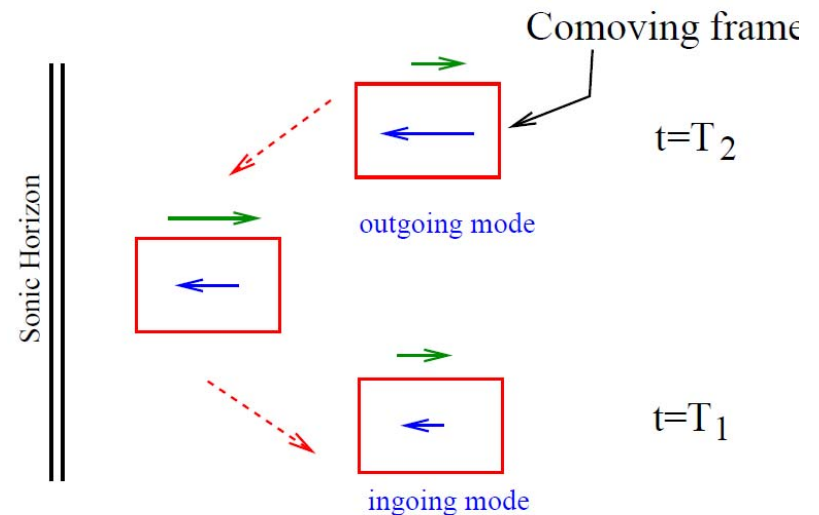
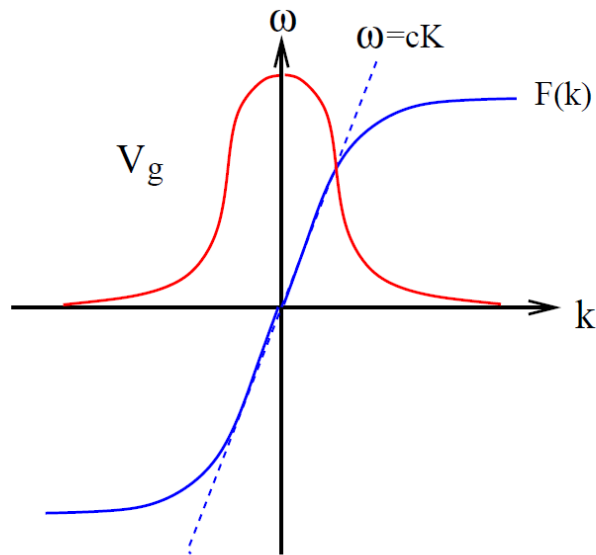
For only nearest neighbor interactions $c = 2e\sqrt{N}$.

or
$$c(\theta) = \sqrt{2(2\pi)^3 n(\theta) e^2 / (4\pi\epsilon_0 \cdot mL^3)}$$

Testing wave packet trajectories

From Unruh's model we expect:

Dispersion relation: $\omega = F(k)$ (fluid rest frame)



- reflection at the horizon
- Hawking's radiation thermal properties remain mostly unaffected
- but in our case k has a maximal value.

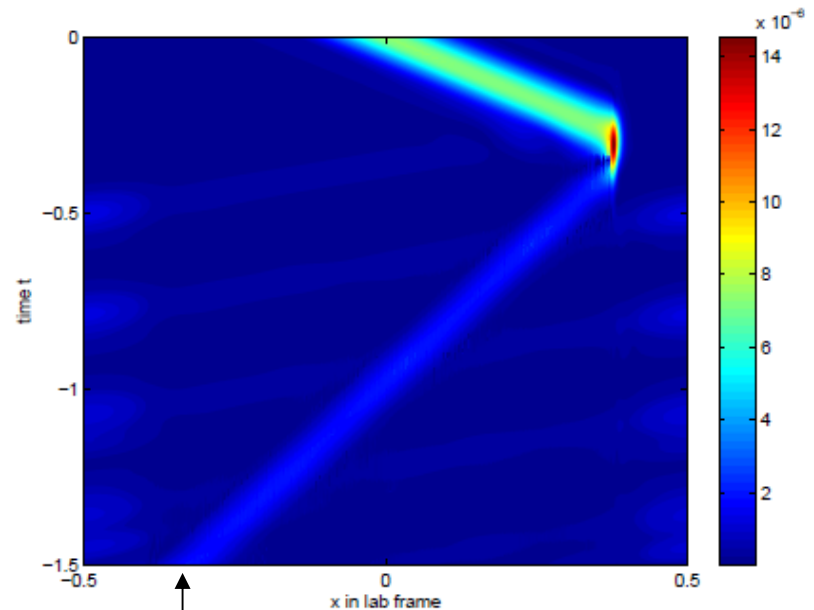
Back in time

$$\delta\theta_k^n(0) = k \cdot e^{-\left(\frac{k-2\pi n^2}{40\pi}\right)}, \quad n = 1, \dots, 20,$$

Values of $\delta\theta_i(t)$ during propagation
backwards in time.

Here we have $v = 0.83$, $k = 10$,
 $N = 1000$, $N2 = 1200$,

Outgoing wave
(negative frequency)



Incoming wave
(Negative and positive frequency)

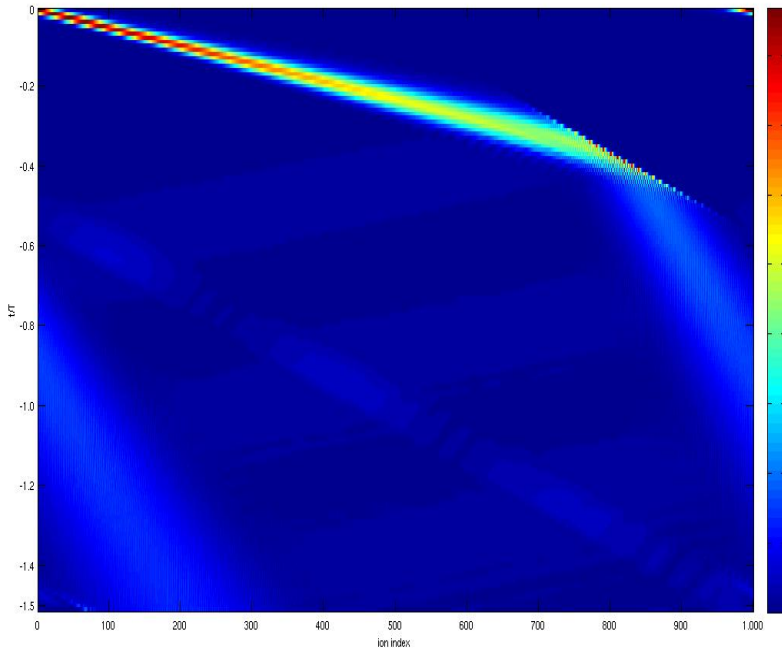
Scattering to Low k modes

We find scattering to a low wavenumber negative frequency mode, on the second branch, which is mostly right moving. This scattering is consistent with the approximate Killing frequency conservation.

It was argued that conformal invariance in 1D prevents scattering.

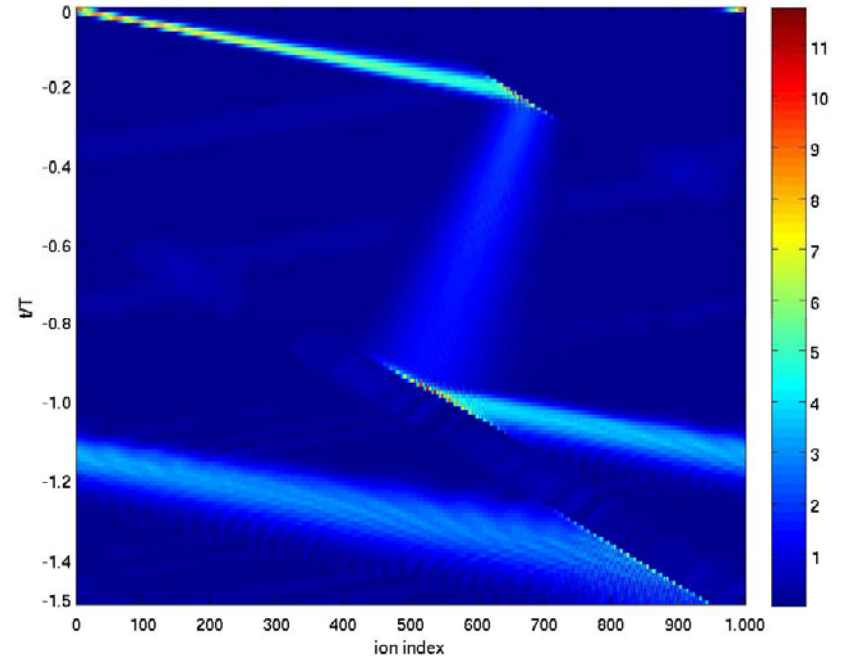
We find that both for the full and truncated models there is a small non-vanishing scattering.

In the commoving frame



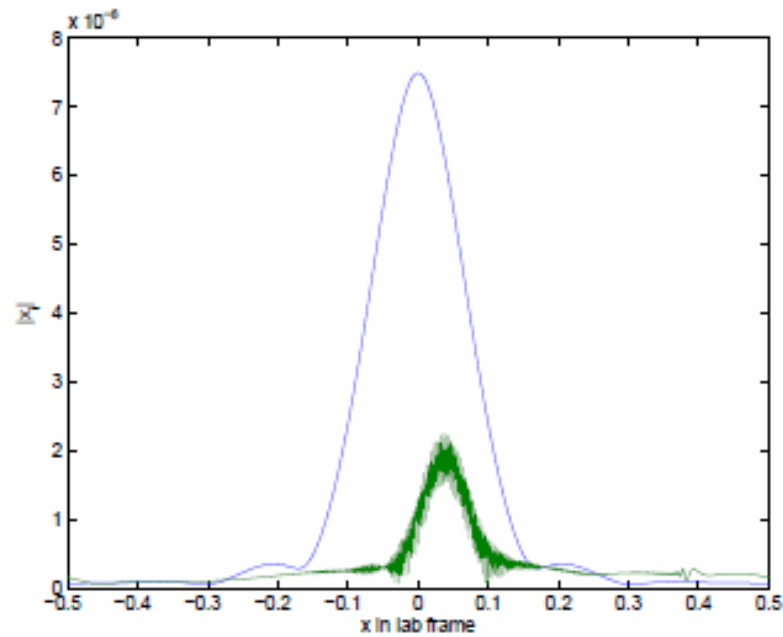
Wave remains a right mover

Unruh's process



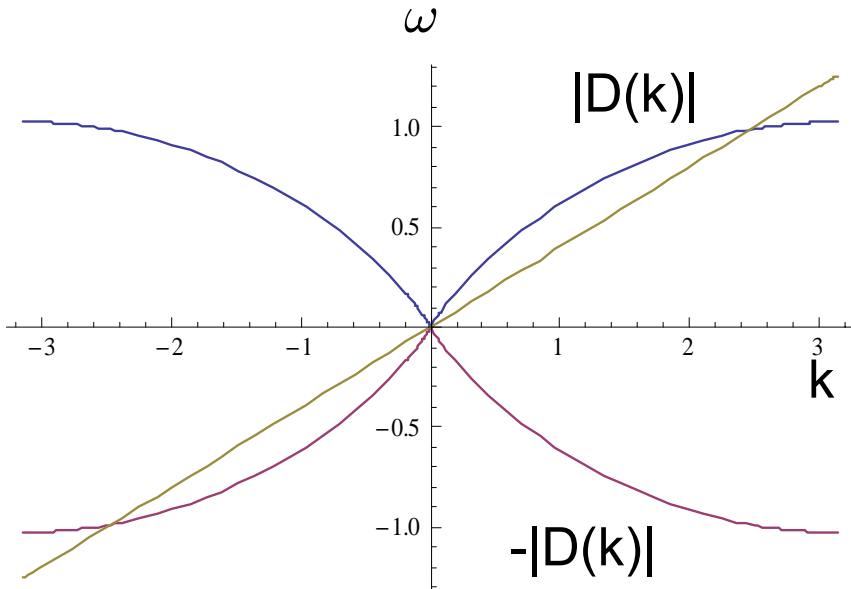
Scattering to a left mover

Corley & Jacobson's Bloch oscillation

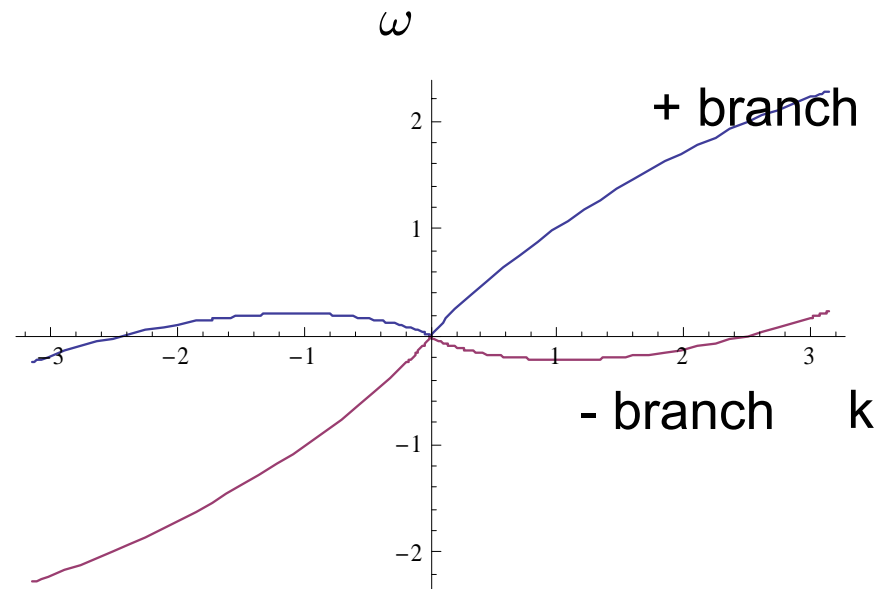


Late time outgoing and early time ingoing distributions $|\Delta x_i(t)|$ in the lab frame.

Positive and negative frequencies



comoving frame



Lab frame

Positivity is determined by the sign of $D(k)$

ω is nearly conserved only if $ka < 1$

$$\omega = kv + |D(k)|$$

$$\omega = kv - |D(k)|$$

Positive and negative modes

If the phononic excitations are localized in the flat subsonic region, the excitations $\delta\theta_i(t) = \langle \delta\theta_i \rangle_t$ and $\delta\dot{\theta}_i(t) = \langle -i\hbar\partial_{\delta\theta_i} \rangle_t$ can be expressed as modes $\delta\theta_k(t)$ and $\delta\dot{\theta}_k(t)$ with wavenumber k . The positive and negative frequency part of these excitations are defined by

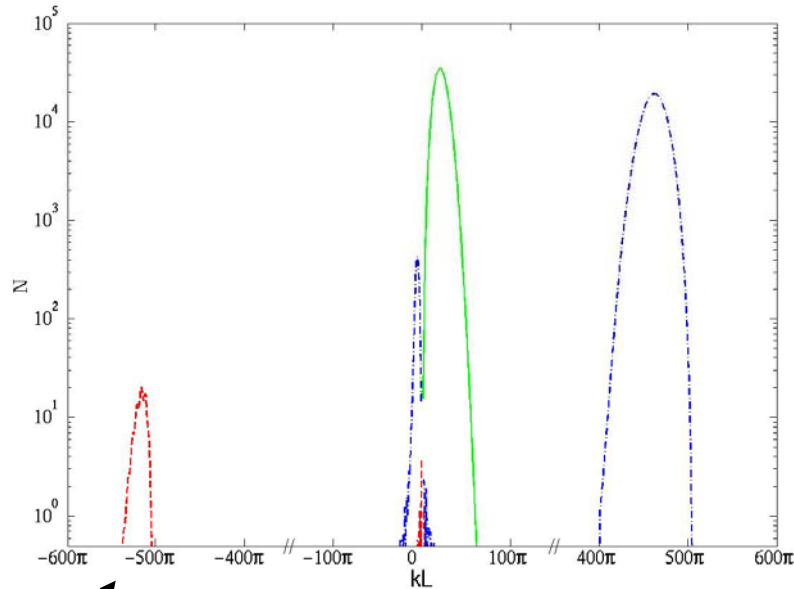
$$\delta\theta_k^\pm(t) = \frac{1}{2} \left(\delta\theta_k(t) \pm i\delta\dot{\theta}_k(t)/\omega_k \right),$$

$$\delta\dot{\theta}_k^\pm(t) = \frac{1}{2} \left(\delta\dot{\theta}_k(t) \mp i\omega_k\delta\theta_k(t) \right).$$

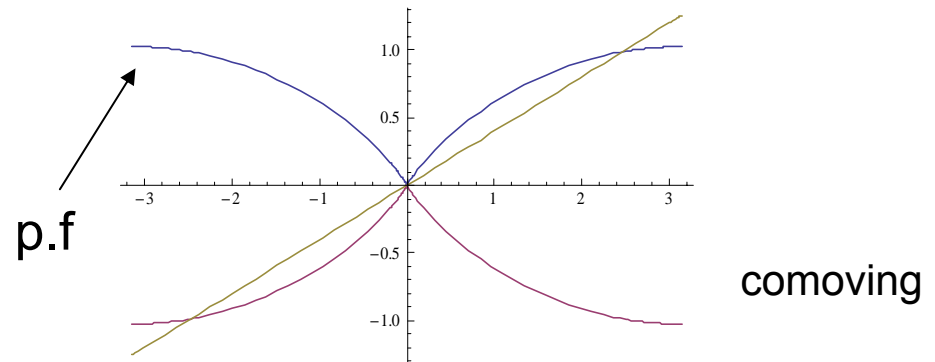
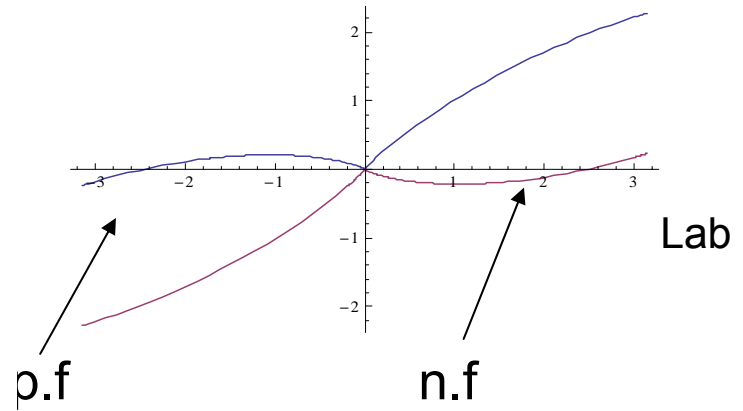
Norm distribution: $\mathcal{N}_k^\pm = \dot{\delta\theta}_k^{\pm*} \delta\theta_k^\pm - \delta\theta_k^{\pm*} \dot{\delta\theta}_k^\pm$

First mechanism: mode conversion

(Unruh 96)
 When the Solutions to $\omega_0 = vk \pm D(k)$ lie within Brillouin zone:

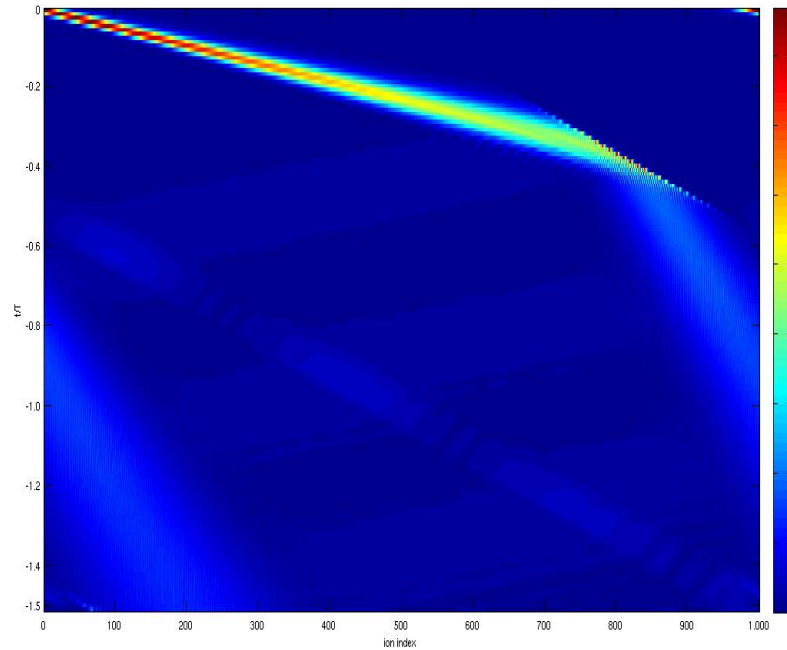


Positive frequency initial mode



Late time (final) negative frequency pulse is depicted in Green. Initial negative frequency pulse in light blue (dash-dotted), and positive frequency pulse in red (dashed).

In the commoving frame

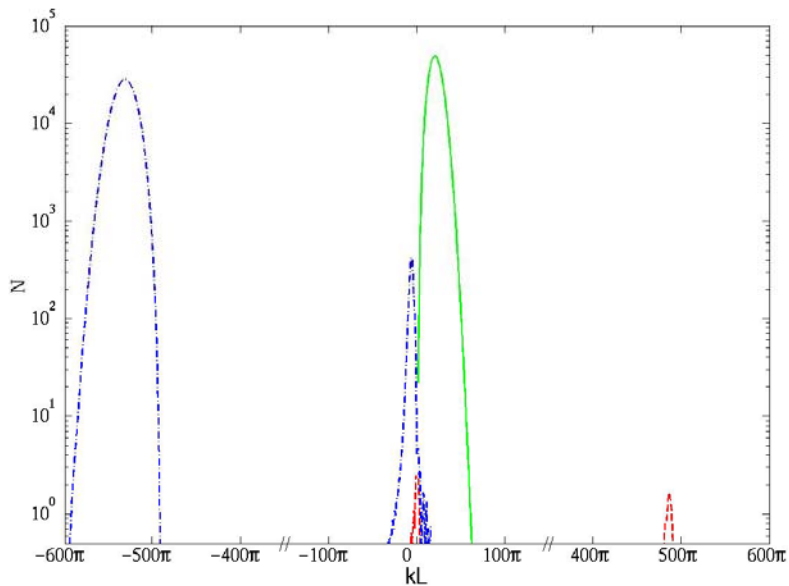


Wave remains a right mover

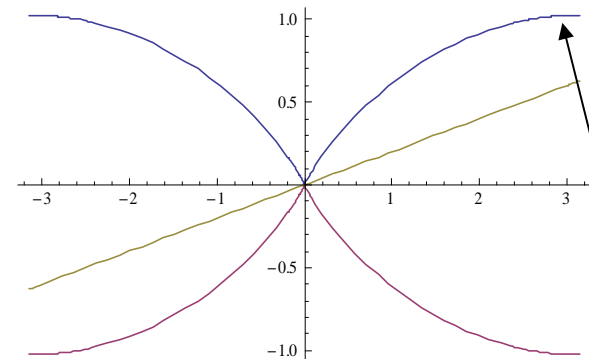
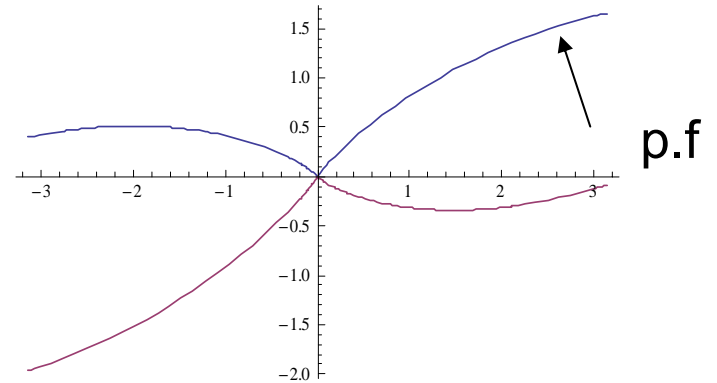
Unruh's process

Second mechanism: Block-like oscillations (Jacobson 98)

Solutions to $\omega_0 = vk \pm D(k)$ lie outside the Brillouin zone:

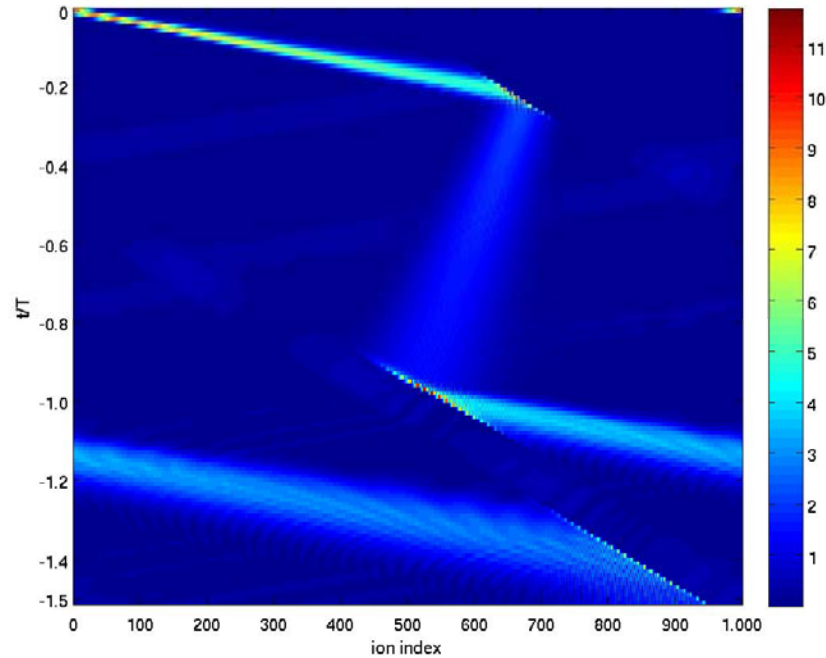


p.f



Smaller velocity

In the commoving frame



Scattering from right to left mover

Testing Hawking's hypothesis

We compare between the occupation number of the outgoing Wave packet under the thermal Hypothesis
And the the Bogoliubov coefficient:

$$\beta_T = \sum_k \frac{\mathcal{N}_k^-(final)}{e^{\frac{\hbar\omega_k}{k_B T}} - 1} = \sum_k \mathcal{N}_k^+(initial)$$

For different late timewaves: $\delta\theta_k^n(0) = k \cdot e^{-\left(\frac{k-2\pi n^2}{40\pi}\right)}$, $n = 1, \dots, 20$,

Which taken to be localized at flat space.

Hawking temperature

The Black Hole horizon is located at $c(\theta_H)=v(\theta_H)$. The temperature T_H of Hawking radiation is given by the expression (c depends on θ)

$$\frac{k_B T_H}{\hbar} = -\frac{1}{4\pi v} \frac{d}{d\theta} (c^2 - v^2) \Big|_H = \frac{1}{2\pi} \frac{d}{d\theta} (v - c) \Big|_H .$$

In the case of a Coulomb chain with nearest neighbor interactions only, where the sound velocity of an homogeneous system $c=\sqrt{2e^2N}$, it can be evaluated in the local density approximation as

$$\frac{k_B T_H}{\hbar} = \frac{3}{4\pi} \frac{g''(g^{-1}(\theta))}{g'(g^{-1}(\theta))} \Big|_{\theta=\theta_H}$$

Numerical results

1. If only nearest neighbor Coulomb interactions are considered, the relative difference between these quantities is lower than $\epsilon < 0.01$ in our calculations with up to $N=1000$ ions.
2. For the long range Coulomb interactions it is of the order of $\epsilon < 0.14$. Here the definition of a global Hawking temperature is difficult because of the non-linear dispersion relation at low wavenumbers. Possibly also due to the “non-conformal” scattering.
3. We have checked, that anharmonic effects do not significantly alter these results for oscillation amplitudes comparable to $\langle \theta^2 \rangle$ at the Hawking temperature.

Therefore qualitatively Hawking radiation (mode mixing) seems to persist even in a fundamentally discrete system with long range interactions and a logarithmically diverging group velocity at low wavenumbers

Experimental Parameters

The ion velocity must lie in the same order of magnitude as the phonon velocity in the proposed experimental setup.

For $N=1000$ singly charged ${}^7\text{Li}$ ions with an average spacing of $L/N=10\mu\text{m}$ the rotation frequency of the ions must be $\omega_{ion}=6.3\text{kHz}$.

The Hawking temperature in this system is

$$k_B T_H / \hbar = 9.8 \omega_{ion} = \underline{62\text{kHz}}.$$

These parameters and such a temperature can be realized experimentally.

Creation of a black hole

Experimental sequence to measure evidence for Hawking radiation on ion rings:

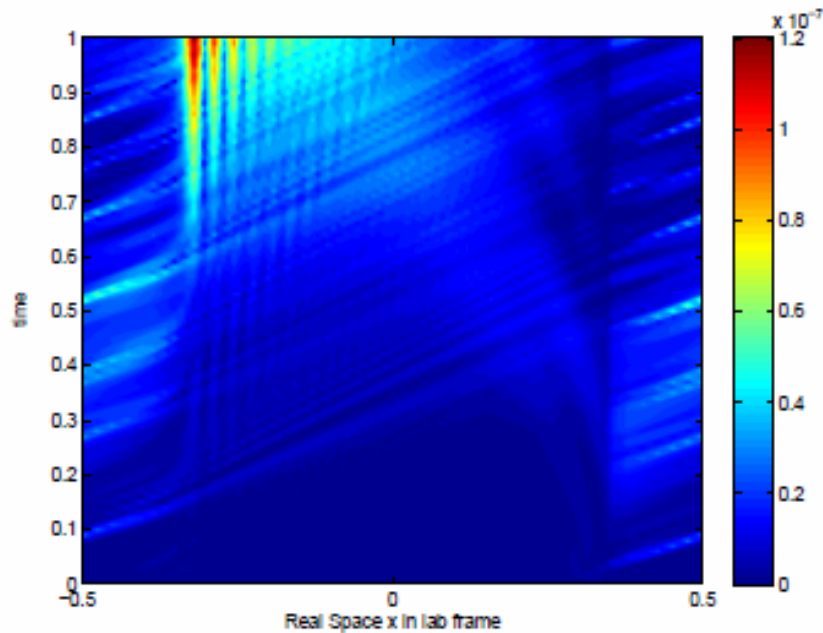
Begin with a thermal state of the excitations around homogeneously spaced subsonic rotating ions.

Create in a short time a supersonic region to avoid white hole and finite size effects. We change the velocity profile parameter in an exponentially smooth way.

In our case we used $0.01 < \frac{T_{creation}}{T} < 0.1$

So we have a time of order up to $0.5 T$ to observe the radiation.

in Stability



Deviation of the ions position relative to equilibrium as function Of time. BH Creation time is here ~ 0.01 .

Correlations

We analyze the correlations obtained from the covariance matrix Γ

Local creation operators:
$$a_i^\dagger = \frac{1}{\sqrt{2\hbar}} (f_{ii}^{1/4} \delta\theta_i - i f_{ii}^{-1/4} \delta p_i)$$

$$C_{ij}(t) = \frac{\langle (\hat{n}_i - \langle \hat{n}_i \rangle) (\hat{n}_j - \langle \hat{n}_j \rangle) \rangle}{\langle \hat{n}_i \rangle \langle \hat{n}_j \rangle}$$

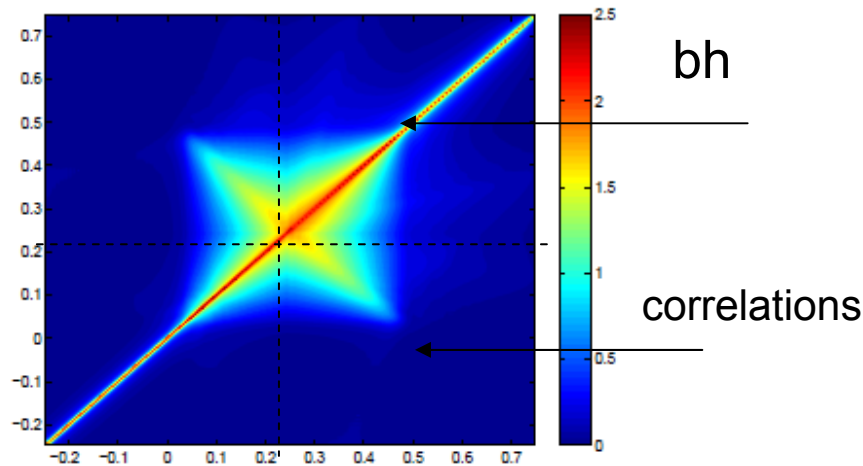
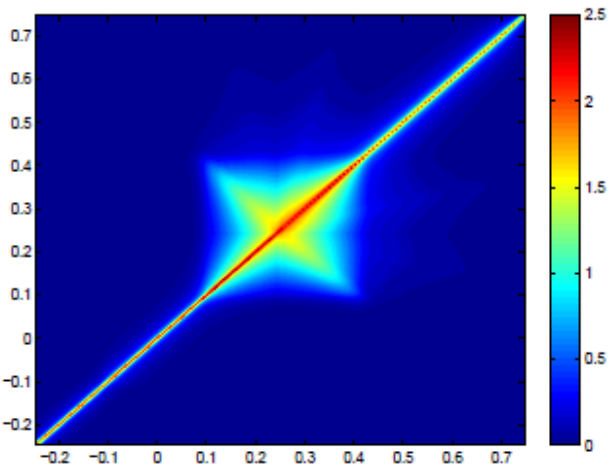
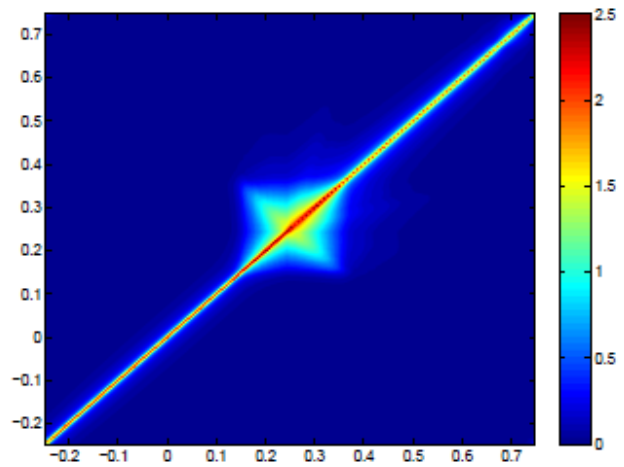
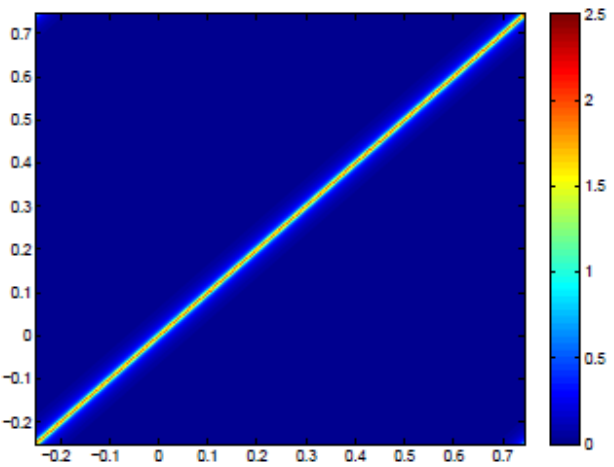
Where we use Wick's theorem:

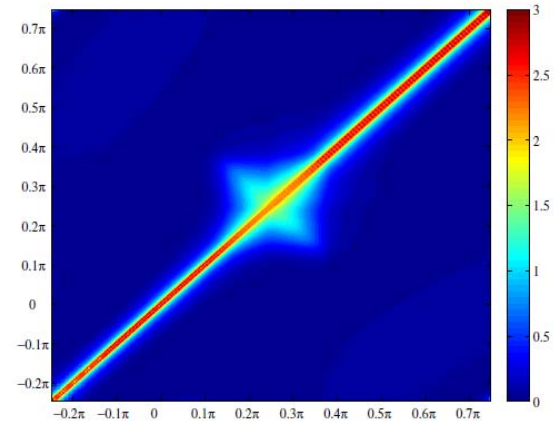
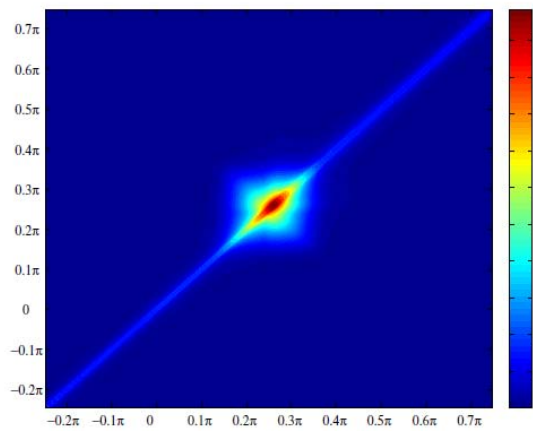
$$\langle a_i^\dagger a_j^\dagger a_i a_j \rangle = \langle a_i^\dagger a_i \rangle \langle a_j^\dagger a_j \rangle + \langle a_i^\dagger a_j^\dagger \rangle \langle a_i a_j \rangle + \langle a_i^\dagger a_j \rangle \langle a_j^\dagger a_i \rangle$$

Note that the interpretation is quite different compared to the BEC case!

Density-density correlations

$t=0,0.1,0.2,0.3,0.4T$ $R=2.25$

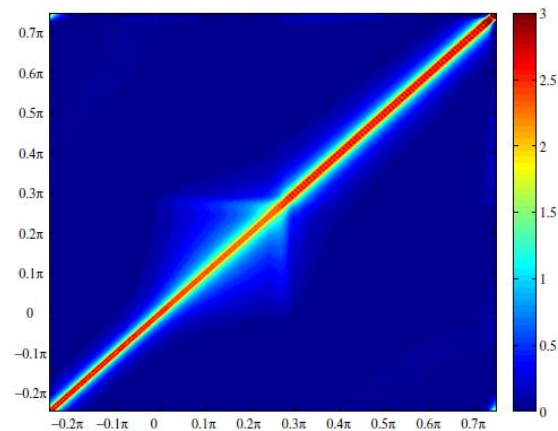
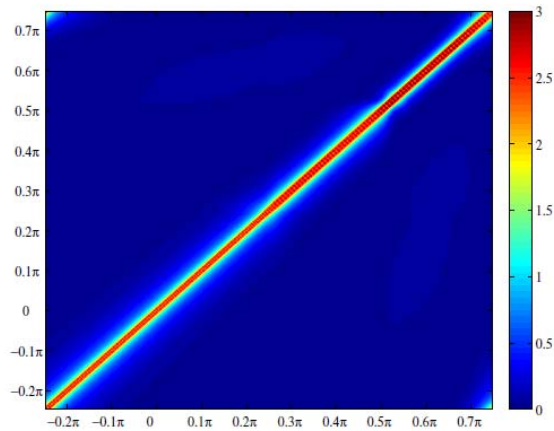




On the left Non-normalized correlations. $t=0.4T$

Possible breakdown of the harmonic approximation.

Testing kinematics by modifying the charge e^2



The Black hole is not created.

Different in/out relative velocities

Summary

- (a) We suggest another avenue towards realization of an analogue BH with cold ions.
- (b) This fully discrete Physical analogue model agrees with previously suggested theoretical models.
- (c) It appears to be accessible in today's experiments with ions.
- (d) Various other tests can be easily done in this model.