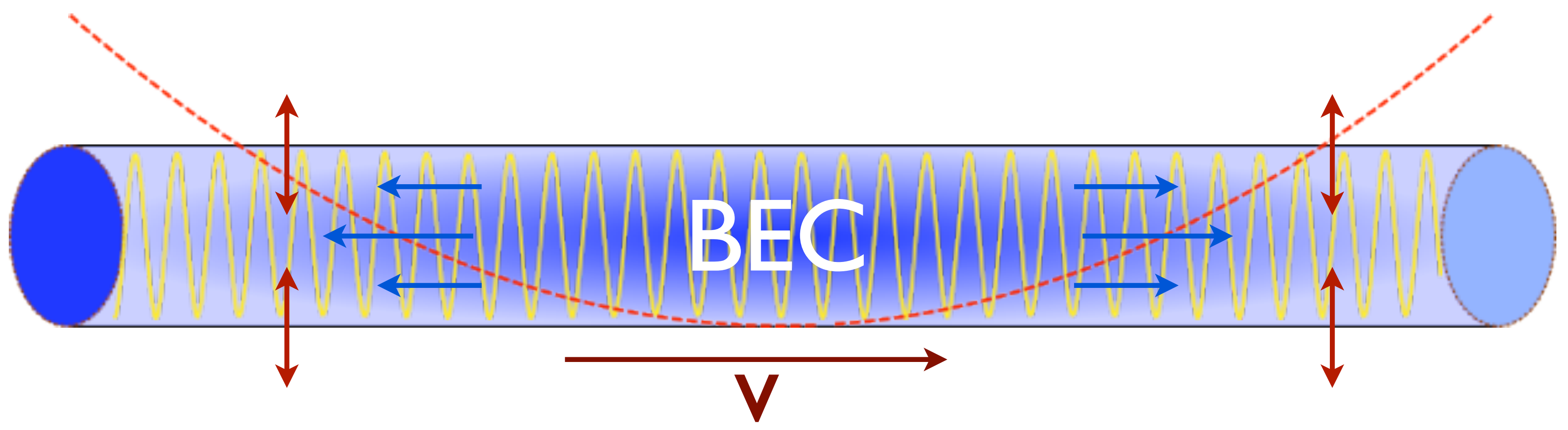
# Instability, dispersion management, and pattern formation in the superfluid flow of a BEC in a cylindrical waveguide

Michele Modugno
LENS & Dipartimento di Fisica, Università di Firenze, Italy

Workshop EHR - Valencia - February 3rd, 2009


BEC
V

# Summary

- *Optical lattices*
  - Instability of the superfluid flow
  - Effect of the radial confinement
  - Managing the dispersion by tuning the lattice velocity
  - Attractive interactions

- *Modulation of the radial confinement*
  - Parametric instability and pattern formation
  - Toroidal geometries, quantized circulation

# Meanfield regime

Gross-Pitaevskii equation:

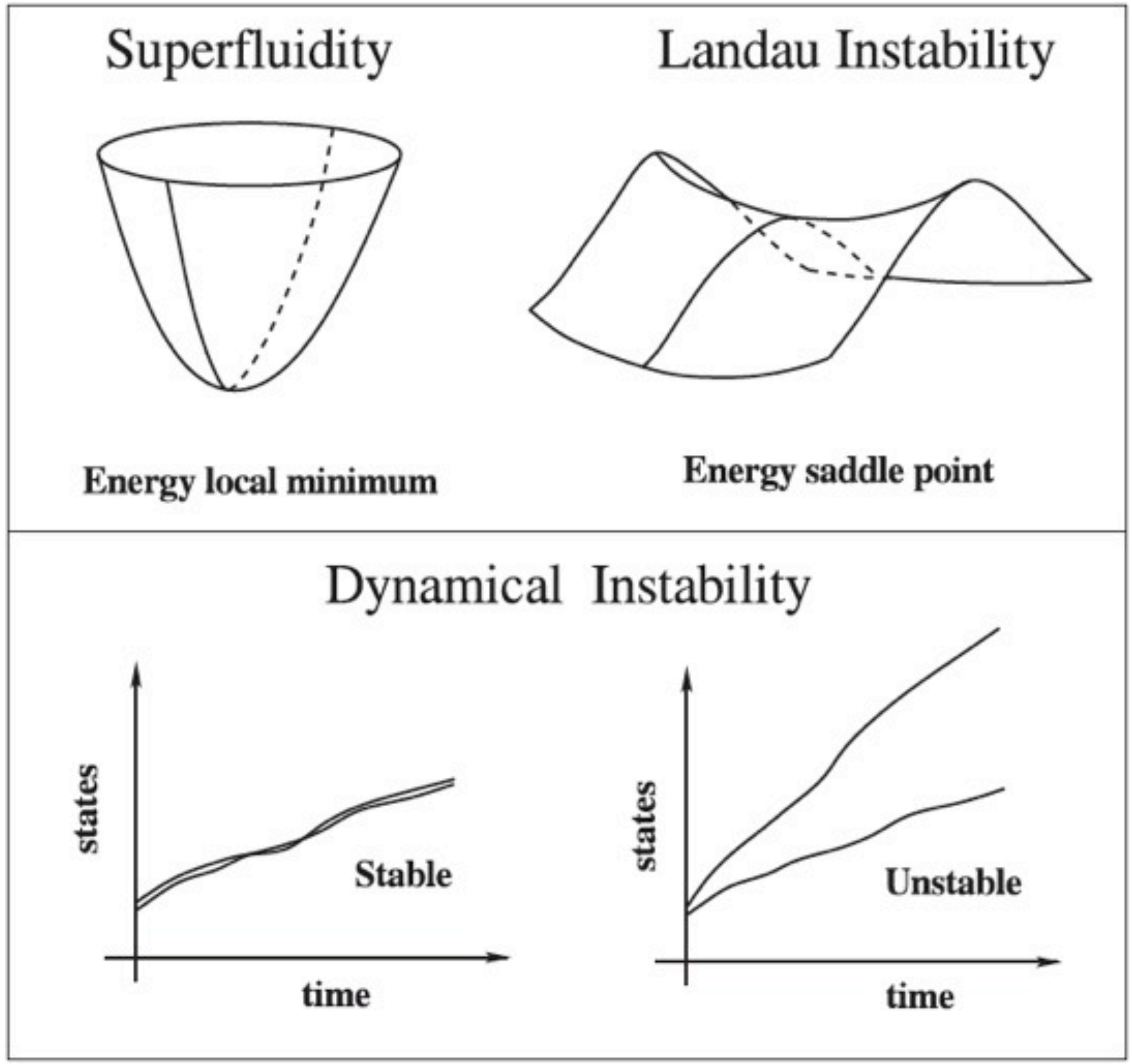
$$i\hbar \frac{\partial}{\partial t}\psi(\mathbf{x},t) = \left( -\frac{\hbar^2}{2m}\nabla^2 + V(\mathbf{x}) + g|\psi(\mathbf{x},t)|^2 \right)\psi(\mathbf{x},t)$$

- 1D GPE: the radial degrees of freedom are frozen
- Effective 1D model (NPSE)
- 3D GPE

# Instabilities

the stationary solution is energetically stable

the system can lower its energy by emitting excitations (dissipative process)




small fluctuations do not perturb the evolution

small fluctuations grow exponentially in time

# Bogoliubov excitations

linear regime:

$$\psi(\boldsymbol{x},t) = \mathrm{e}^{-i\mu t}\,[\,\phi_0(\boldsymbol{x}) + \delta\phi(\boldsymbol{x},t)\,]$$

$$\delta\phi(\boldsymbol{x},t) = \sum_j u_j(\boldsymbol{x})\mathrm{e}^{-i\omega_j t} + v_j^*(\boldsymbol{x})\mathrm{e}^{i\omega_j t}$$

Bogoliubov equations:

$$\begin{aligned}\left[H_0 - \mu + 2g|\phi_0|^2\right] u + g\phi_0^2 v &= \hbar\omega u \\ \left[H_0 - \mu + 2g|\phi_0|^2\right] v + g\phi_0^{*2} u &= -\hbar\omega v\end{aligned}$$

normalization: $\int d\boldsymbol{x}(u_i^* u_j - v_i^* v_j) = \delta_{ij}$

# Landau (energetic) instability

$$E[\phi] = E[\phi_0] + \int d^3x (\delta\phi^*, \delta\phi) M \begin{pmatrix} \delta\phi \\ \delta\phi^* \end{pmatrix}$$

$$M = \begin{pmatrix} H_0 + 2g|\phi_0|^2 & g\phi_0^2 \\ g\phi_0^{*2} & H_0 + 2g|\phi_0|^2 \end{pmatrix}$$

negative eigenvalues → instability

# Dynamical instability

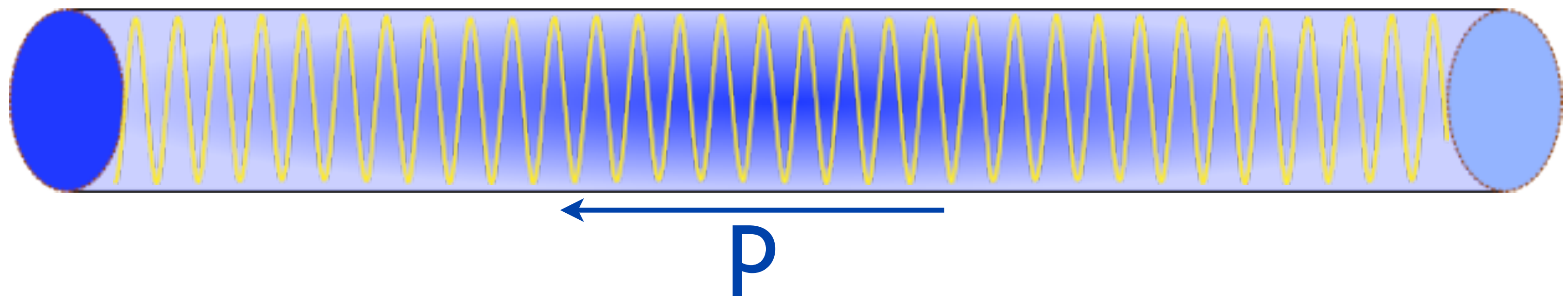
Bogoliubov equations:

$$\sigma_z M \begin{pmatrix} \delta\phi \\ \delta\phi^* \end{pmatrix} = \hbar\omega \begin{pmatrix} \delta\phi \\ \delta\phi^* \end{pmatrix}$$

imaginary frequencies → excitations grow exponentially in time

$$\delta\phi \sim A\mathrm{e}^{-i\omega t} \qquad Im(\omega) \neq 0$$

# Stability of a Bloch wave

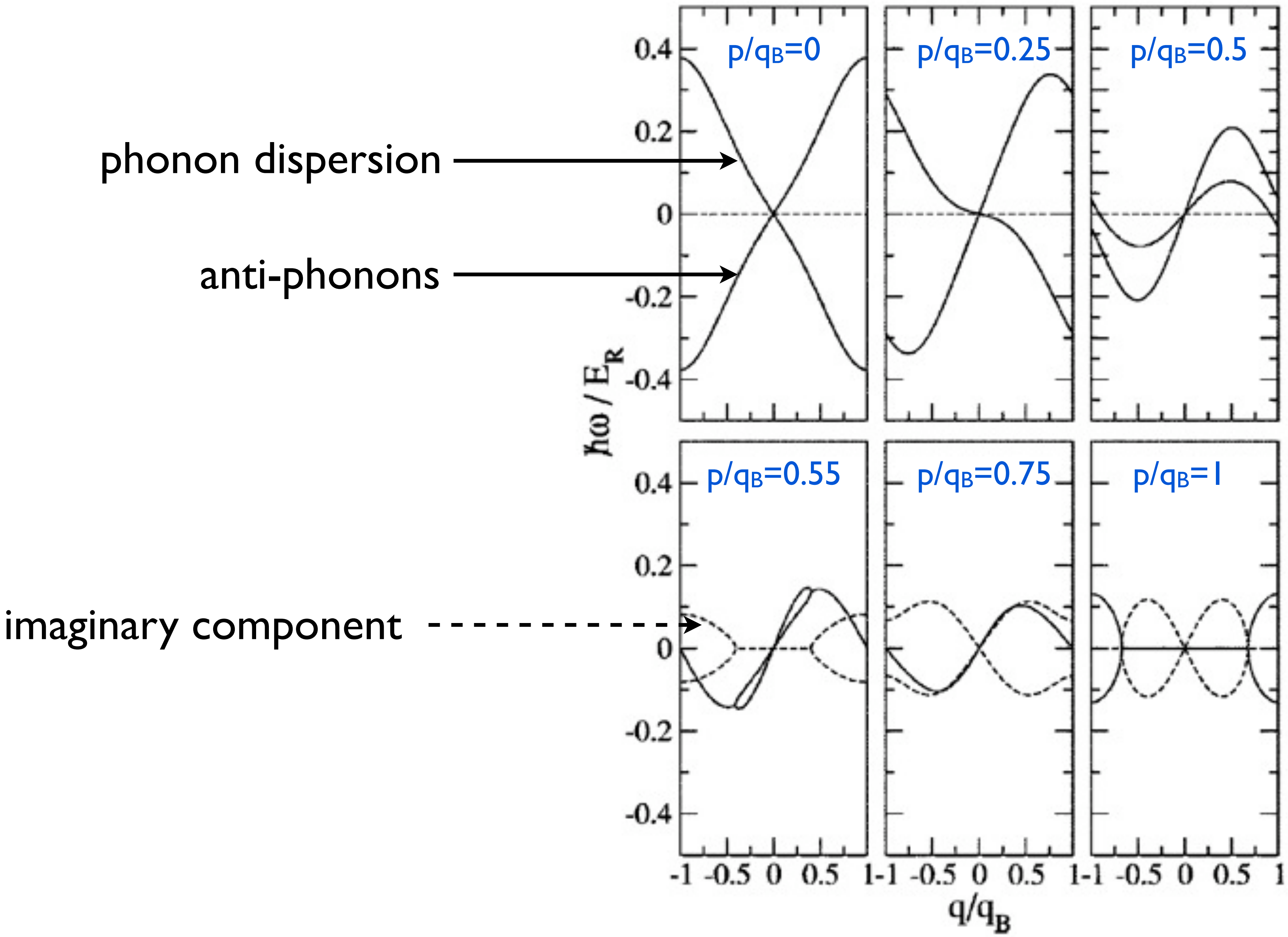



periodic system → Bloch waves

$$\varphi(z,t) = \mathrm{e}^{i(\mu t - pz)}\left[\phi_p(z) + \delta\phi_p(z,t)\right]$$

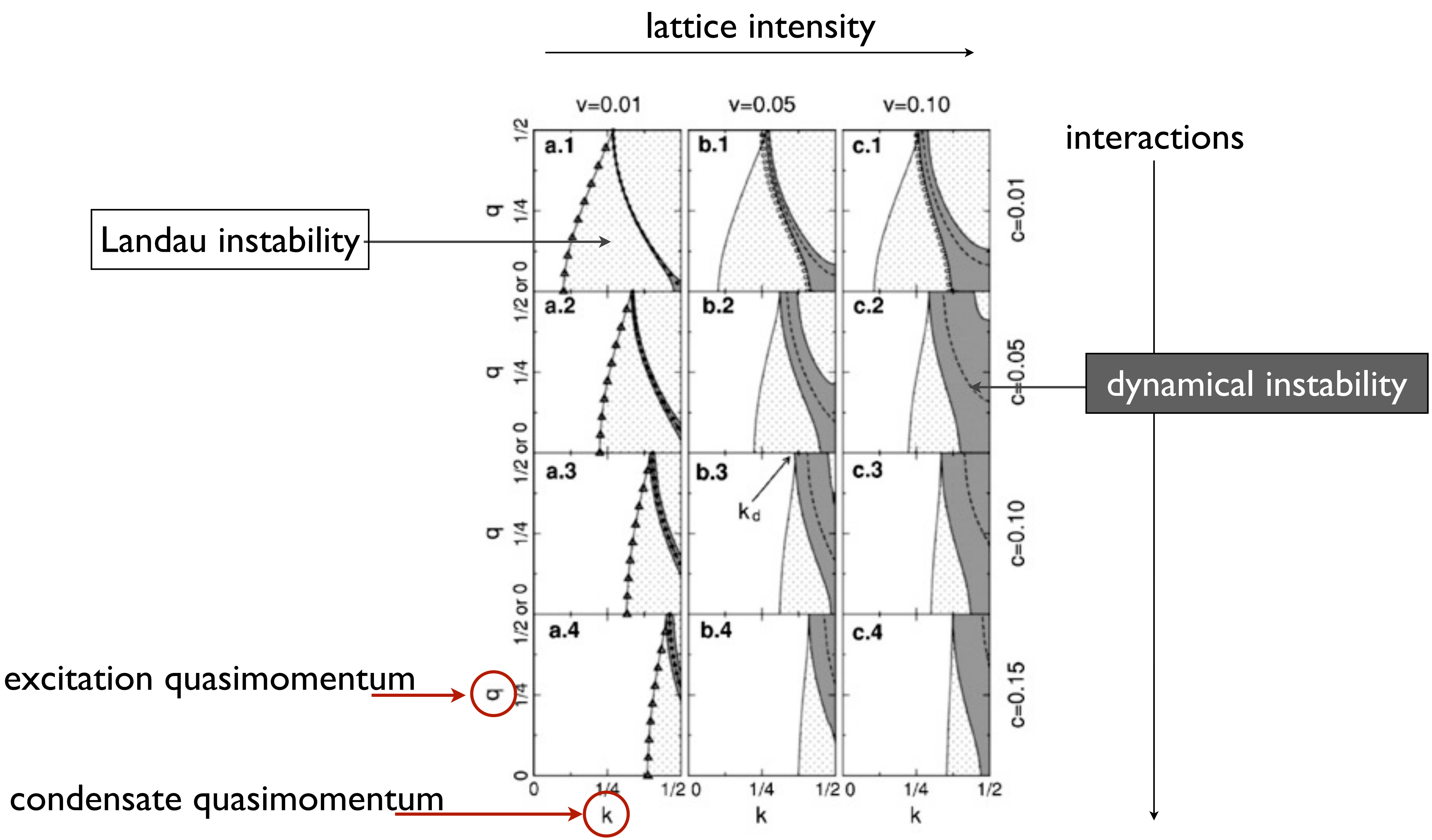
$$\delta\phi_p = \sum_{q,n}\left[u_{pq,n}\mathrm{e}^{i(qz-\omega_{pq,n}t)} + v^*_{pq,n}\mathrm{e}^{-i(qz-\omega_{pq,n}t)}\right]$$

# Dynamical instability: phonon-antiphonon resonance



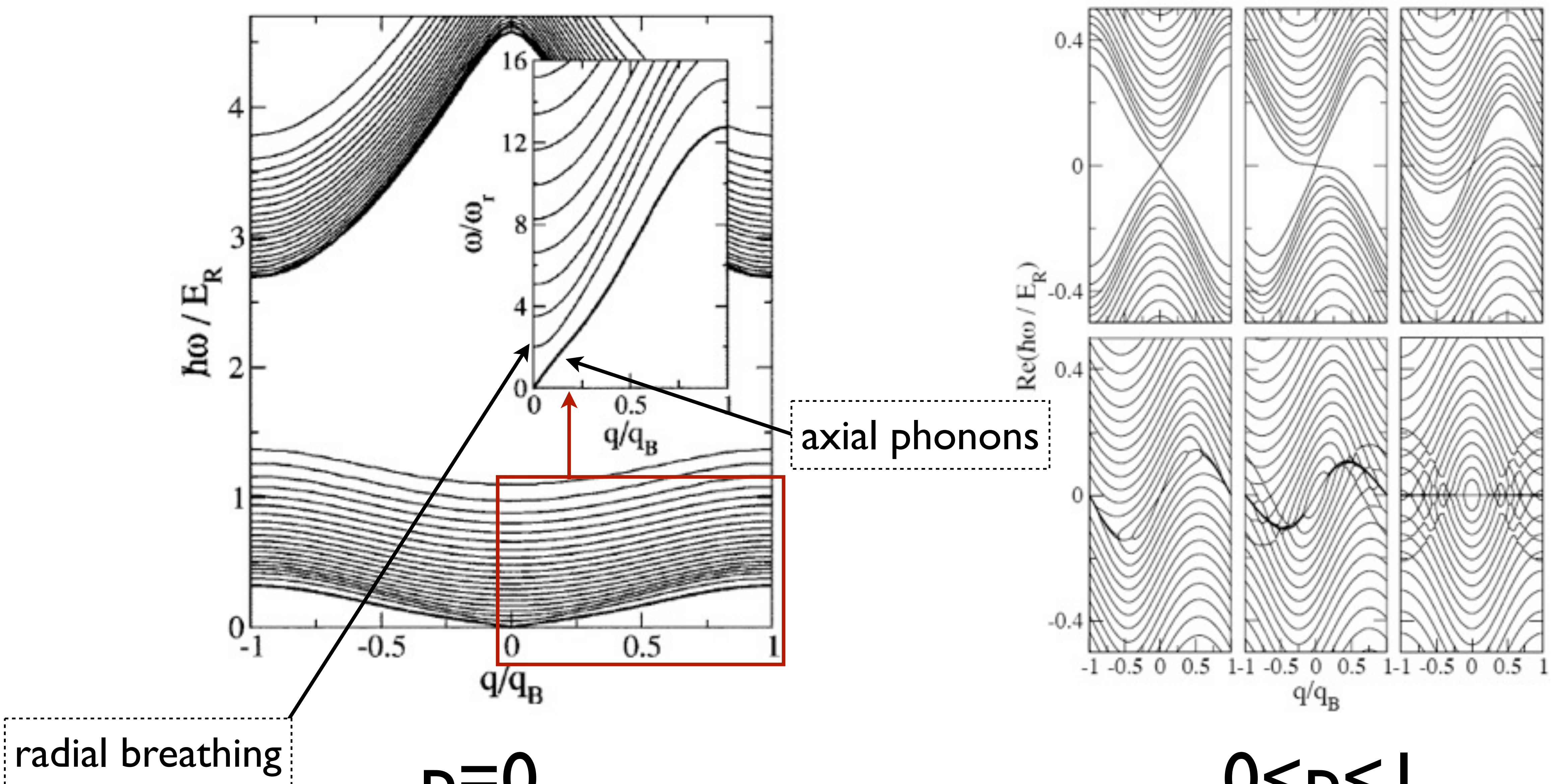

M. Modugno, *et al.*, PRA **70**, 043625 (2004)
[B. Wu and Q. Niu, PRA **64,** 061603(R) (2001); C. Menotti *et al.*, New J. Phys. **5**, 112 (2003)]

# Stability diagrams for Bloch waves (1D)




B. Wu and Q. Niu, PRA **64**, 061603(R) (2001)

# Effect of the radial degrees of freedom

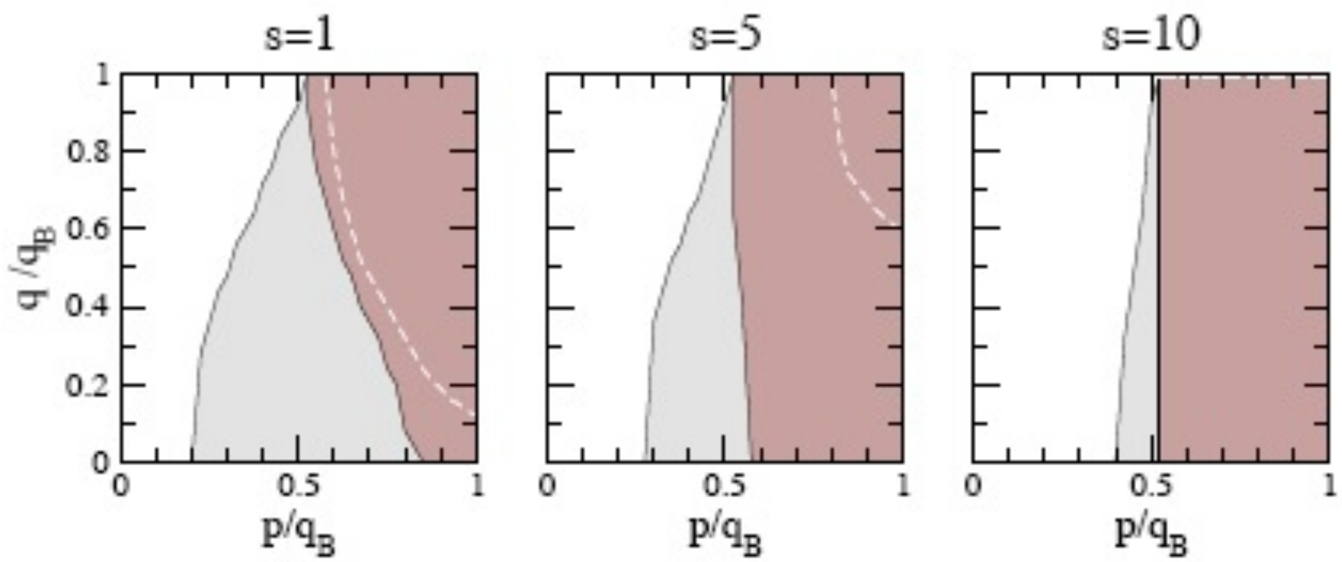


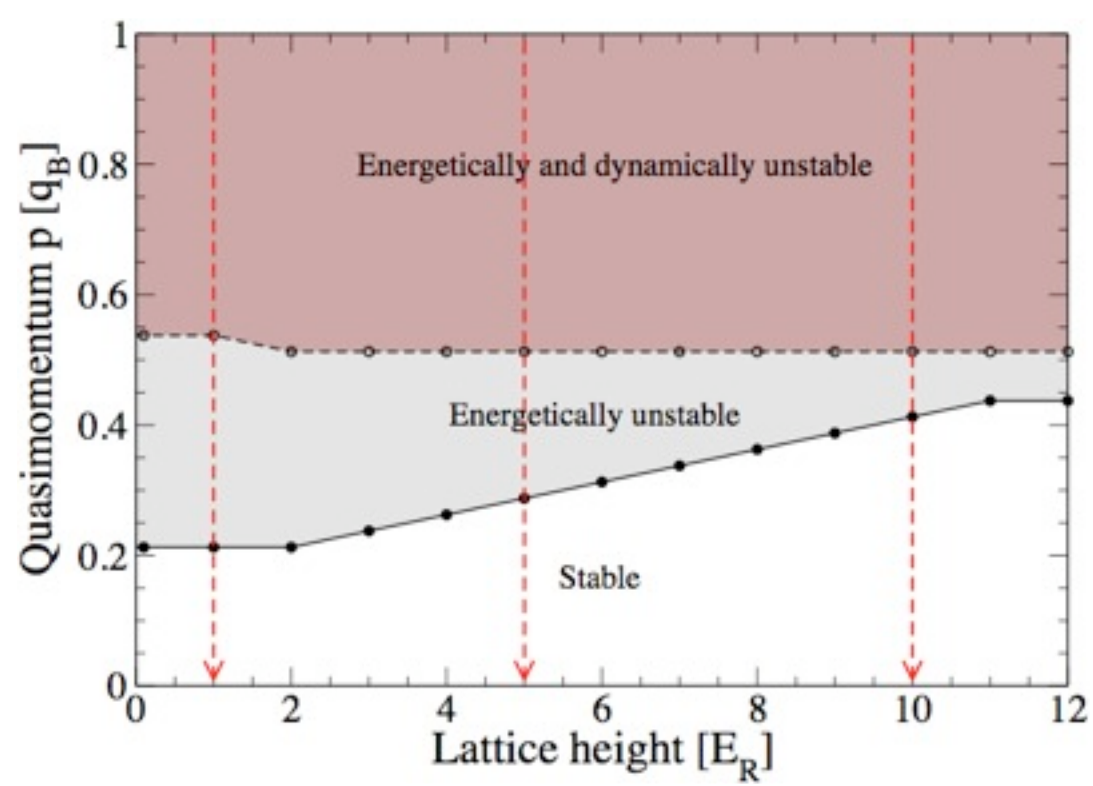

p=0

0<p<1

M. Modugno, C. Tozzo, and F. Dalfovo, PRA **70**, 043625 (2004)

# 3D stability diagrams

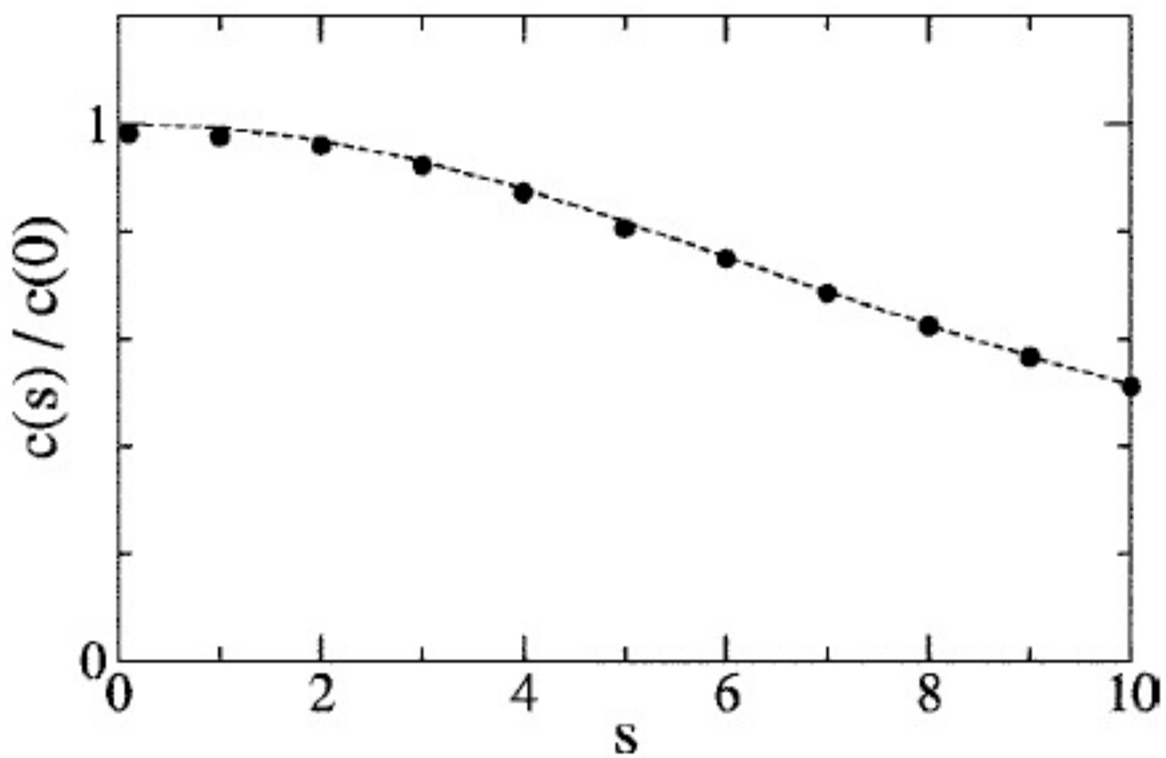



Same onset for instability from GPE 3D and effective 1D (NPSE)

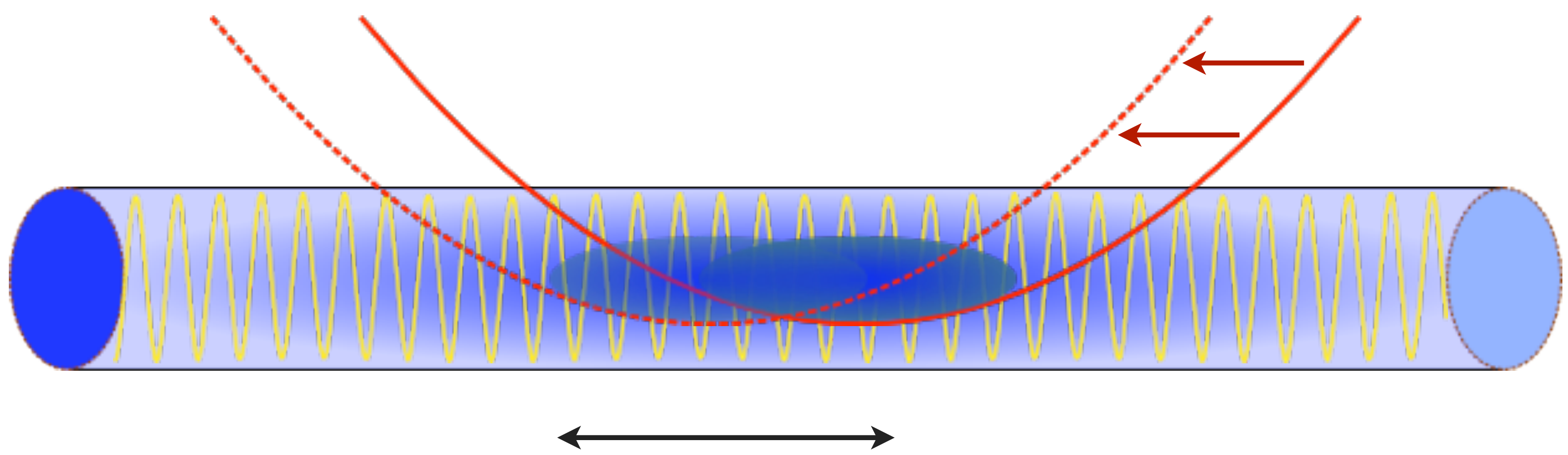
# sound velocity

$$c = \sqrt{\frac{N}{m^*} \frac{\partial \mu(N)}{\partial N}}$$




M. Krämer, C. Menotti and M. Modugno, J. of Low Temp. Phys., Vol. 138 (2005)

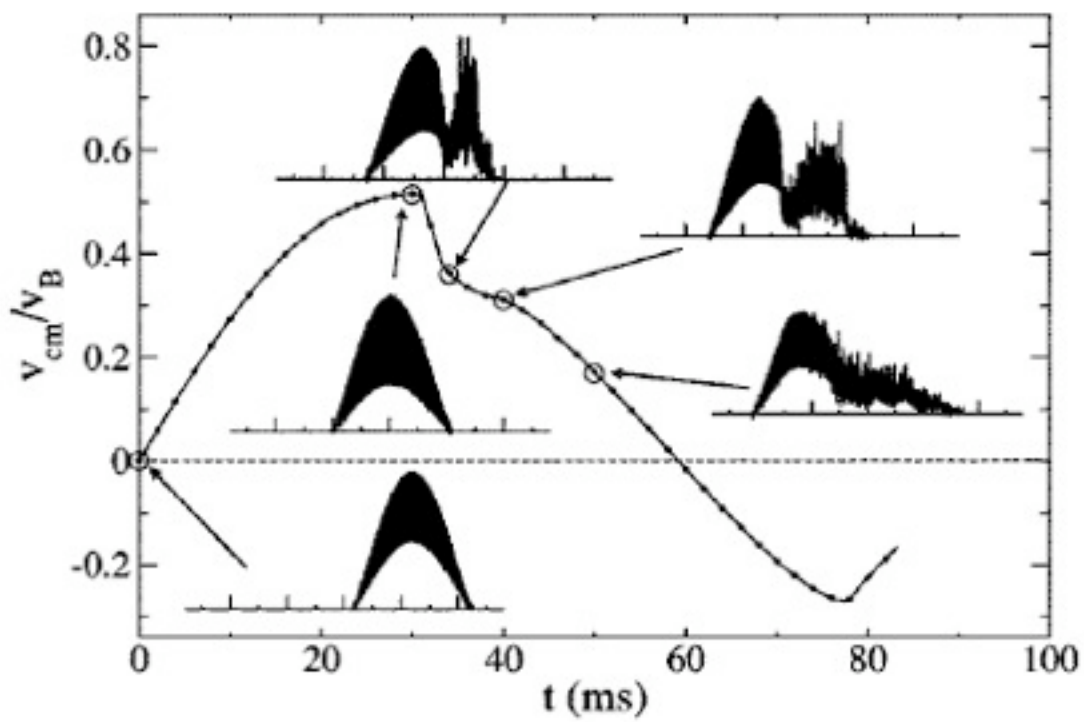
# Dipole mode of a BEC



experiment S. Burger *et al.*, PRL **86**, 4447 (2001)

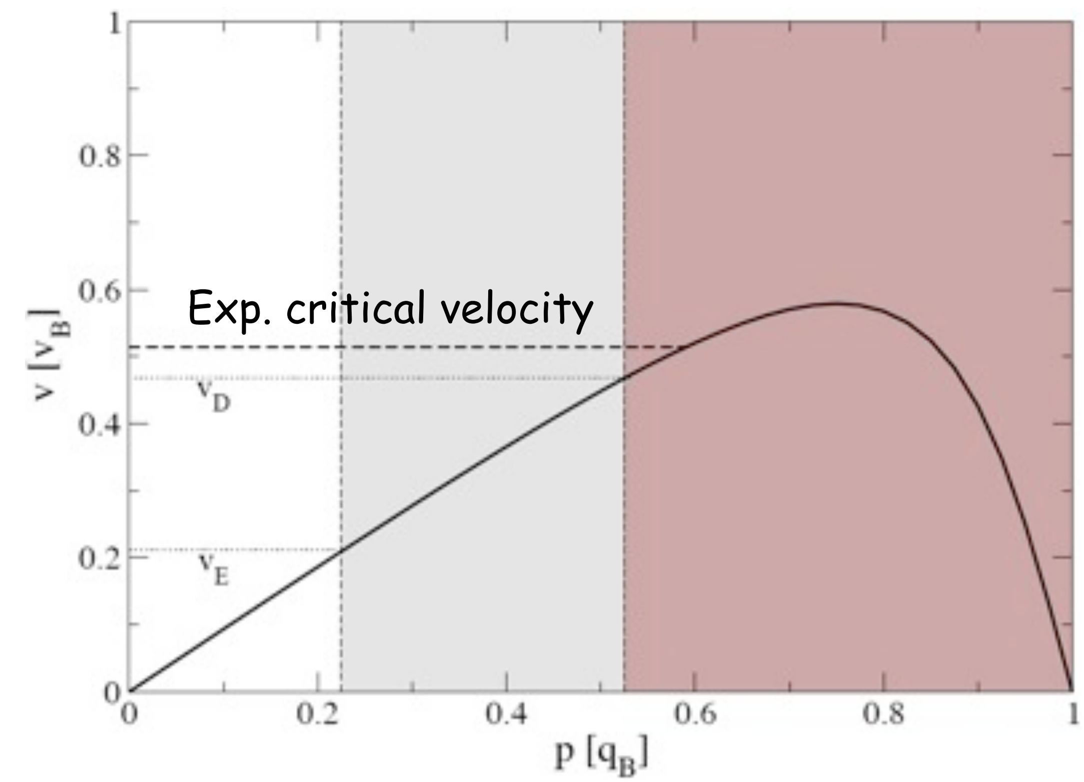
# Dipole mode of a BEC

3D GPE




Center-of-mass velocity vs. time.

Linear stability analysis:

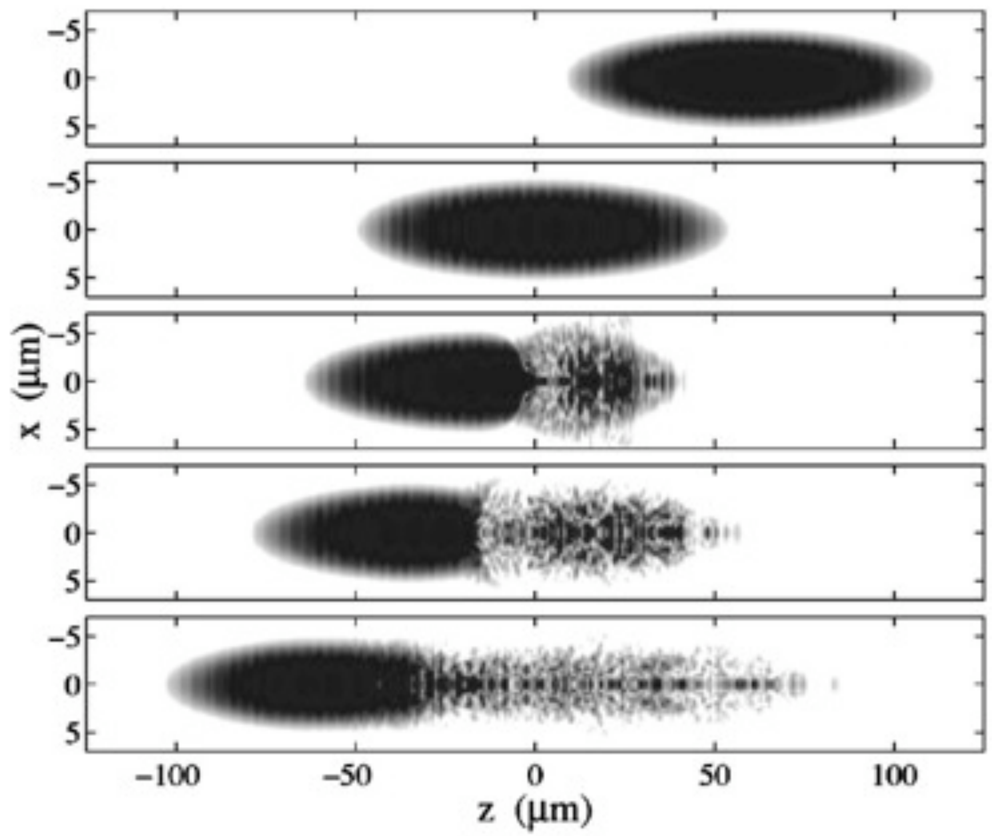



Center-of-mass velocity vs. BEC quasimomentum in the first Bloch band.

M. Modugno, C. Tozzo, and F. Dalfovo, PRA **70**, 043625 (2004):
simulation of the experiment in S. Burger *et al.*, PRL **86**, 4447 (2001)

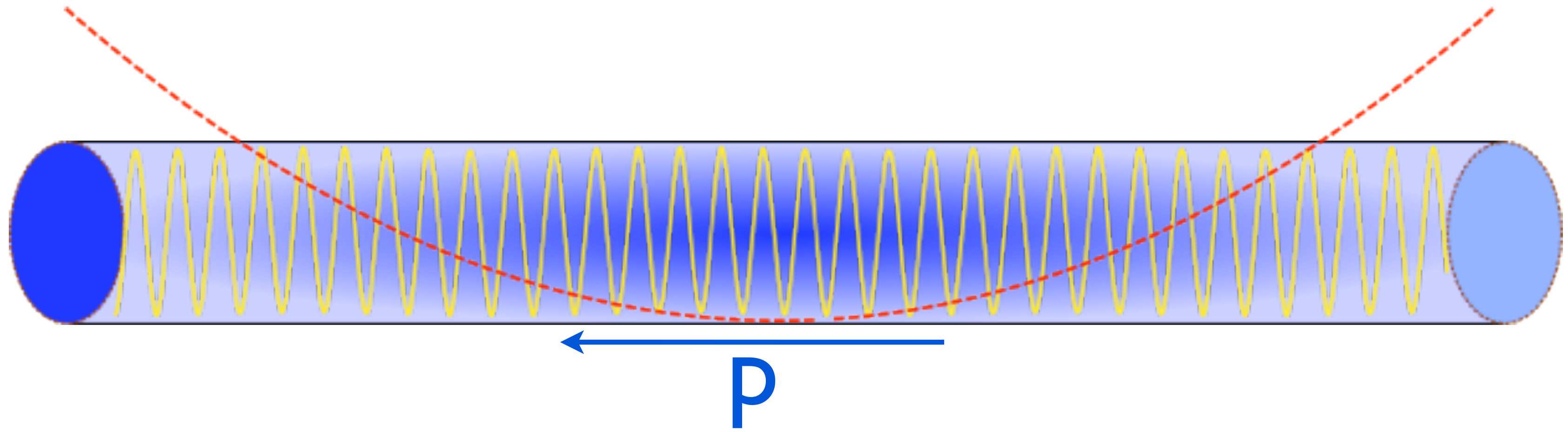
# 1D vs 3D

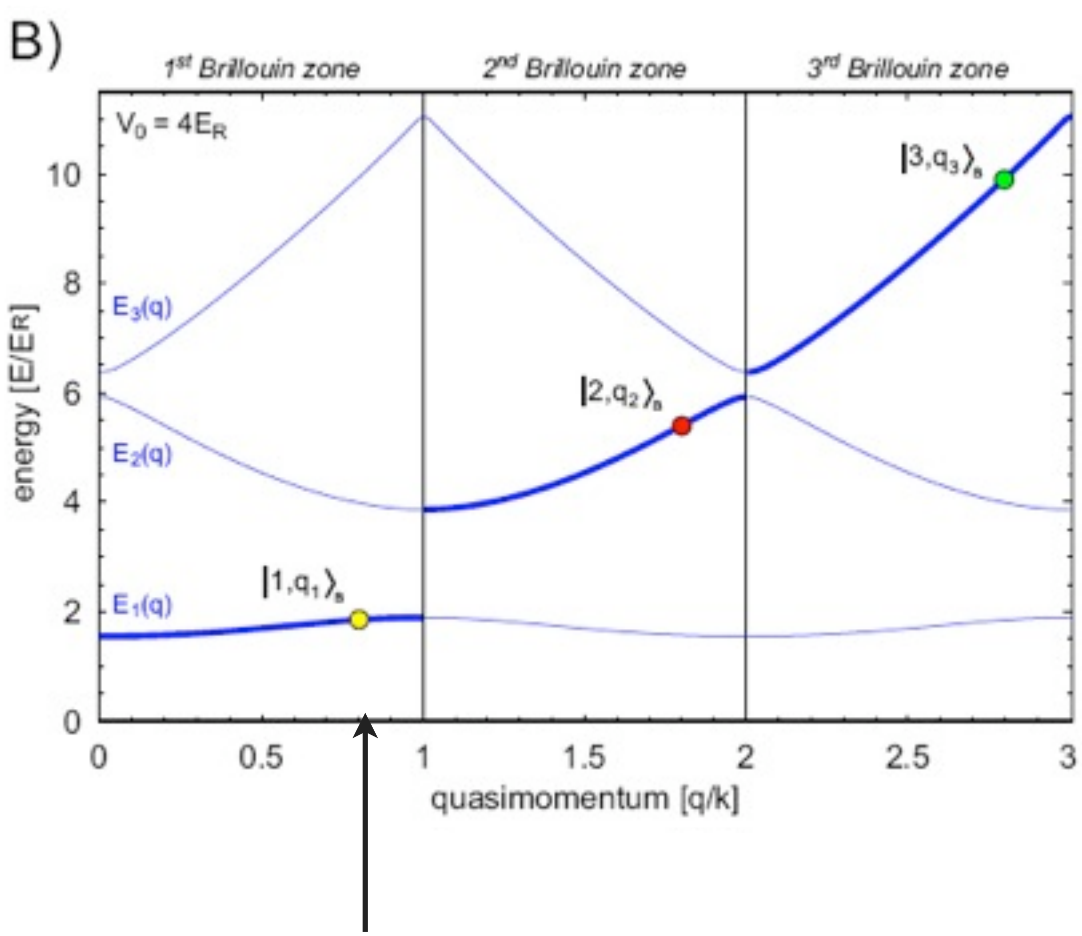
3D:

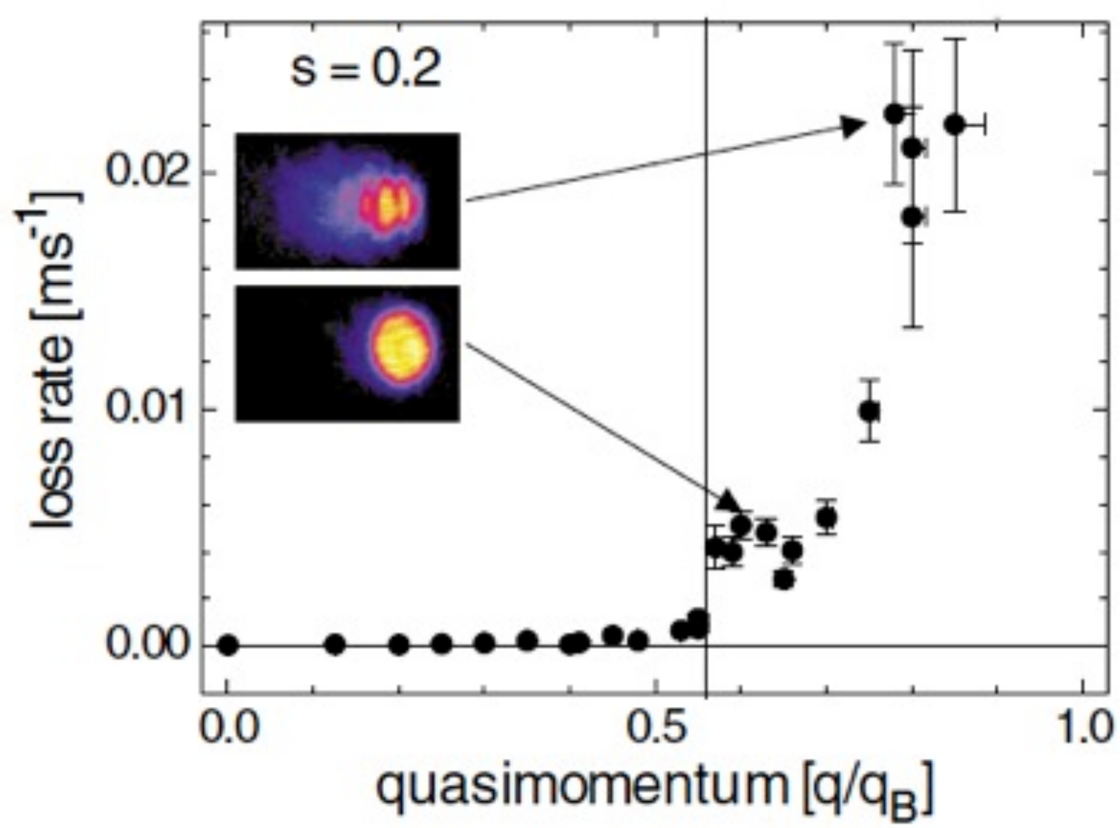



1D, effective 1D : complete loss of coherence

# A BEC in a moving lattice

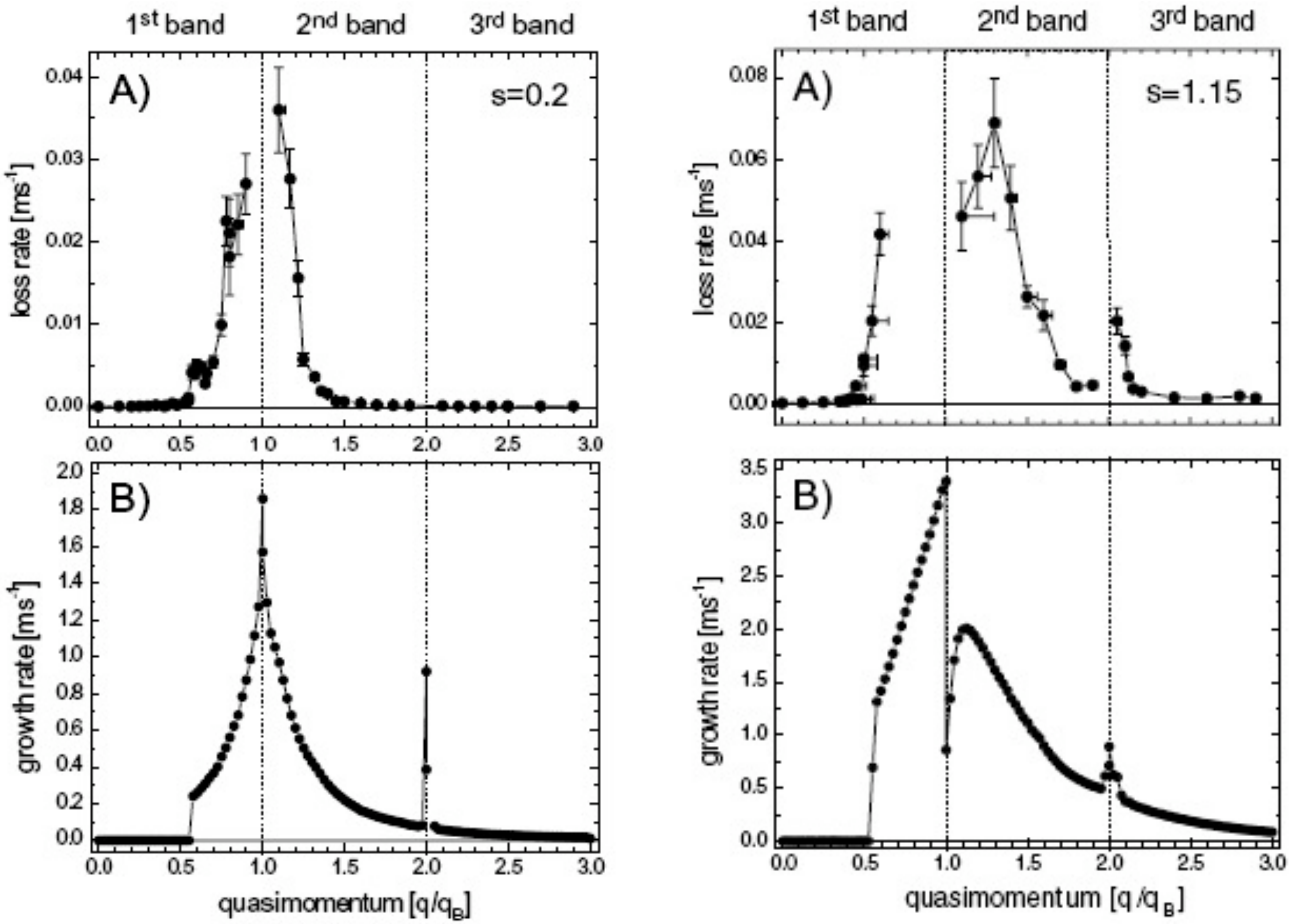


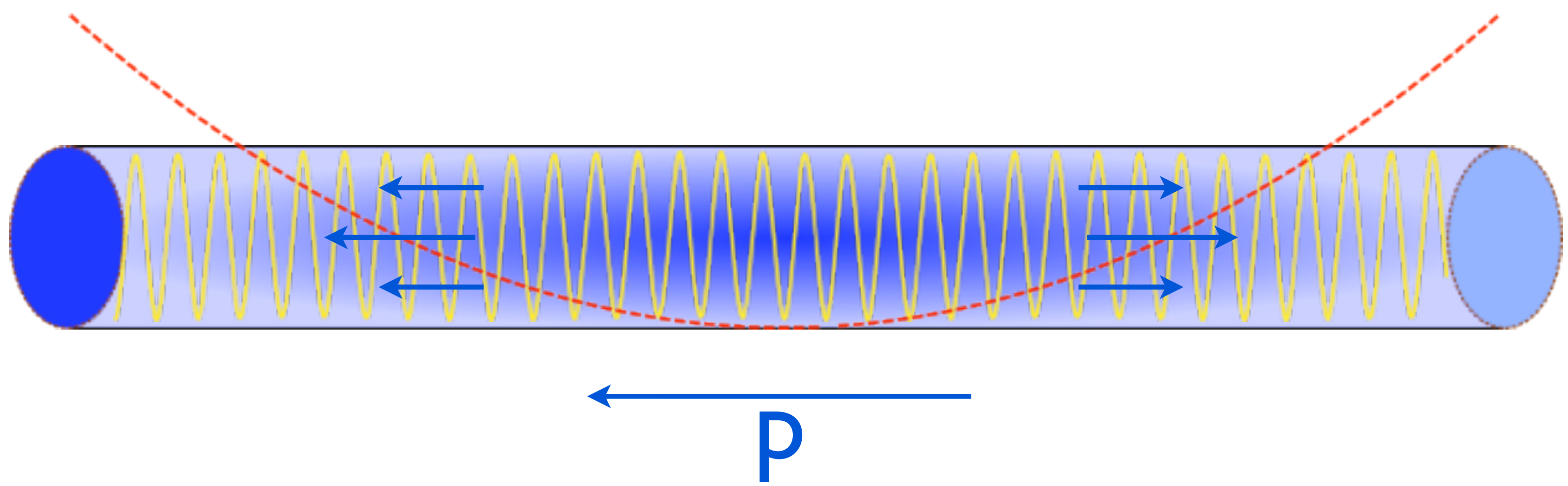

a lattice with fixed velocity
is ramped up adiabatically

L. Fallani *et al.*, PRL **93**, 140406 (2004)



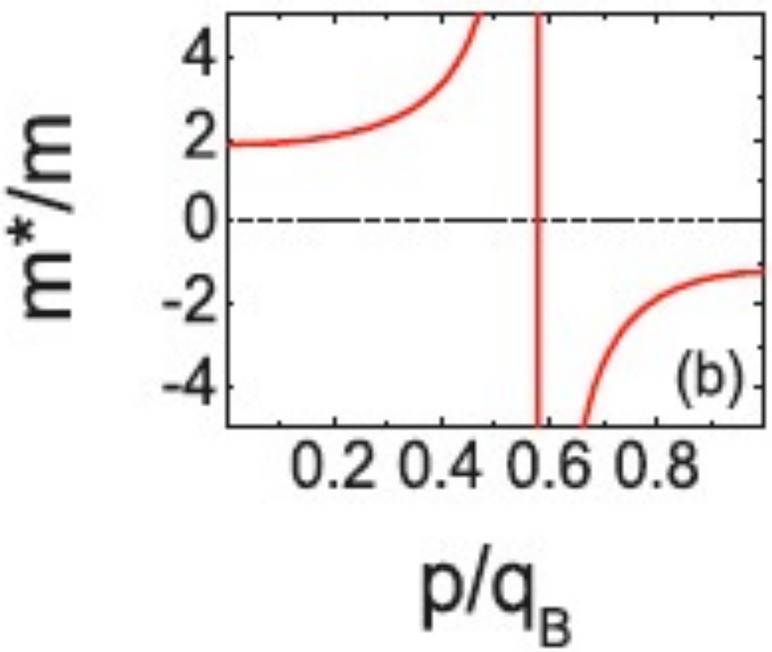

L. Fallani *et al.*, PRL **93**, 140406 (2004)

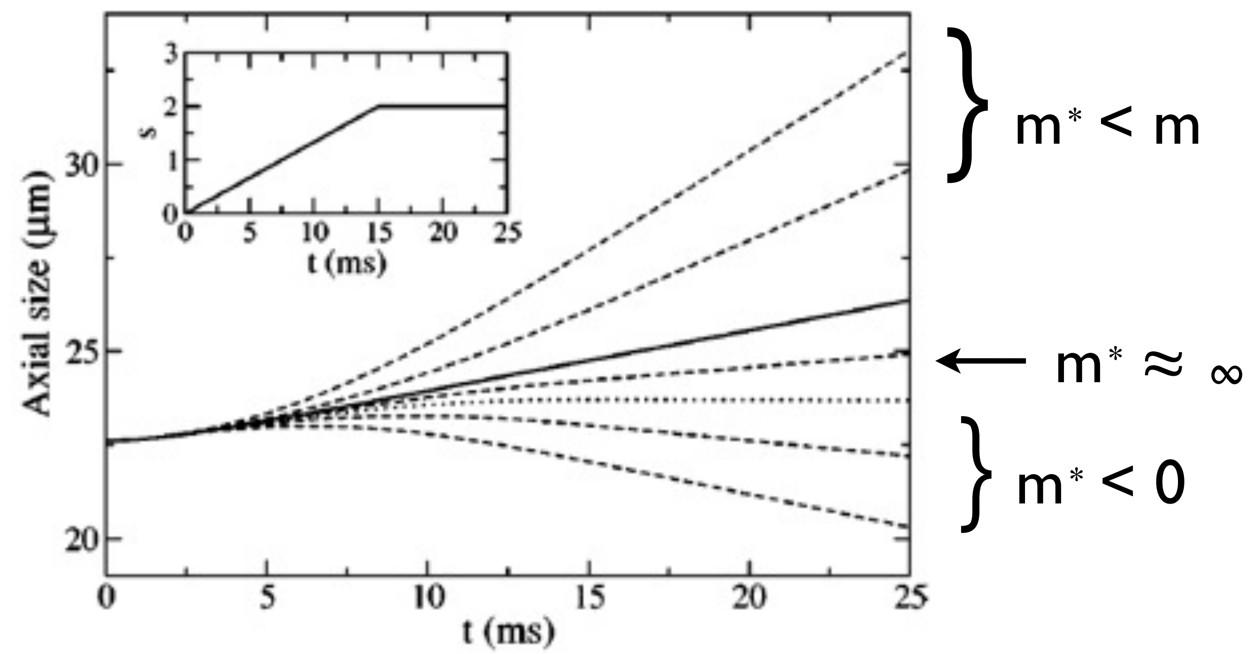
# Dispersion management




control the BEC dispersion by tuning the lattice velocity
(for weak nonlinearity)

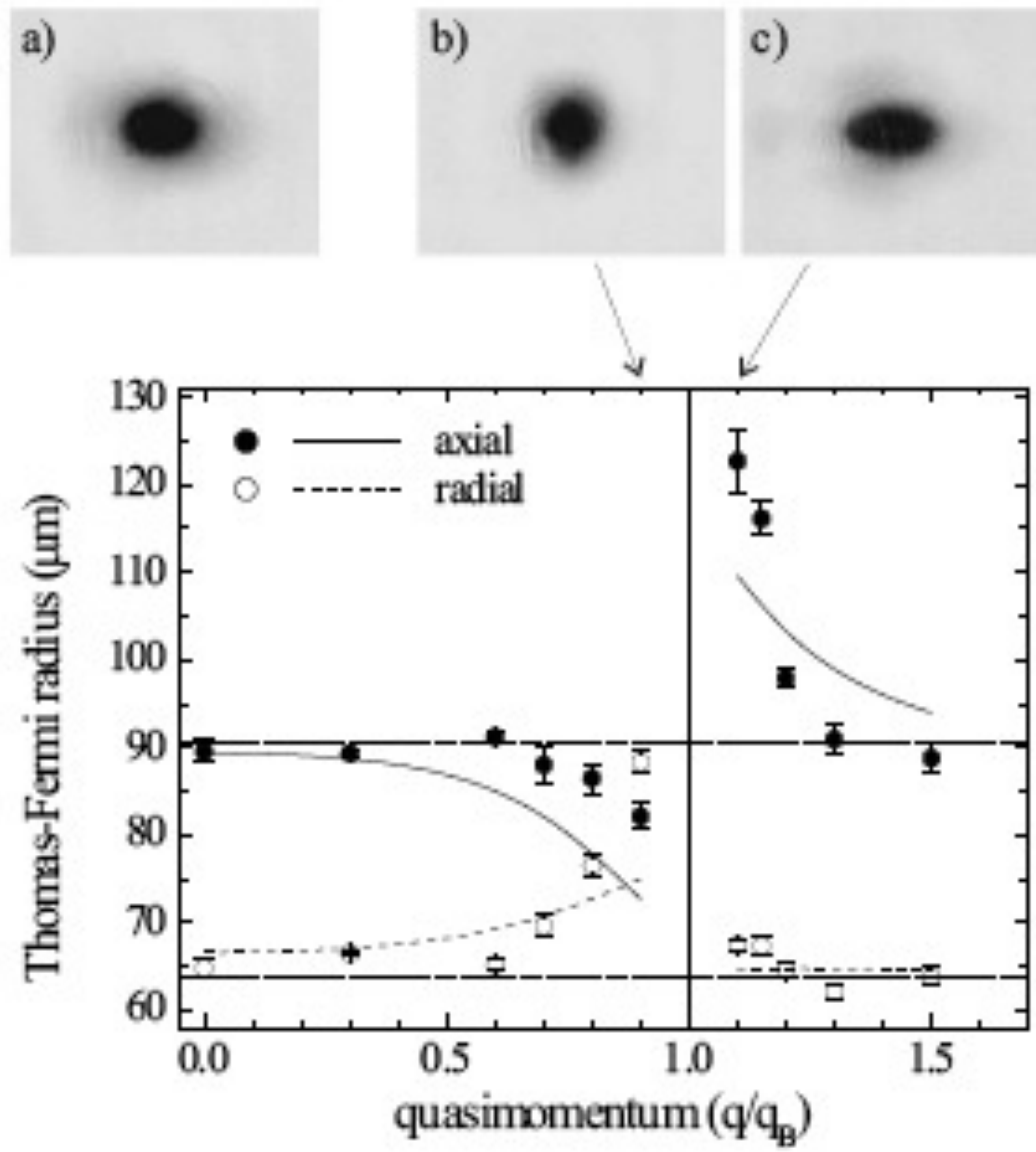
# Dispersion management




free expansion: $m^* < 0 \Rightarrow$ time reversed evolution

P. Massignan and M. Modugno, Phys. Rev. A **67**, 023614 (2003)

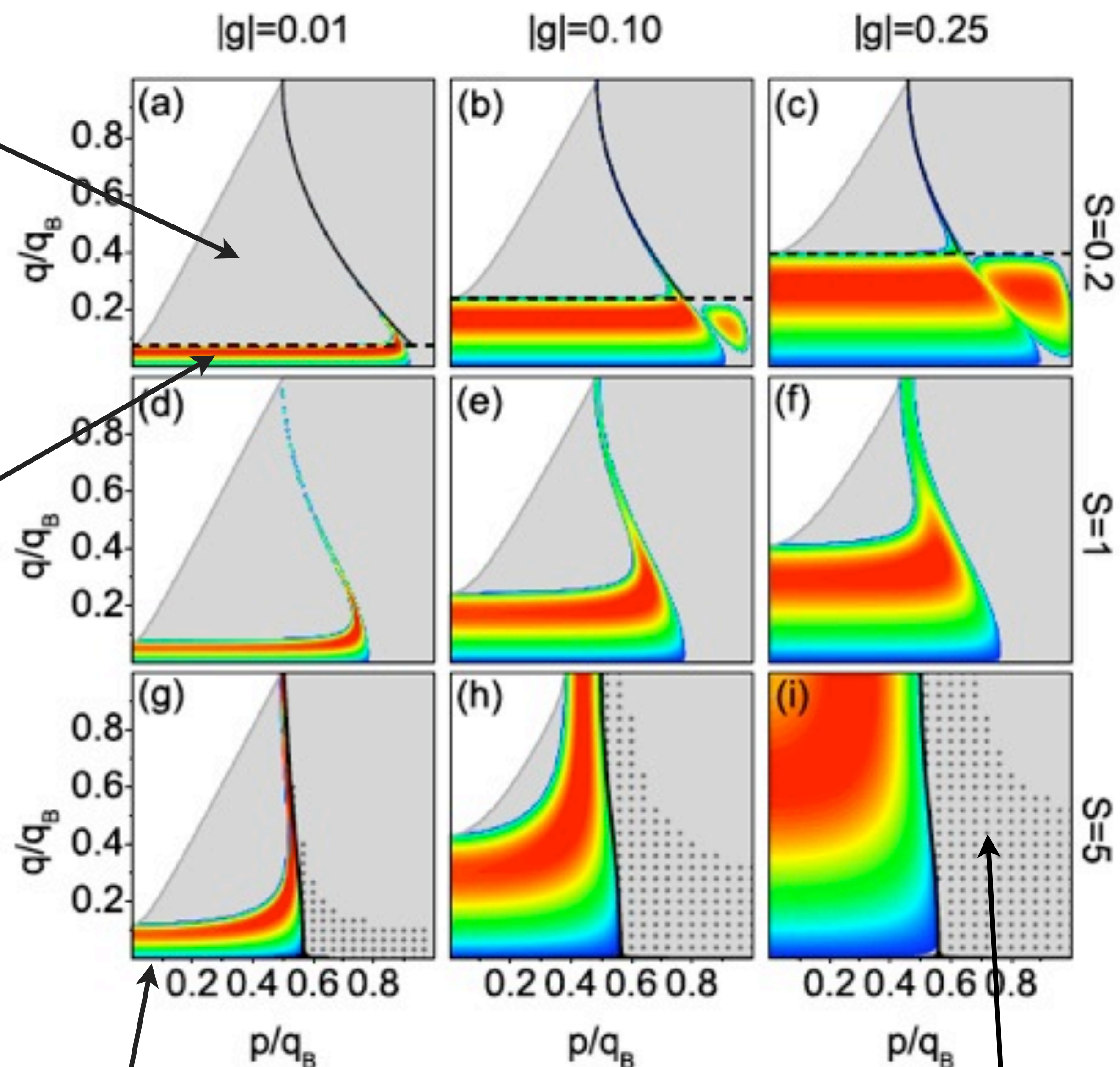



L. Fallani *et al.*, Phys. Rev. Lett. **91**, 240405 (2003)
(see also B. Eiermann *et al.*, Phys. Rev. Lett. 91, 060402 (2003))

# Attractive interactions

G. Barontini and M. Modugno, Phys. Rev. A 76, 041601(R) (2007)

Gray area: Landau instability
The system is energetically unstable for any velocity
→ no superfluidity according to the Landau criterion.




Colored regions: DI
(Color scale ~ growth rate of the unstable modes = Im($\omega_{pq}$))

DI at low p, can be stabilized above a critical threshold
(opposite behaviour of that for repulsive BECs)

*naive explanation*: changing the sign of g ≃ change of sign of m

Weak interactions/shallow lattices: DI takes place via long wavelength (low q) excitations → no site-to-site dephasing, collective oscillations

Dotted region in (g-i): regimes of DI for the repulsive case

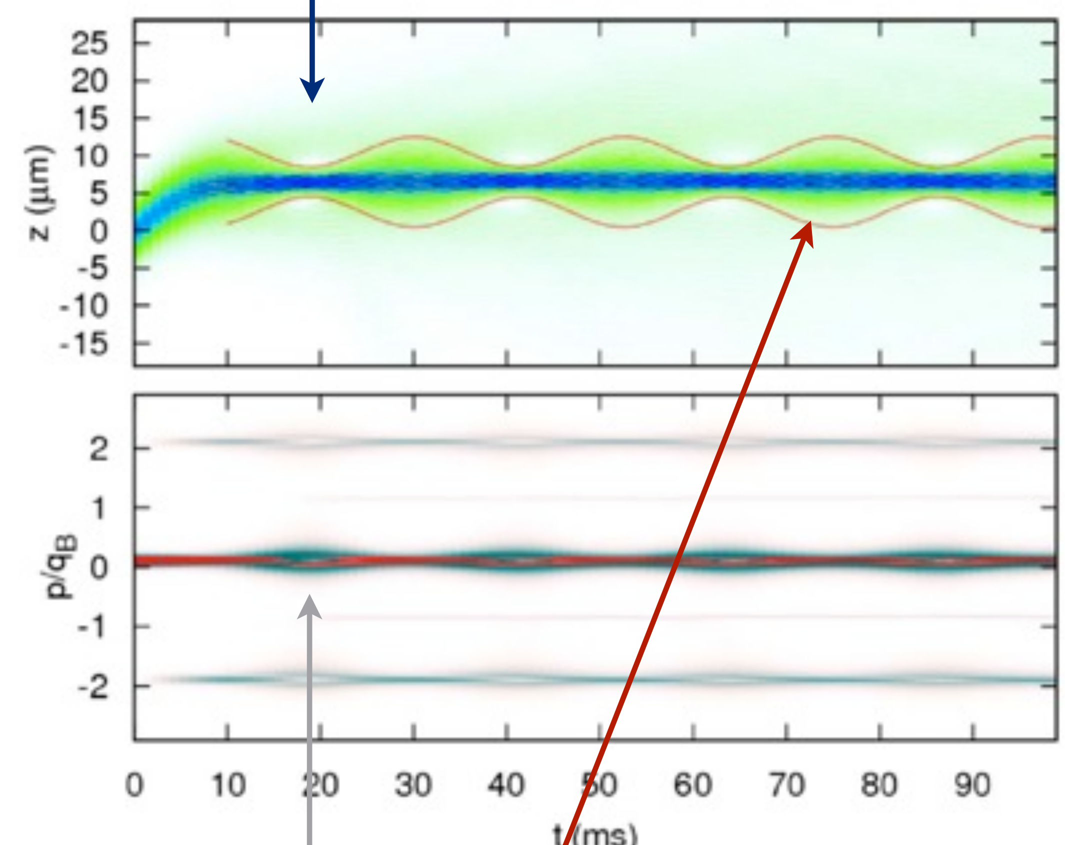
Negative m and DI appear in separate regions
→ tune the dispersion relation to negative values avoiding the effects of DI

# Waveguide expansion: $v=0.2\ v_B$

density modulations over several sites of the lattice

Axial density plot of the BEC
as a function of time during the
expansion in the waveguide....

......and its momentum distribution

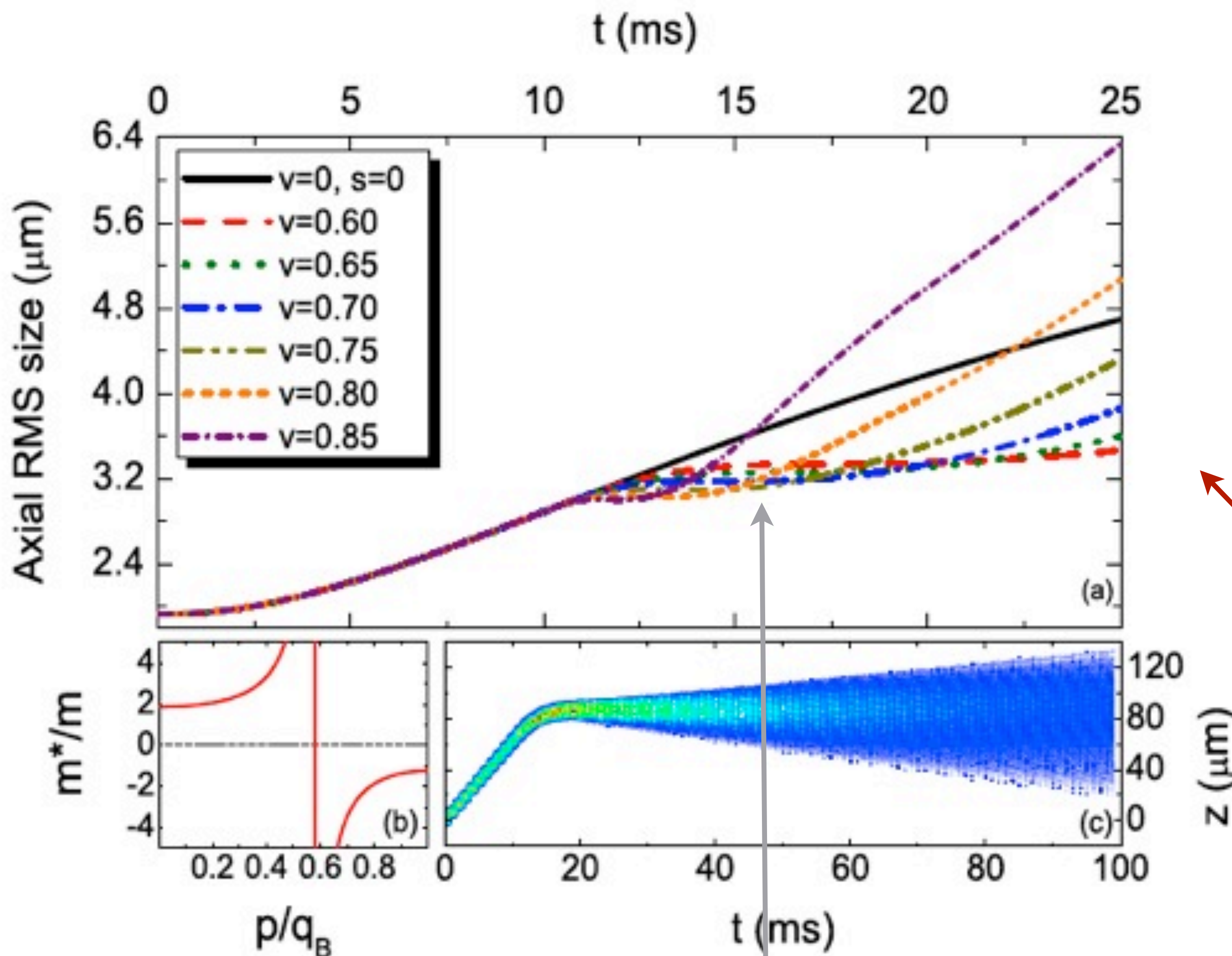



rapid population of modes at small q ($\simeq 0.058 q_B$)
(most unstable modes of the uniform system)

breathing-like oscillation, no decoherence
as observed so far with repulsive BECs

oscillation accounted for by the <u>real part of the excitations spectrum</u> + momentum spread due to finite size (fitted frequency = 44.5 Hz ~ real part of frequency of the most unstable modes)

# $0.5v_B < v < v_B$




$|m^*| < m$ :
enhancement of the expansion near the band edge

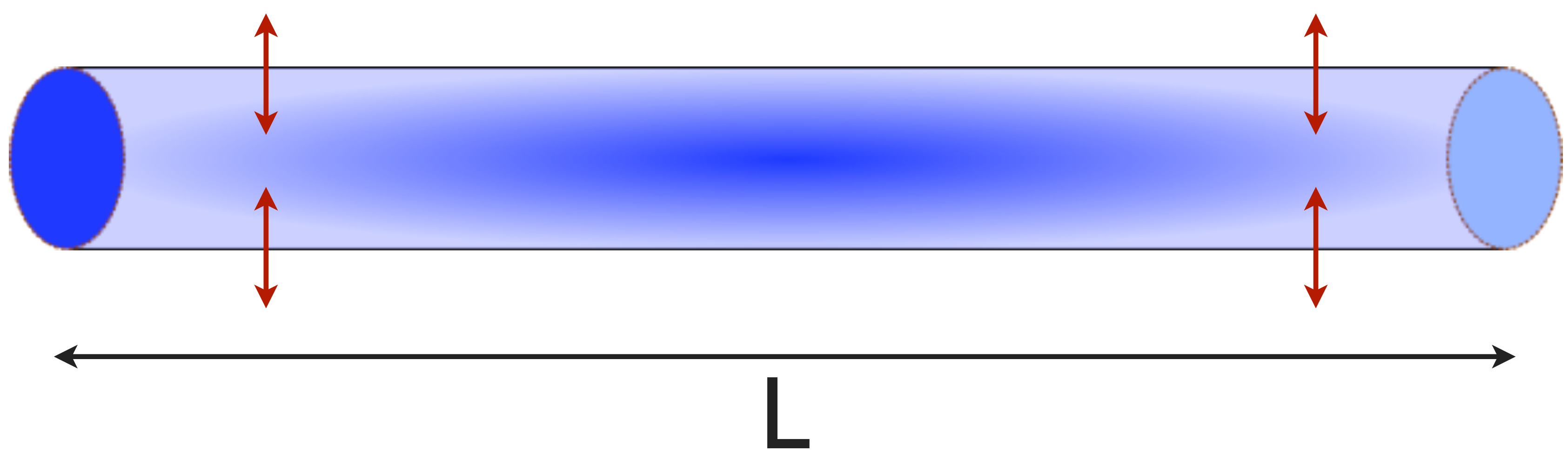
attractive interactions are turned into an effective repulsion: boost of the expansion with respect to the free case even when $|m^*|>m$

reduced expansion for very large $|m^*|$

change of sign of $m^*$ $\Rightarrow$ time-reversed evolution

$\Rightarrow$ contraction of a BEC initially expanding outwards

# Parametric instability & pattern formation




Modulation of the transverse confinement at frequency $\Omega$
(GPE + initial quantum/thermal fluctuations)

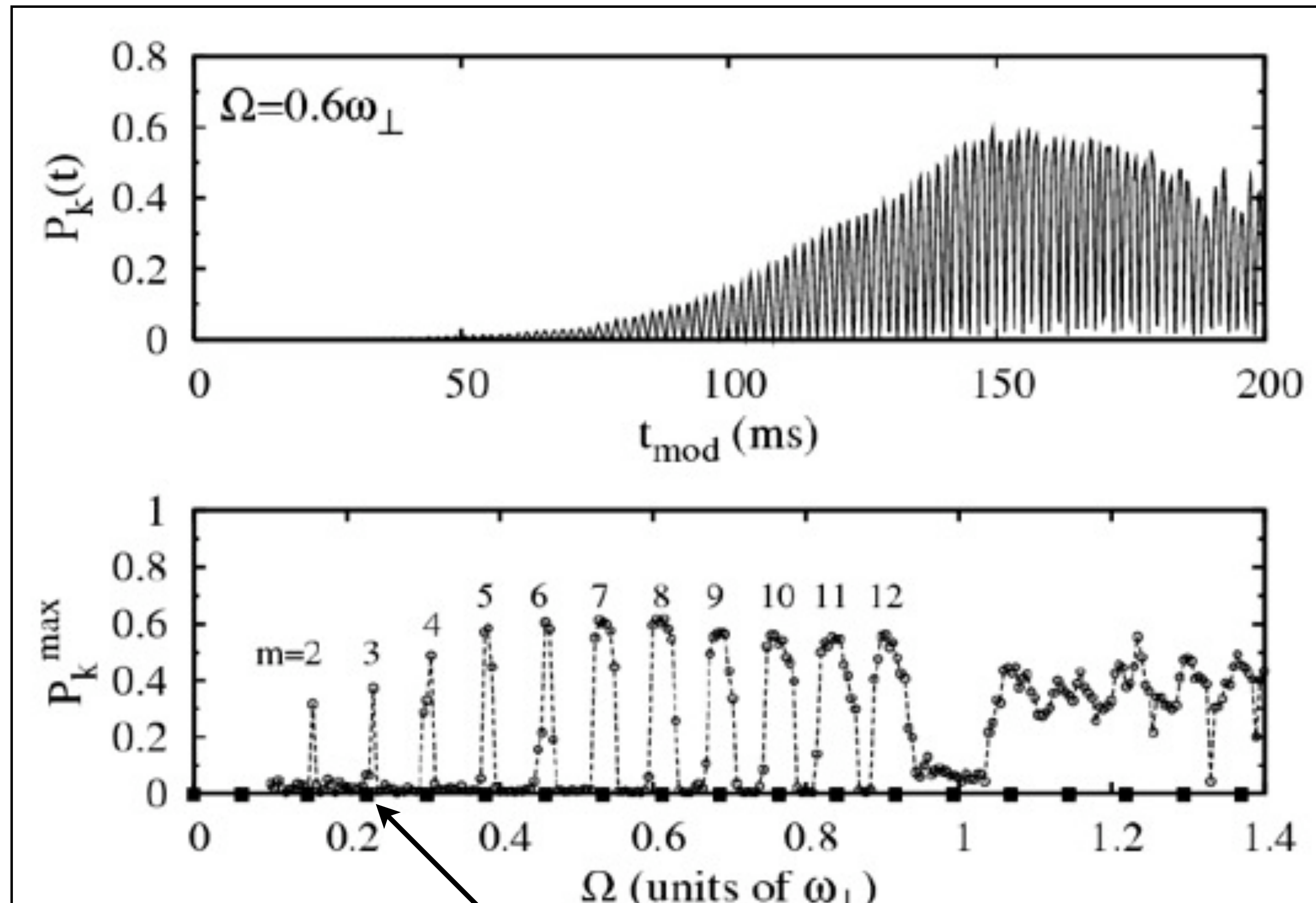
$\rightarrow$ parametric amplification of counter-propagating axial phonons of frequency $\omega(k) = \Omega/2$

M. Modugno, C. Tozzo and F. Dalfovo, Phys. Rev. A **74**, 061601(R) (2006)

periodic boundary contitions → discrete spectrum, $k = m2\pi/L$
→ resonance behaviour

amplitude of the ±k axial phonons
as a function of $t_{mod}$
for $\Omega=0.6\omega_\perp$ (resonance *m*=8)

maximum value of $P_k$
as a function of $\Omega$




black squares: $2\omega(k)$ of the Bogoliubov excitations parametric amplification of phonons → spontaneous pattern formation of standing waves with *m*-periodicity

analogous to Faraday's instability

M. C. Cross and P. P. Hohenberg, Rev. Mod. Phys. 65, 851 (1993)

K. Staliunas et al., Phys. Rev. Lett. 89, 210406 (2002)

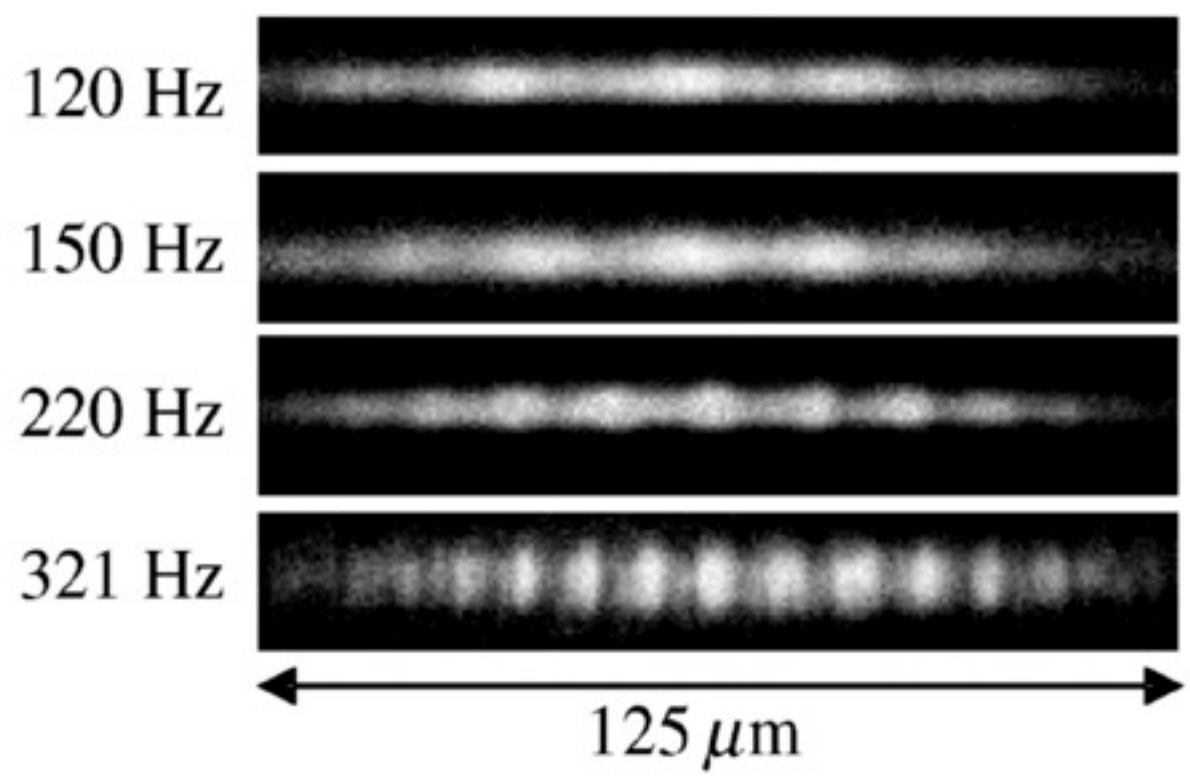
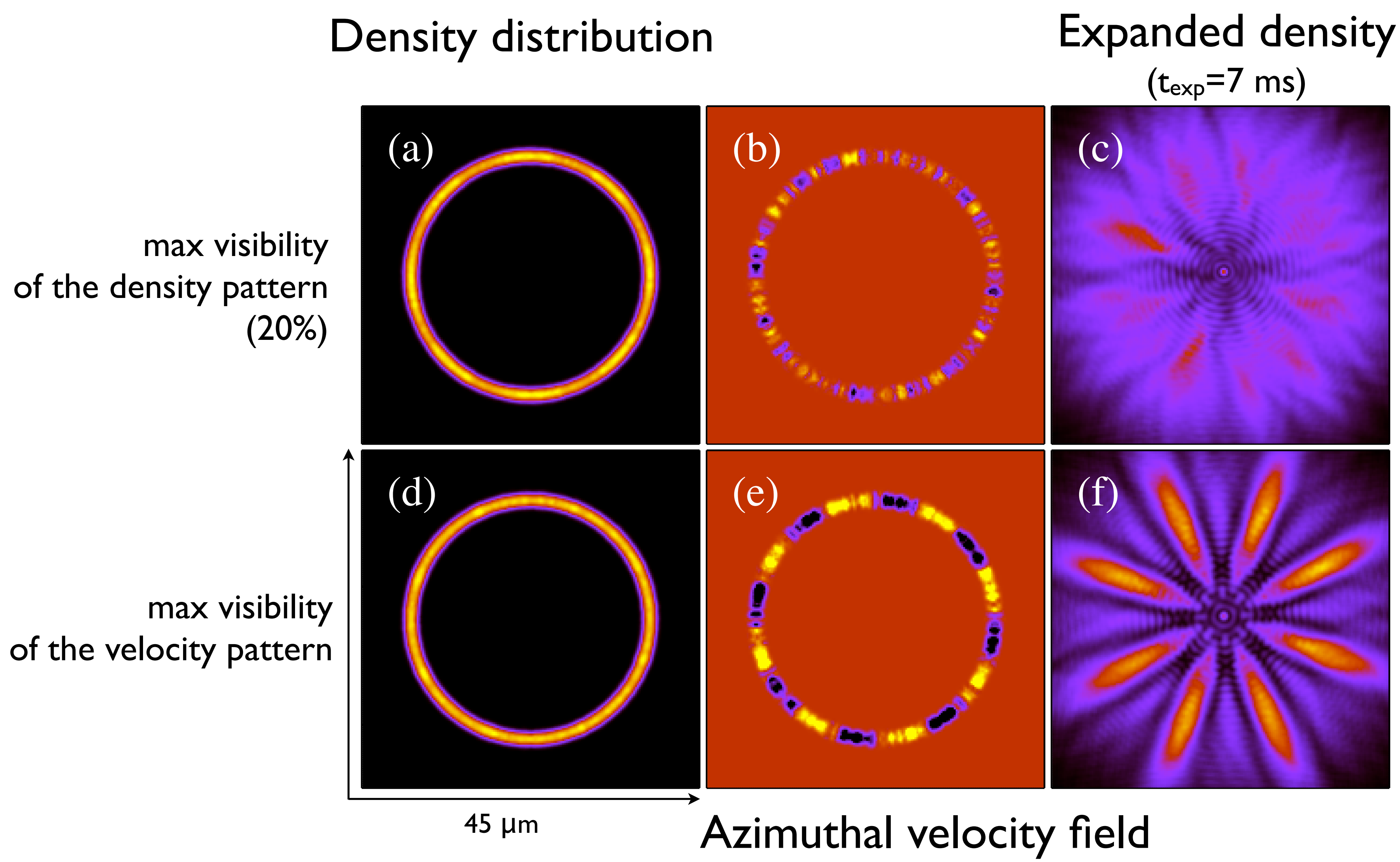



FIG. 1. In-trap absorption images of Faraday waves in a BEC. Frequency labels for each image represent the driving frequency at which the transverse trap confinement is modulated.

P. Engels et al., Phys. Rev. Lett. 98, 095301 (2007)

# toroidal geometry

- periodic boundary conditions
- produced in current experiments
  S. Gupta *et al.*, Phys. Rev. Lett. **95**, 143201 (2005)
  A. S. Arnold *et al.*, Phys. Rev. A **73**, 041606(R) (2006)
  C. Ryu *et al.*, Phys. Rev. Lett. **99**, 260401 (2006)
- tools for observing fundamental properties: quantized circulation, persistent currents,
  matter-wave interference, sound waves and solitons in low-D, rotation sensors

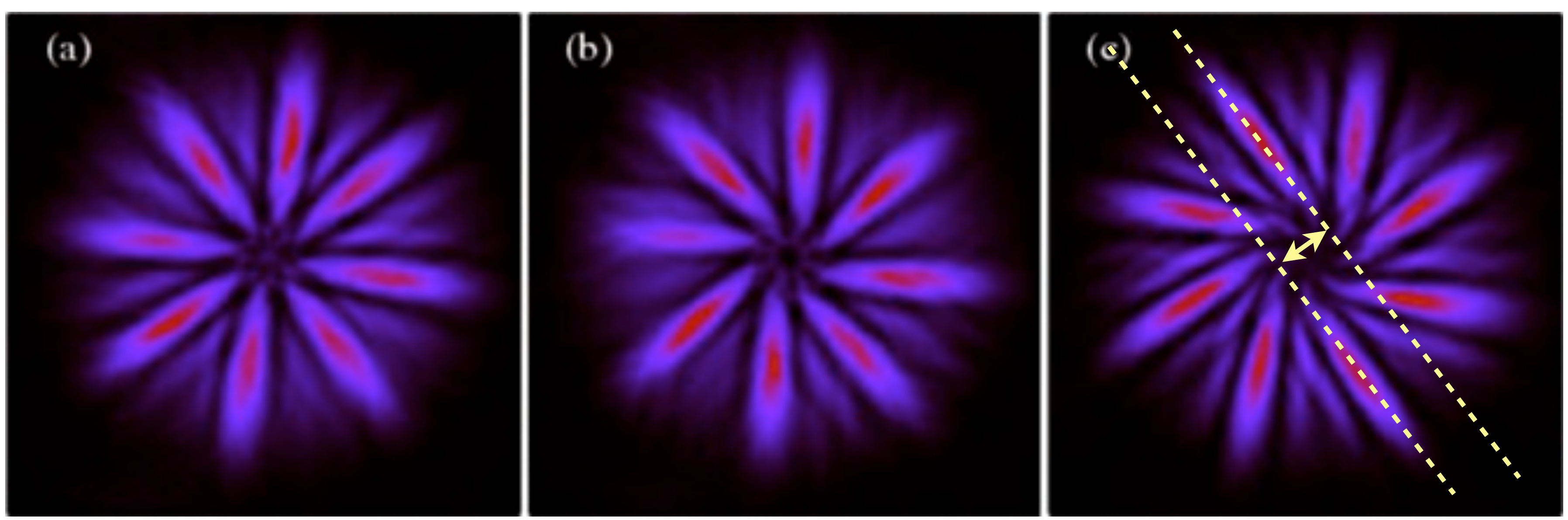



Periodic pattern in the velocity field → interference fringes of atoms expanding in preferred directions: flower-like structure with *m* "petals" in the expanded density profile, reflecting the periodicity of the initial pattern.

# quantized circulation

The pattern formation is affected by the presence of quantized circulation:
if the condensate is initially rotating with angular momentum $L_z = \kappa\hbar$ per particle:

- in-situ pattern: rotates at the same angular velocity of the condensate
- expandend pattern: misalignment of opposite petals proportional to $\kappa$




→ sensitive detection of quantized circulation