Spectral Properties of Hawking Radiation in BECs (work in progress)

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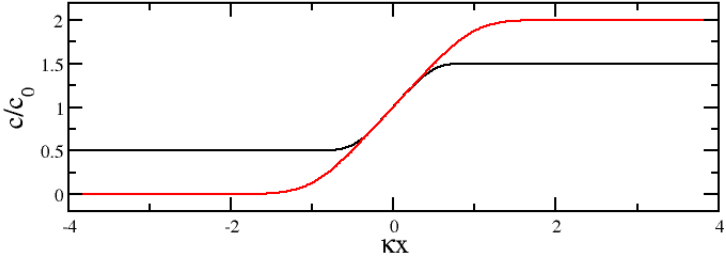
Workshop: *Toward observation of HR in BECs*, Valencia 2009

Set up (Carusotto et al., 2008)

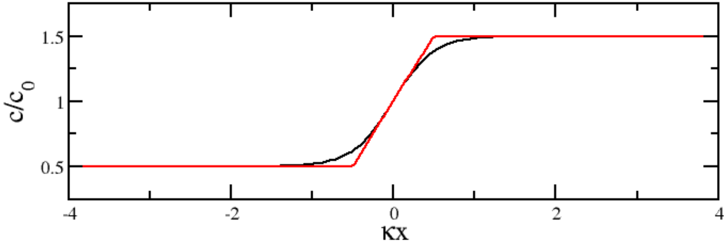
1D BEC, uniformly flowing with $v_0 = -c_0$, constant $V + g\rho \rightarrow$ constant density ρ_0, with varying sound speed $c(x)$ (*i.e.* varying coupling constant $g(x)$)

$$\frac{c}{c_0}(x; \kappa, n, D) = 1 + D\,\mathrm{sign}(x)\,\tanh^{1/n}\left[\left(\frac{\kappa|x|}{c_0 D}\right)^n\right].$$

n=2, D=0.5 and D=1



D=0.5, n=0.5 and n=20



Bogolubov-de Gennes equations.

- Production of quasi-particles governed by the Bogolubov-de Gennes equations. At fixed $\tilde{\omega} = \omega/\kappa$, in coordinate $\tilde{x} = \kappa x/c_0$:

$$
\begin{aligned}
(\tilde{\omega} - i\partial_{\tilde{x}})\phi_{\tilde{\omega}} &= \left(-\frac{1}{\lambda}\partial_{\tilde{x}}^2 + \frac{\lambda\tilde{c}^2}{2}\right)\phi_{\tilde{\omega}} + \frac{\lambda\tilde{c}^2}{2}\varphi_{\tilde{\omega}} \\
(\tilde{\omega} - i\partial_{\tilde{x}})\varphi_{\tilde{\omega}} &= \left(\frac{1}{\lambda}\partial_{\tilde{x}}^2 - \frac{\lambda\tilde{c}^2}{2}\right)\varphi_{\tilde{\omega}} - \frac{\lambda\tilde{c}^2}{2}\phi_{\tilde{\omega}}
\end{aligned}
$$

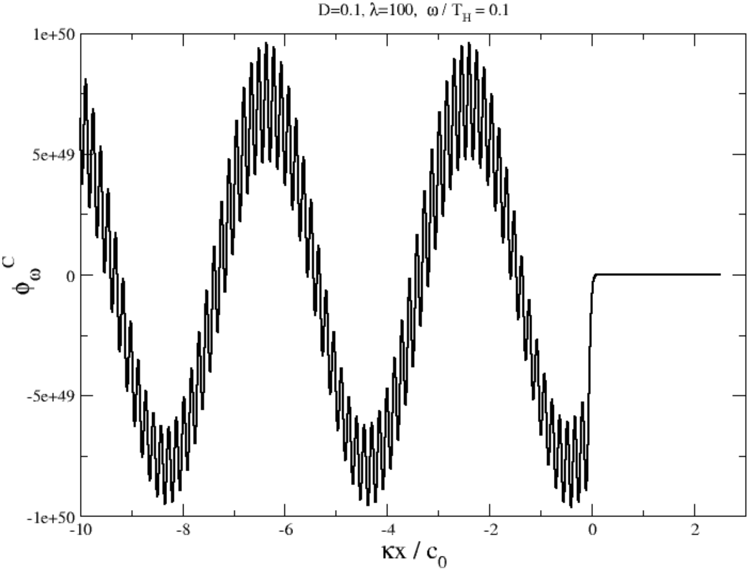
- Adimensional parameters: $\kappa\lambda = \Lambda = 2mc_0^2/\hbar$, related to the healing length: $\Lambda = c_0/\xi$.
- κ does not appear explicitely $\longrightarrow$ results true for all κ, need only rescaling to get actual values for given κ.

Asymptotic solutions

- In the asymptotic regions $x \to \pm\infty$, normalized solutions

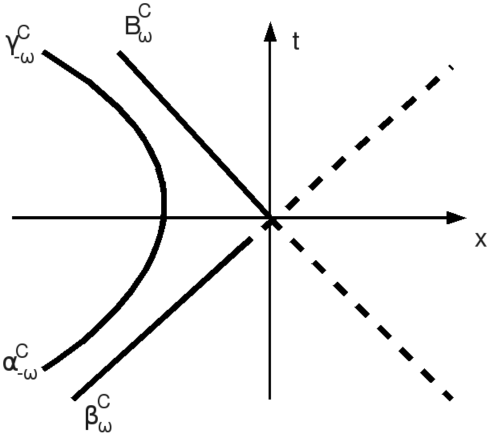
$$
\begin{aligned}
\phi_\omega &= \frac{1}{\sqrt{|d\omega/dk|}\sqrt{1-D_{k,\omega}^2}}e^{ikx} \\
\varphi_\omega &= \frac{D_{k,\omega}}{\sqrt{|d\omega/dk|}\sqrt{1-D_{k,\omega}^2}}e^{ikx} \\
D_{k,\omega} &= 2(\omega+k)/\lambda c^2 - 2k^2/\lambda^2 c^2 - 1
\end{aligned}
$$

- k solution of the dispersion relation ($c_\pm = 1 \pm D$): $(\omega + k)^2 = k^2 c_\pm^2 + k^4/\lambda^2$.
- In supersonic region, for $\omega < \omega_{max}$, 4 oscill. solutions; in subsonic region, 2 oscill.+ growing + decaying.



D=0.1, λ=100, ω / T_H = 0.1
1e+50
5e+49
0
-5e+49
-1e+50
-10
-8
-6
-4
-2
0
2
ϕ_ω^C
$\kappa x / c_0$

Corley mode (Corley & Jacobson, 1996, and subsequent papers by Corley)



Corley mode.

- Link between β_ω^C, β_ω, and/or $\beta_{-\omega}$?
- A bit of algebra gives

$$\frac{|\beta_\omega^C|^2 - |\beta_{-\omega}|^2}{|\beta_\omega^C|^2} = O\left[\frac{|B_\omega^C|^2}{|\beta_\omega^C|^2}\right]$$

(with the reasonable hypothesis that $|\tilde{B}_\omega|$ is of the same order of magnitude as $|B_\omega^C|$).

- $\rightarrow$ *sufficient* criterium to have $|\beta_\omega^C|^2 \simeq |\beta_\omega|^2 \simeq |\beta_{-\omega}|^2$:

$$\frac{|B_\omega^C|^2}{|\beta_\omega^C|^2} \ll 1$$

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Outline

Typical values of the parameters (Dalfovo et al., 1999)

- For ^{87}Rb, typical $c_0 \simeq 5\mathrm{mm}\cdot\mathrm{s}^{-1}$, $m \simeq 1.5 \cdot 10^{-25}\mathrm{kg}$. Gives

$$\Lambda = \frac{2mc_0^2}{\hbar} \simeq 10^4\mathrm{s}^{-1}$$

.

- Parameters considered in Carusotto *et al.* 2008 give $D = 0.3$ and $\kappa \simeq 10^4\mathrm{s}^{-1}$.
- $\lambda = \frac{\Lambda}{\kappa} \simeq 1$!
- In the following, when the goal is to predict experimental results, $\lambda = 1, 10, 100$, $D \simeq 0.1$.

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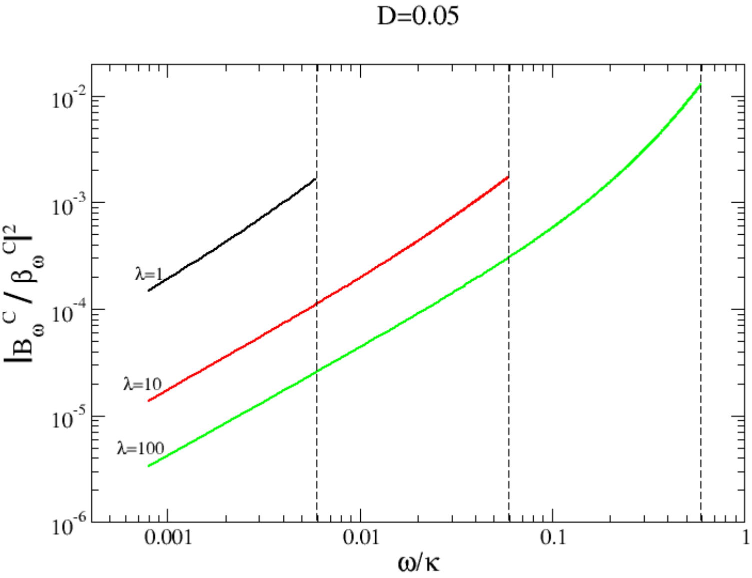
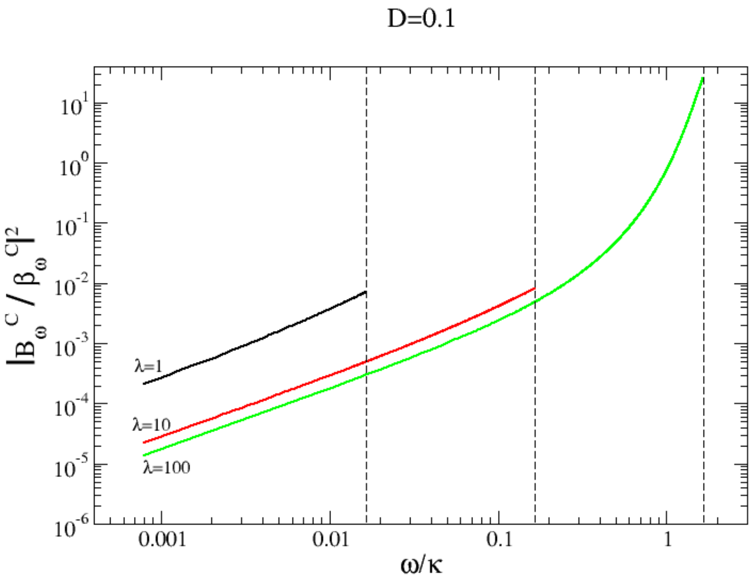
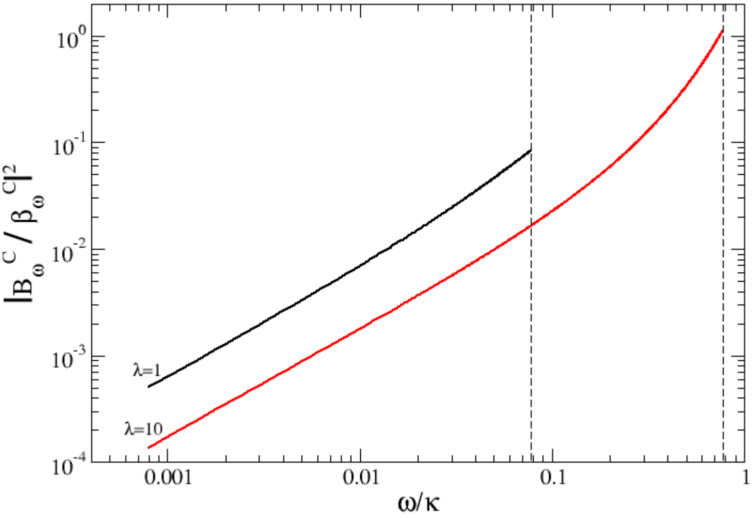
Outline


D=0.05
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
$|B_\omega^C/\beta_\omega^C|^2$
λ=1
λ=10
λ=100
0.001
0.01
0.1
1
ω/κ


D=0.1
$|B_\omega^C/\beta_\omega^C|^2$
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
λ=1
λ=10
λ=100
0.001
0.01
0.1
1
ω/κ

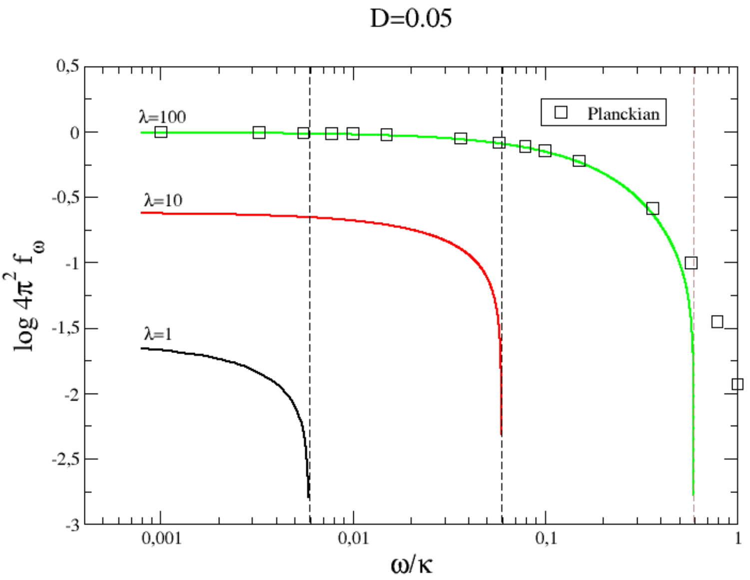
D=0.3


$|B_\omega^C/\beta_\omega^C|^2$
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
λ=1
λ=10
0.001
0.01
0.1
1
ω/κ

Outline

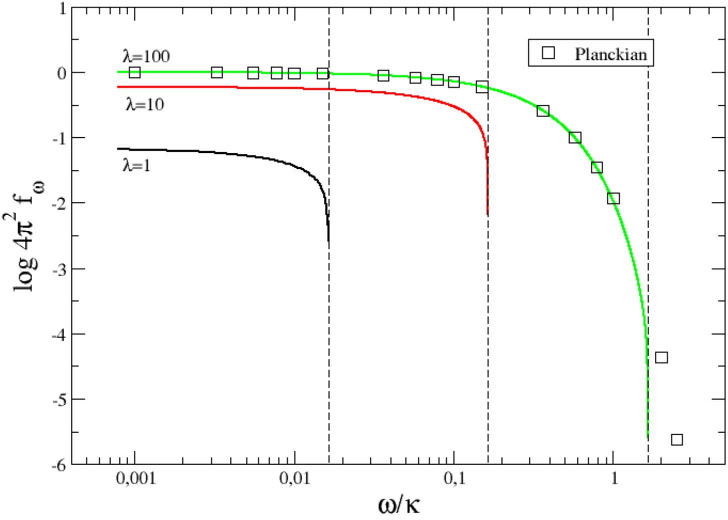
In the following, f_ω designates the energy flux per frequency interval $d\omega$:

$$f_\omega = \frac{\tilde{\omega}}{2\pi} |\beta_\omega^C|^2$$

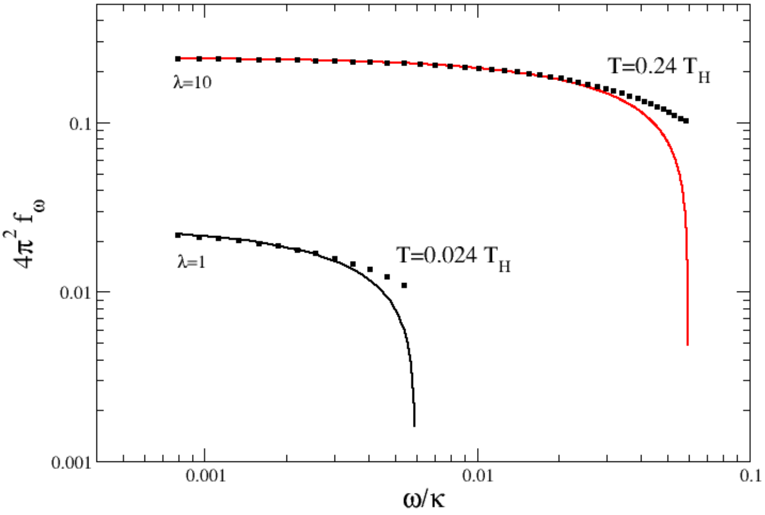


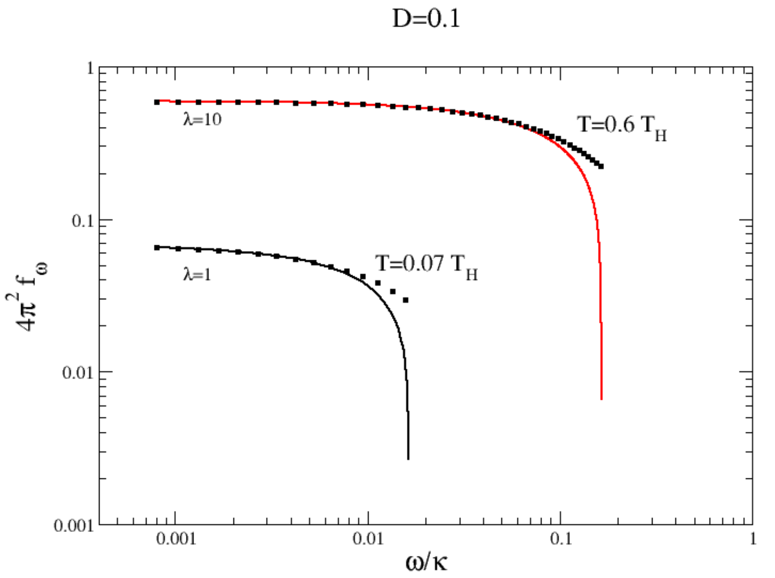
D=0.05
Planckian
λ=100
λ=10
λ=1
0,5
0
-0,5
-1
-1,5
-2
-2,5
-3
0,001
0,01
0,1
1
log 4π² f_ω
ω/κ

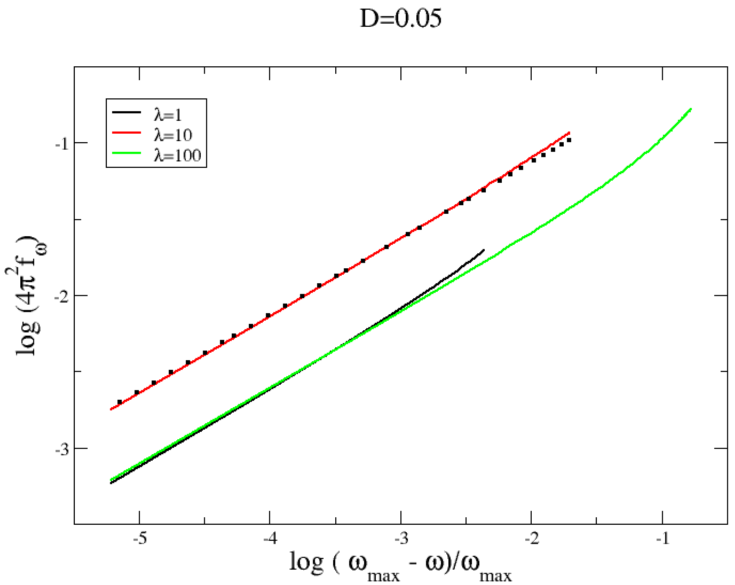
D=0.1



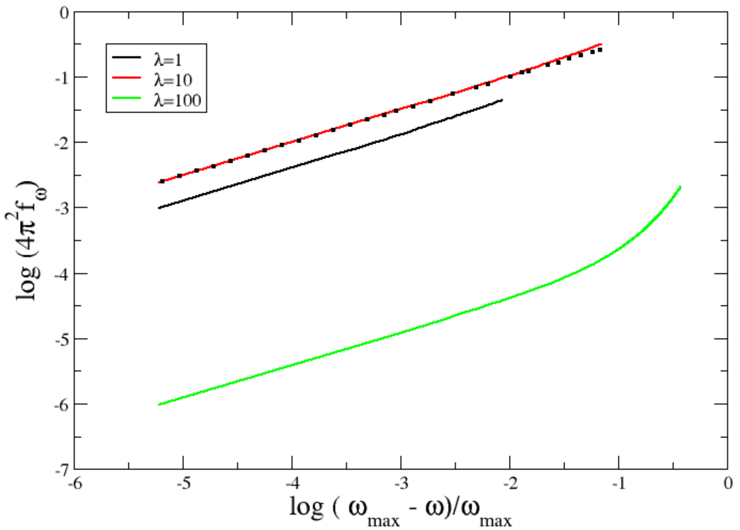
D=0.05



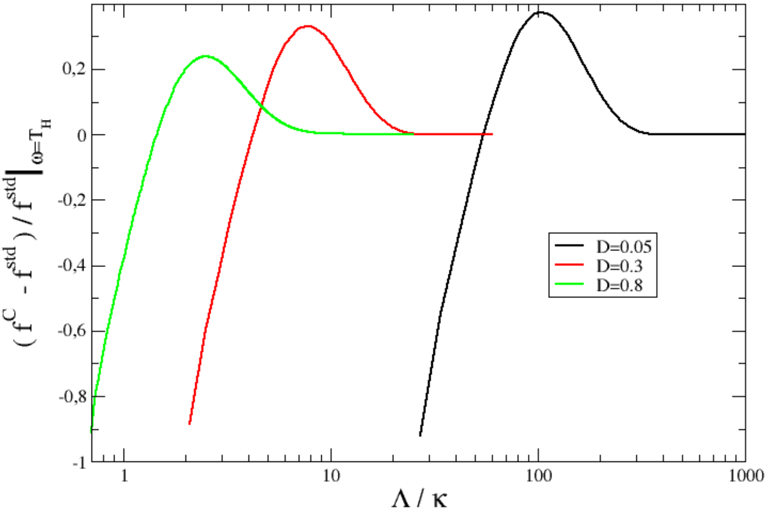

D=0.1
1
0.1
0.01
0.001
$4\pi^2 f_\omega$
λ=10
λ=1
T=0.6 T$_H$
T=0.07 T$_H$
0.001
0.01
0.1
1
ω/κ

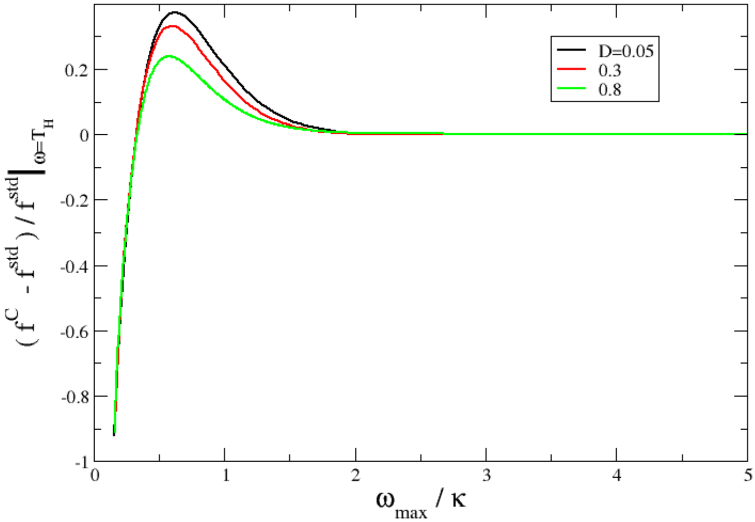

D=0.05
λ=1
λ=10
λ=100
-1
-2
-3
-5
-4
-3
-2
-1
log (4π²f_ω)
log (ω_max - ω)/ω_max

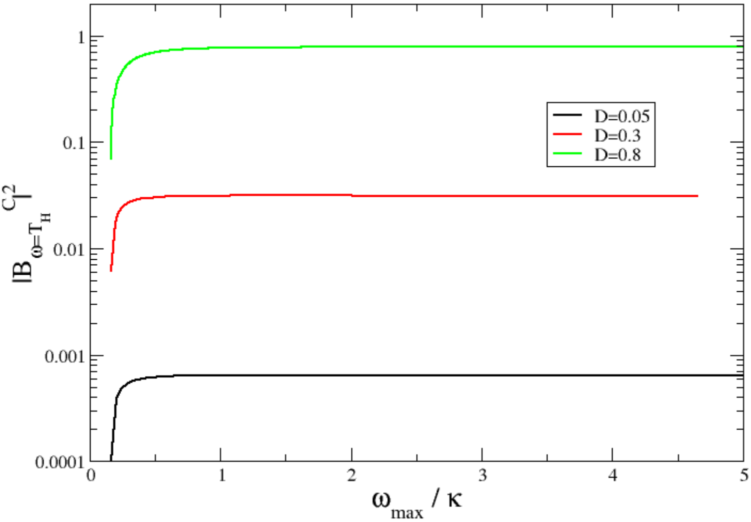
D=0.1


λ=1
λ=10
λ=100
log $(4\pi^2 f_\omega)$
log $(\omega_{max} - \omega)/\omega_{max}$
0
-1
-2
-3
-4
-5
-6
-7
-6
-5
-4
-3
-2
-1
0

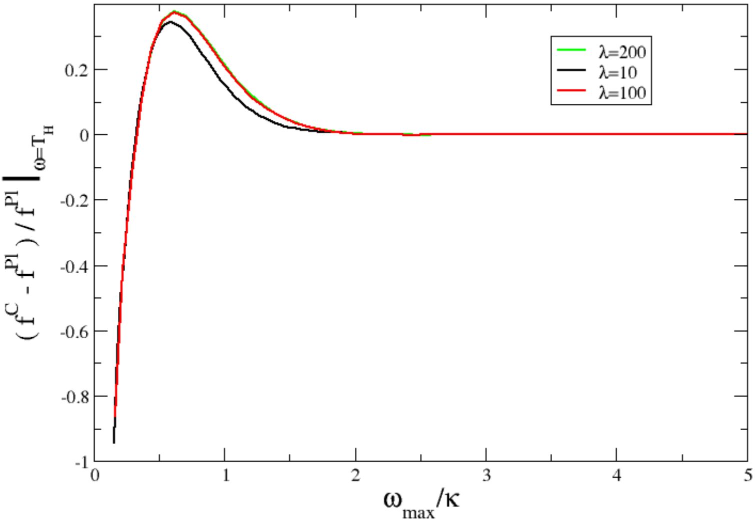
Outline

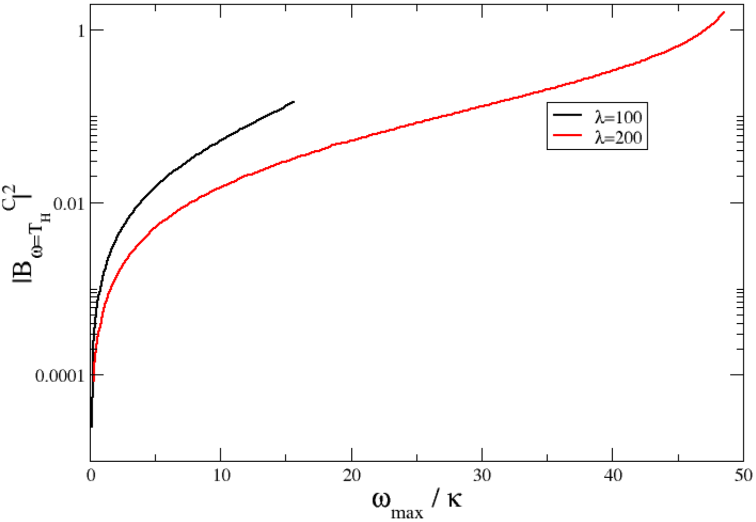

0,2
0
-0,2
-0,4
-0,6
-0,8
-1
1
10
100
1000
Λ / κ
$(f^C - f^{std}) / f^{std} \big|_{\omega = T_H}$
D=0.05
D=0.3
D=0.8


D=0.05
0.3
0.8
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
1
2
3
4
5
ω_{max} / κ
$(f^{C} - f^{std}) / f^{std} |_{\omega=T_H}$

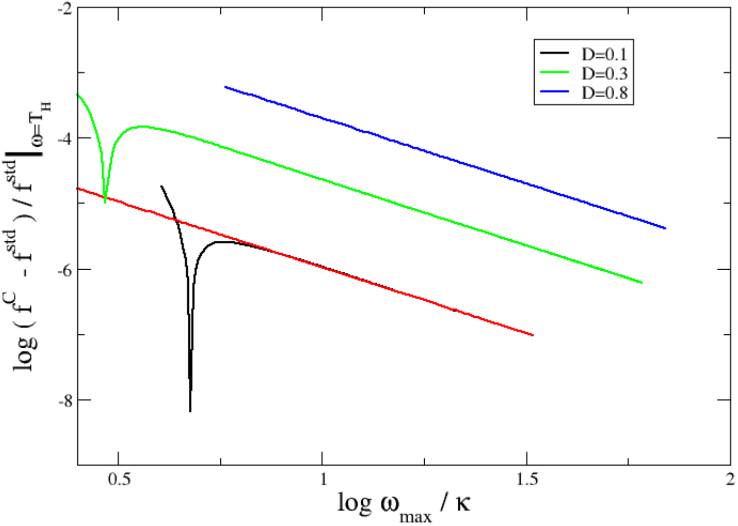


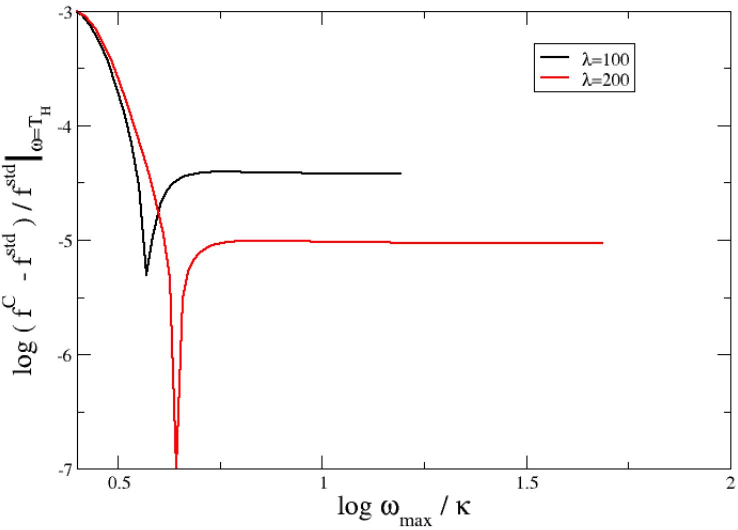
$|B_{\omega=T_H}^C|^2$
ω_{max} / κ
D=0.05
D=0.3
D=0.8
1
0.1
0.01
0.001
0.0001
0
1
2
3
4
5


λ=200
λ=10
λ=100
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
1
2
3
4
5
$(f^C - f^{Pl}) / f^{Pl} |_{\omega=T_H}$
ω_{max}/κ


λ=100
λ=200
1
0.01
0.0001
0
10
20
30
40
50
$|B_{\omega=T_H}^C|^2$
ω_{max} / κ

Outline


D=0.1
D=0.3
D=0.8
-2
-4
-6
-8
0.5
1
1.5
2
log (f^C - f^std) / f^std |ω=T_H
log ω_max / κ



λ=100
λ=200
-3
-4
-5
-6
-7
0.5
1
1.5
2
$\log\,(f^C - f^{std})\,/\,f^{std}\,\big|_{\omega=T_H}$
$\log\,\omega_{max}\,/\,\kappa$

Conclusions

- In BEC experiments the spectra could be non-standard ($T \neq T_H$), unless one works with very small κ.
- Closer to dispersion-free spectrum if D close to 1 (large variation of $c(x)$), but LARGE non-adiabatic effects and expected grey-body factors+strong v particle creation.
- ω_{max}/κ, NOT Λ/κ governs the influence of dispersion.
- The corrections to the dispersion-free case, scale as λ^{-2}.

- Limitations of our approach:
  - Corley mode is not what we would have liked...
  - No quantum backaction.

Numerical procedure

- In subsonic region: $\phi^C_{-\omega} = u_0 e^{ik^C_{-\omega}x}$, $\varphi^C_{-\omega}$ fixed by the BdG eqs.
- Integrate deep into supersonic region$\rightarrow \phi^C_{-\omega} = \sum_j c^j e^{ik^j_\omega x}$,

$$
\begin{aligned}
c^{u,out}_{-\omega} &= N\frac{\gamma^C_{-\omega}}{\sqrt{|d(-\omega)/dk^{u,out}_{-\omega}|}\sqrt{1-D^2_{k^{u,out}_{-\omega}}}} \\
c^{v,out}_{\omega} &= N\frac{B^C_\omega D_{k^{v,out}_\omega}}{\sqrt{|d\omega/dk^{v,out}_\omega|}\sqrt{1-D^2_{k^{v,out}_\omega}}}
\end{aligned}
$$

- normalized mode: $|\gamma^C_{-\omega}|^2 - |B^C_\omega|^2 = 1$, so:

$$
N^2 = |c^{u,out}_{-\omega}|^2\left|\frac{d(-\omega)}{dk^{u,out}_{-\omega}}\right|(1-D^2_{k^{u,out}_{-\omega}}) - |c^{v,out}_\omega|^2\left|\frac{d\omega}{dk^{v,out}_\omega}\right|\frac{1-D^2_{k^{v,out}_\omega}}{D^2_{k^{v,out}_\omega}}
$$

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$$

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$$N^2 = |c^{u,out}_{-\omega}|^2 \left|\frac{d(-\omega)}{dk^{u,out}_{-\omega}}\right| (1-D^2_{k^{u,out}_{-\omega}}) - |c^{v,out}_{\omega}|^2 \left|\frac{d\omega}{dk^{v,out}_{\omega}}\right| \frac{1 - D^2_{k^{v,out}_\omega}}{D^2_{k^{v,out}_\omega}}$$

- Once N is known, $|B_{\omega}^{C}|^2$ and $|\beta_{-\omega}^{C}|^2$ given by:

$$
\begin{aligned}
|\beta_{-\omega}^{C}|^2 &= \frac{1}{N^2}|c_{\omega}^{u,in}|^2 \left|\frac{d\omega}{dk_{\omega}^{u,in}}\right| \frac{1 - D^2_{k_{\omega}^{u,in}}}{D^2_{k_{\omega}^{u,in}}} \\
|B_{\omega}^{C}|^2 &= \frac{1}{N^2}|c_{\omega}^{v,out}|^2 \left|\frac{d\omega}{dk_{\omega}^{v,out}}\right| \frac{1 - D^2_{k_{\omega}^{v,out}}}{D^2_{k_{\omega}^{v,out}}}
\end{aligned}
$$