Sensitivity of Hawking radiation to superluminal dispersion relations

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Modified dispersion relations

- Effects of (Lorentz breaking) superluminal dispersion relations on Hawking radiation produced by collapsing configurations.
- Hawking's original derivation rested on the assumption that the low-energy laws of physics, and in particular Lorentz invariance, are preserved up to arbitrarily large scales.
- Robustness: Analyze effective field theories with high-energy modifications of the dispersion relations.
  - ✓ Subluminal modifications (under reasonable assumptions)
    - dampen the influence of ultra-high frequencies;
    - do not explore arbitrarily large frequencies.
  - ? Superluminal modifications magnify the influence of ultra-high energies.

Superluminal dispersion relations

- Superluminal is qualitatively different to subluminal:
  - The horizon lies ever closer to the singularity for increasing frequencies. This causes the interior of the (zero-frequency) horizon to be exposed to the outside world.
  - Boundary conditions at the horizon $\Rightarrow$
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  - Moreover, if quantum effects remove the general relativistic singularity, a critical frequency might appear above which no horizon would be experienced at all.

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  - Moreover, if quantum effects remove the general relativistic singularity, a critical frequency might appear above which no horizon would be experienced at all.
- Our approach:
  - Hawking derivation through the relation between the asymptotic past and future in a collapsing configuration.
  - No extra assumptions on the asymptotic regions (only standard ones: Minkowski geometry in the past and flatness at spatial infinity also in the future).

Summary of results

Differences in the late-time radiation (superluminal vs. relativistic)

- At any instant, above a critical frequency, there is no horizon. This induces a cutoff in the modes contributing to radiation.
  - Intensity is lower even if the critical frequency is well above the Planck scale.
  - Radiation will extinguish as time advances.

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  - Intensity is lower even if the critical frequency is well above the Planck scale.
  - Radiation will extinguish as time advances.
- Surface gravity is frequency-dependent and the radiation depends on the physics inside the black hole.
  The radiation spectrum undergoes a strong qualitative modification:
  - High-frequency radiation is not negligible compared to the low-frequency thermal part, but can even become dominant.
  - This effect becomes more important with increasing critical frequency.

Standard Hawking radiation

Geometry — collapse

- Painlevé-Gullstrand $1+1$ spacetime

$$\mathrm{d}s^2 = -[c^2 - v^2(t,x)]\mathrm{d}t^2 - 2v(t,x)\mathrm{d}t\mathrm{d}x + \mathrm{d}x^2 \ ,$$

  - regular at the horizon
  - c = speed of light, v = velocity of free-fall
  - acoustic models: c = speed of sound, v = flow velocity

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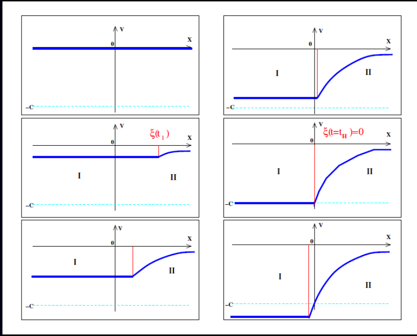
- Schwarzschild-type velocity profile $\bar{v}(x)$ (only qualitative features are relevant)

$$\bar{v}(x) = -c\sqrt{\frac{2M/c^2}{x + 2M/c^2}}$$

$$v(t,x) = \begin{cases} \bar{v}(\xi(t)), & x \leq \xi(t), \\ \bar{v}(x), & x \geq \xi(t). \end{cases}$$



Wave equation — inner product

[Standard Hawking radiation]

- Wave equation: $(\partial_t + \partial_x v)(\partial_t + v\partial_x)\phi = c^2\partial_x^2\phi$
- Equivalent to $3+1$ spherical symmetry if backscattering (grey-body factors) is ignored
- Klein-Gordon product: $(\varphi_1, \varphi_2) \equiv -i \int_{\Sigma_t} \mathrm{d}x \; \varphi_1 \overleftrightarrow{\partial_t} \varphi_2^*$

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- Future null coordinates

$$u(t,x) \to t - x/c\ , \quad w(t,x) \to t + x/c, \quad \text{when} \quad t, x \to +\infty$$

- Independent of Σ_t. For $t \to \infty$,

$$(\varphi_1, \varphi_2) = -\frac{ic}{2}\left\{\int_{-\infty}^{+\infty} \mathrm{d}u\ \left[\varphi_1\partial_u\varphi_2^* - \varphi_2^*\partial_u\varphi_1\right]_{w=+\infty} + \int_{-\infty}^{+\infty} \mathrm{d}w\ \left[\varphi_1\partial_w\varphi_2^* - \varphi_2^*\partial_w\varphi_1\right]_{u=+\infty}\right\}.$$

- Likewise for past null coordinates U, W and $t \to -\infty$.

Bogoliubov coefficient β (i) [Standard Hawking radiation]

- Right-moving positive frequency past and future modes:

$$\psi'_{\omega'} = \frac{1}{\sqrt{2\pi c\ \omega'}} e^{-i\omega' U}, \qquad \psi_{\omega} = \frac{1}{\sqrt{2\pi c\ \omega}} e^{-i\omega u}.$$

- Hawking radiation is encoded in $\beta_{\omega\omega'} \equiv (\psi'_{\omega'}, \psi^*_{\omega})$.
- Mode mixing happens in the right-moving sector. Therefore, we only need the first term of the previous KG expression:

$$\beta_{\omega\omega'} = -\frac{ic}{2}\int_{-\infty}^{+\infty} \mathrm{d}u\ \left[\psi'_{\omega'}\partial_u\psi_\omega - \psi_\omega\partial_u\psi'_{\omega'}\right]_{w=+\infty}$$
$$= \frac{1}{2\pi}\sqrt{\frac{\omega}{\omega'}}\int \mathrm{d}u\ e^{-i\omega' U(u)} e^{-i\omega u}.$$

- All the info is contained in $U = U(u) \equiv U(u, w \to +\infty)$.

Bogoliubov coefficient β (ii)

- At late times, $U = U_H - Ae^{-\kappa u/c}$, where U_H, A and the surface gravity $\kappa \equiv c\left|\frac{\mathrm{d}\bar{v}}{\mathrm{d}x}\right|_{x_H}$ are constants.
- We can define a threshold time u_I at which an asymptotic observer will start to detect thermal radiation from the black hole. This retarded time corresponds to the moment at which the function $U(u)$ enters the exponential regime.
- Rewrite as $U = U_H - A_0 e^{-\kappa(u-u_I)/c}$, valid for $u > u_I$.
- Obtain $\beta_{\omega\omega'}$. [Dirac-delta normalization]

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- Obtain $\beta_{\omega\omega'}$. [Dirac-delta normalization]
- Narrow wave packets:

$$P_{\omega_j,u_l}(\omega) \equiv \begin{cases} \frac{e^{i\omega u_l}}{\sqrt{\Delta\omega}}, & -\frac{1}{2}\Delta\omega < \omega - \omega_j < \frac{1}{2}\Delta\omega, \\ 0, & \text{otherwise;} \end{cases}$$

  - centered at $u_l \equiv u_0 + 2\pi l/\Delta\omega$, with u_0 an overall reference;
  - central frequency: $\omega_j \equiv j\Delta\omega$; width: $\Delta\omega \ll \omega_j$.

Bogoliubov coefficient β (iii) [Standard Hawking radiation]

- Define
$$\beta_{\omega_j,u_l;\omega'} \equiv \int \mathrm{d}\omega\, \beta_{\omega\omega'} P_{\omega_j,u_l}(\omega),$$
$$z = (c\Delta\omega/2\kappa)\ln(\omega' A_0)\ , \qquad z_l = (\Delta\omega/2)(u_l - u_I).$$
- Number of particles with frequency ω_j detected at time u_l by an asymptotic observer:
$$N_{\omega_j,u_l} = \int_0^{+\infty} \mathrm{d}\omega' |\beta_{\omega_j,u_l;\omega'}|^2 = \int_{-\infty}^{+\infty} \mathrm{d}z \frac{\sin^2(z - z_l)}{\pi(z - z_l)^2} \frac{1}{\exp(2\pi c\omega_j/\kappa) - 1}$$
$$= \frac{1}{\exp(2\pi c\omega_j/\kappa) - 1}.$$
- Hawking's formula (in the absence of backscattering): Planckian spectrum with temperature $T_H = \kappa/(2\pi c)$.

Hawking radiation with superluminal dispersion
Modified wave equation (i)

- Quartic modification to wave equation:

$$(\partial_t + \partial_x v)(\partial_t + v\partial_x)\phi = c^2 \left(\partial_x^2 + \frac{1}{k_P^2}\partial_x^4 \right) \phi \ ,$$

- Dispersion relation: $(\omega - vk)^2 = c^2 k^2 \left(1 + k^2/k_P^2\right).$
  - k_P: 'Planck scale' — non-relativistic deviations.
  - In BEC, $k_P = 2/\xi$ (inverse of the healing length)

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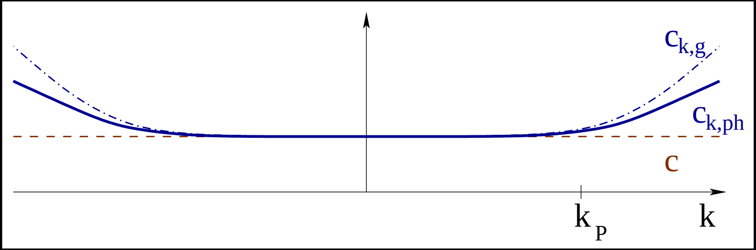
  - In BEC, $k_P = 2/\xi$ (inverse of the healing length)
- Modification in the phase and the group velocities,

$$v_{k,ph} \equiv \omega/k = c_{k,ph} + v \ , \qquad v_{k,g} \equiv \mathrm{d}\omega/\mathrm{d}k = c_{k,g} + v \ ,$$

due to k-dependent phase and group speeds of light/sound

$$c_{k,ph} = c\sqrt{1 + k^2/k_P^2} \ , \qquad c_{k,g} = c\frac{1 + 2k^2/k_P^2}{\sqrt{1 + k^2/k_P^2}}.$$

Modified wave equation (ii)



- Both speeds $c_{k,g}$ and $c_{k,ph}$ show the same qualitative behaviour. Our results are independent of the choice $\rightarrow c_k$.
- Frequency-dependent horizon when $c_k + v = 0$.

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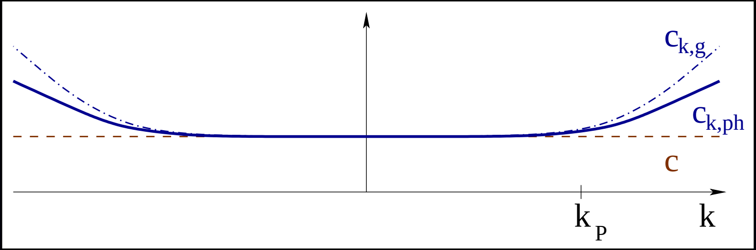


- Both speeds $c_{k,g}$ and $c_{k,ph}$ show the same qualitative behaviour. Our results are independent of the choice $\rightarrow c_k$.
- Frequency-dependent horizon when $c_k + v = 0$.
- Since c_k becomes arbitrarily high for increasing wave number, there will be a critical ω'_c such that waves with an initial frequency $\omega' > \omega'_c$ do not experience a horizon at all. The only exception occurs when the velocity profile ends in a singularity $\bar{v} \rightarrow -\infty$, which implies $\omega'_c \rightarrow \infty$.

Scalar product

- Same as before for t =constant. It is well-defined and conserved:

$$\partial_t(\varphi_1, \varphi_2) = \int \mathrm{d}x \; \varphi_1 \; \overleftrightarrow{\partial_t^4} \; \varphi_2^* = 0.$$

- There is a preferred time frame: the 'laboratory' time t.
- Perform the same change of coordinates $(t, x \to u, w)$ as before and evaluate at $t \to +\infty$.
- The relevant part (right-moving sector) of the inner product is

$$-\frac{ic}{2} \int_{-\infty}^{+\infty} \mathrm{d}u \; \left[\varphi_1 \partial_u \varphi_2^* - \varphi_2^* \partial_u \varphi_1\right]_{w=+\infty}.$$

- Invariant under change of integration variable $u \to f(u)$.

Bogoliubov coefficient β (i)

- With slowly varying profiles, the past and future right-moving positive-energy modes are (up to grey-body factors)

$$\psi'_{\omega'} \approx \frac{1}{\sqrt{2\pi c\omega'}} e^{-i\omega' \mathcal{U}_{\omega'}(u,w)}, \qquad \psi_{\omega} \approx \frac{1}{\sqrt{2\pi c\omega}} e^{-i\omega u_{\omega}(u,w)},$$

where $\mathcal{U}_{\omega'}(u,w)$ and $u_\omega(u,w)$ can be obtained by integration of the ray equation

$$\mathrm{d}x/\mathrm{d}t = c_{k(\omega')}(t,x) + v(t,x).$$

  - Integration for an initial frequency ω' starting from the past left infinity towards the right gives $\mathcal{U}_{\omega'}(u,w)$ =constant.
  - Starting from the future, we can define $u_\omega(u,w)$.
  - *Ditto* for $\mathcal{W}_{\omega'}$ and w_ω.
- $\mathcal{U}_{\omega'}$ and u_ω are not null (geometric) coordinates, since they are frequency-dependent, but share many properties with them.
- Simple (piecewise) profile $\rightarrow$ explicit integration.

Bogoliubov coefficient β *(ii)*

- When calculating $\beta_{\omega,\omega'}$, change integration variables

$$u, w \to u_\omega(u,w), w_\omega(u,w)$$

- $w \to \infty$ implies $w_\omega \to \infty$, $\quad u_\omega \to u_\omega(u)$, $\quad \mathcal{U}_{\omega'} \to \mathcal{U}_{\omega'}(u)$.
- Then, up to grey-body factors,

$$\begin{aligned}\beta_{\omega\omega'} &= -\frac{ic}{2}\int_{-\infty}^{+\infty} \mathrm{d}u_\omega [\psi'_{\omega'}\partial_{u_\omega}\psi_\omega - \psi_\omega\partial_{u_\omega}\psi'_{\omega'}]_{w_\omega=+\infty} \\ &\approx \frac{1}{2\pi}\sqrt{\frac{\omega}{\omega'}}\int \mathrm{d}u_\omega\; \mathrm{e}^{-i\omega'\mathcal{U}_{\omega'}(u_\omega)}\mathrm{e}^{-i\omega u_\omega}.\end{aligned}$$

Bogoliubov coefficient β (iii)

- Integration of the ray equation provides the relation between $\mathcal{U}_{\omega'}$ and u_ω where ω' is the initial frequency of a ray at the past left infinity and $\omega = \omega(\omega')$ is its final frequency when reaching the future right infinity.
- Result: $\mathcal{U}_{\omega'} = \mathcal{U}_{H,\omega'} - A_0 e^{-\kappa_{\omega'}(u_\omega - \bar{u}_{I,\omega'_c})/c}$;
  - valid for $\omega' < \omega'_c$ (for which a horizon is experienced;
  - valid for times $u_\omega > u_{I,\omega'_c}$, where u_{I,ω'_c} is the largest threshold time (this induces a slight underestimation of the effect).
- The term carrying $\mathcal{U}_{H,\omega'}$ is moduloed away, so the only relevant frequency-dependent factor that we are left with is the surface gravity $\kappa_{\omega'}$.

Modified Hawking spectrum (i) [Hawking radiation with superluminal dispersion]

- Smear with narrow packets.
- Change integration variable from ω' to z: $[\kappa_0 \equiv \kappa_{\omega'=0}]$

$$z = \frac{c\Delta\omega}{2\kappa_0}\ln(\omega' A_0), \qquad z_{l,\omega'} = \frac{\kappa_{\omega'}}{\kappa_0}\frac{\Delta\omega}{2}(u_l - \bar{u}_{I,\omega'_c}).$$

- Number of particles (with frequency ω_j at time u_l):

$$N_{\omega_j,u_l} = \int_{-\infty}^{z_c} \mathrm{d}z \frac{\kappa_0}{\kappa_{\omega'}} \frac{\sin^2\left[\frac{\kappa_0}{\kappa_{\omega'}}(z - z_{l,\omega'})\right]}{\pi\left[\frac{\kappa_0}{\kappa_{\omega'}}(z - z_{l,\omega'})\right]^2} \frac{1}{\exp(2\pi c\omega_j/\kappa_{\omega'}) - 1}.$$

  - Dependence on the critical frequency ω'_c (through z_c)
  - Importance of the frequency-dependent $\kappa_{\omega'}$.
  - As u_l increases, a smaller part of the central peak will be integrated over, so the radiation will die off.

Modified Hawking spectrum (ii) [Hawking radiation with superluminal dispersion]

- Given a concrete profile, we can explicitly deduce the relation between $\kappa_{\omega'}$ and ω' as follows:
  - The horizon for a particular initial frequency ω' is formed when $c_k^2(x_{H,\omega'}) = v^2(x_{H,\omega'})$. This is an equation for $x_{H,\omega'}$.
  - Use this value in the expression for the surface gravity.
- For a Schwarzschild profile,

$$\kappa_{\omega'} \equiv c\left|\frac{d\bar{v}}{dx}\right|_{x_{H,\omega'}} = \kappa_0 \frac{1}{2\sqrt{2}}\left(1+\sqrt{1+4\frac{\omega'^2}{c^2 k_P^2}}\right)^{3/2}.$$

- Everything is ready for evaluation.

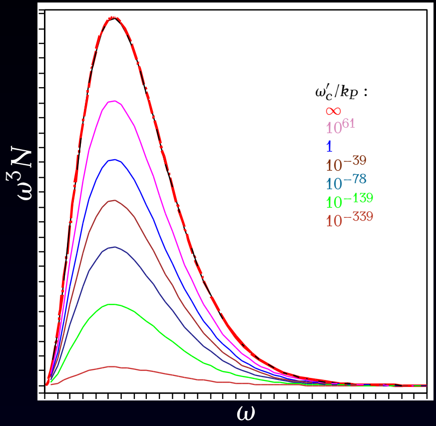
Modified Hawking spectrum (iii) [Hawking radiation with superluminal dispersion]

- Frequencies $\omega' > \omega'_c$ do not contribute to the radiation at all, since they do not experience a horizon.
- This cut-off is not imposed *ad hoc*, but appears explicitly because of the superluminal character of the system at high frequencies.
- The critical frequency depends directly on the physics inside the horizon, i.e. on the velocity profile, and can be calculated from the dispersion relation.

Results

Dependence on ω_c'

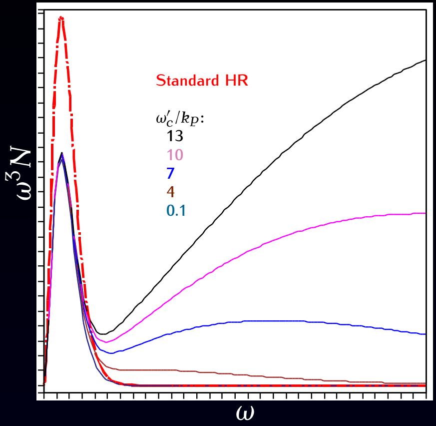
Profile with $\kappa_{\omega'} = \kappa_0$ constant

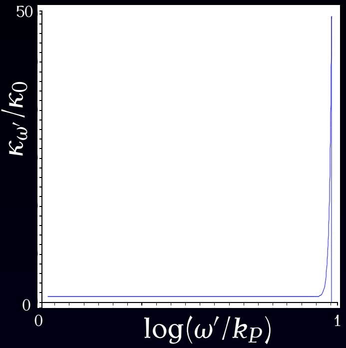


- Important decrease even when $\omega_c' \geq k_P$
- Contributions from extremely wide range of frequencies

Frequency dependence of $\kappa_{\omega'}$ [Results]

Schwarzschild profile ($\omega'_c \sim k_P$)

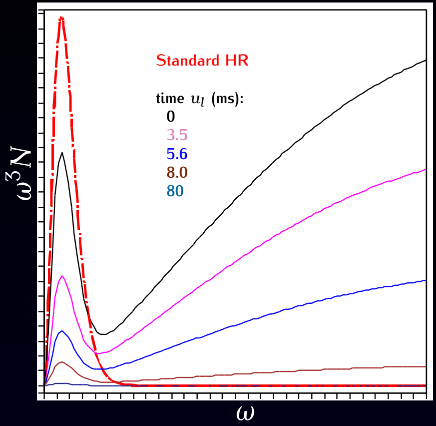




- Important ultraviolet contribution
- 'Interior' of black hole is explored

Dependence on time

For a solar-mass black hole:



- As time increases, radiation dies off
- Decay rate ~ 0.3 ms^{-1}

Conclusions
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  - interior of the (zero-frequency) horizon is probed
  - at every moment of collapse: critical ω_c' above which there is no horizon (unless we allow for an untamed singularity)

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- Hawking radiation:
  - ω'_c-dependent: radiation fainter than standard HR
    radiation dies off
  - $\kappa_{\omega'}$-dependent: high-frequency contribution

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- Hawking radiation:
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    radiation dies off
  - $\kappa_{\omega'}$-dependent: high-frequency contribution
- Superluminal dispersion leads to strong modification of standard Hawking spectrum, even if $\omega'_c \gg k_P$
- Schwarzschild profile does not reproduce standard spectrum

Summary (ii)

- Recovering standard Hawking radiation
  - If the velocity profile is such that the surface gravity is frequency independent, then the thermal form is preserved.
  - If we do not regularize the singularity, there is no critical frequency and the Hawking spectrum is unchanged:
    - full intensity
    - stationarity

Remarks

- Conditions for robustness of Hawking radiation
  1. freely falling frame is preferred
  2. high-frequency excitations start off in ground state at the horizon (w.r.t. freely falling frame)
  3. adiabatic evolution

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- Conditions for robustness of Hawking radiation
  1. freely falling frame is preferred
  2. high-frequency excitations start off in ground state at the horizon (w.r.t. freely falling frame)
  3. adiabatic evolution
- Superluminal dispersion:
  - Lorentz breaking $\longrightarrow$ preferred frame: the lab frame
    - Assumption 1 is not satisfied
  - Horizon approaches singularity as ω' increases
    - Conditions at horizon $\rightsquigarrow$ conditions at singularity!
  - Low-frequency modes couple to the collapsing geometry
    Ultrahigh-frequency modes couple to the lab frame
    - Assumption 2 is not satisfied

the end