Hawking radiation in a BEC: problems & open questions

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Bill Unruh's Idea

Sound waves in irrotational flow $\delta \boldsymbol{v} = \boldsymbol{\nabla}\phi$

$$\left(\frac{\partial}{\partial t} + \boldsymbol{\nabla}\cdot\boldsymbol{v}_0\right)\frac{\varrho_0}{c_{\rm s}^2}\left(\frac{\partial}{\partial t} + \boldsymbol{v}_0\cdot\boldsymbol{\nabla}\right)\phi = \boldsymbol{\nabla}\cdot(\varrho_0\boldsymbol{\nabla}\phi)$$

Scalar field ϕ in curved space-time

$$\Box_{\rm eff}\phi = \frac{1}{\sqrt{-g_{\rm eff}}}\partial_\mu\left(\sqrt{-g_{\rm eff}}\, g_{\rm eff}^{\mu\nu}\partial_\nu\phi\right) = 0$$

Painlevé-Gullstrand-Lemaître metric

$$g_{\rm eff}^{\mu\nu} = \frac{1}{\varrho_0 c_{\rm s}}\begin{pmatrix} 1 & \boldsymbol{v}_0 \\ \boldsymbol{v}_0 & \boldsymbol{v}_0\otimes\boldsymbol{v}_0 - c_{\rm s}^2\mathbf{1}\end{pmatrix}$$

Phonons (quantized)	$\leftrightarrow$	Quantum fields
Fluid flow (classical)	$\leftrightarrow$	Gravitational field
Euler equation	$\neq$	Einstein equations

W. G. Unruh, Phys. Rev. Lett. **46**, 1351 (1981)

Event Horizon

Collapsing matter

Singularity

Light cones, light rays

Event horizon

Distortion of quantum fluctuations

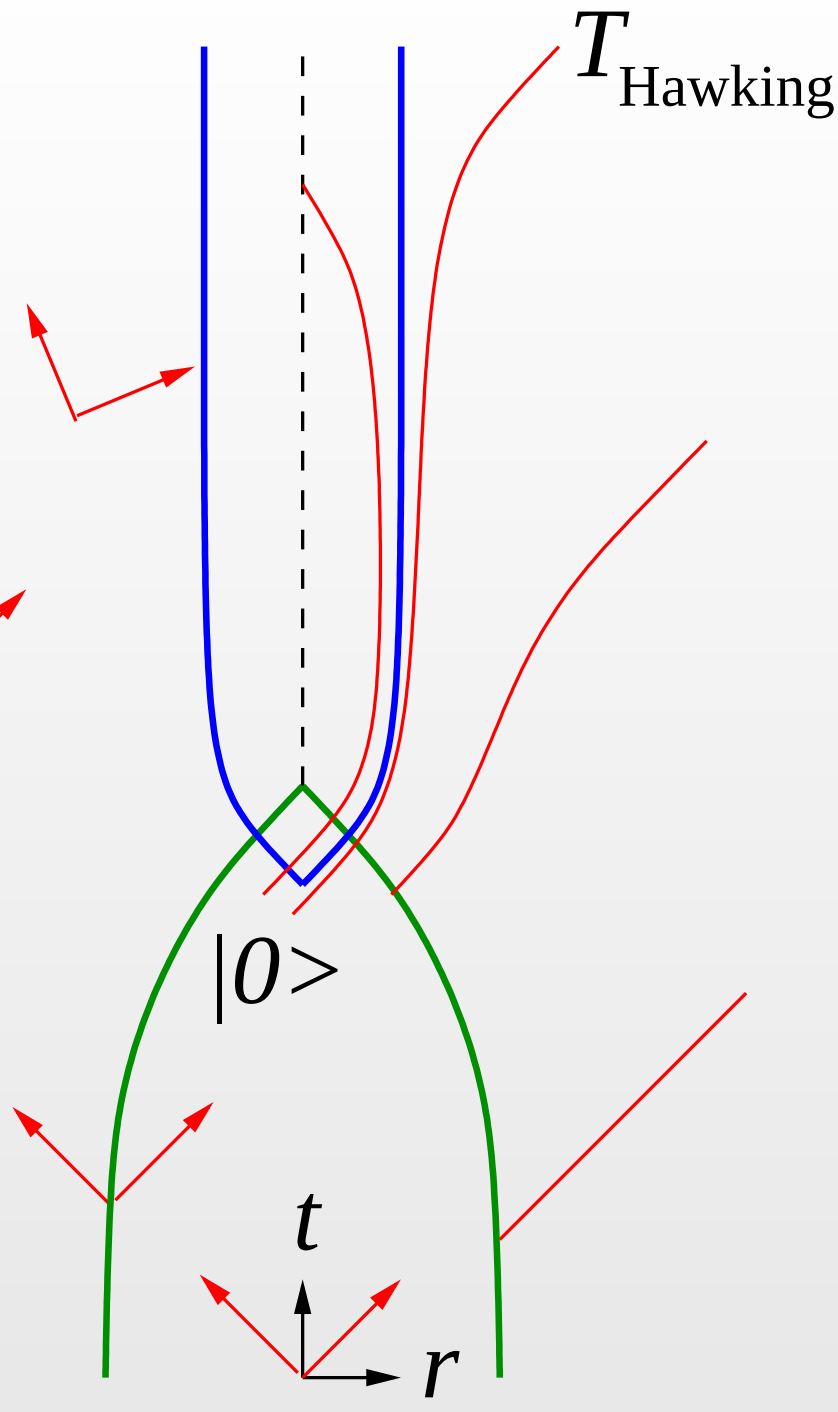
$\longrightarrow$ Hawking effect

$$T_{\text{Hawking}} = \frac{1}{8\pi M}\frac{\hbar c^3}{G_{\text{N}} k_{\text{B}}}$$

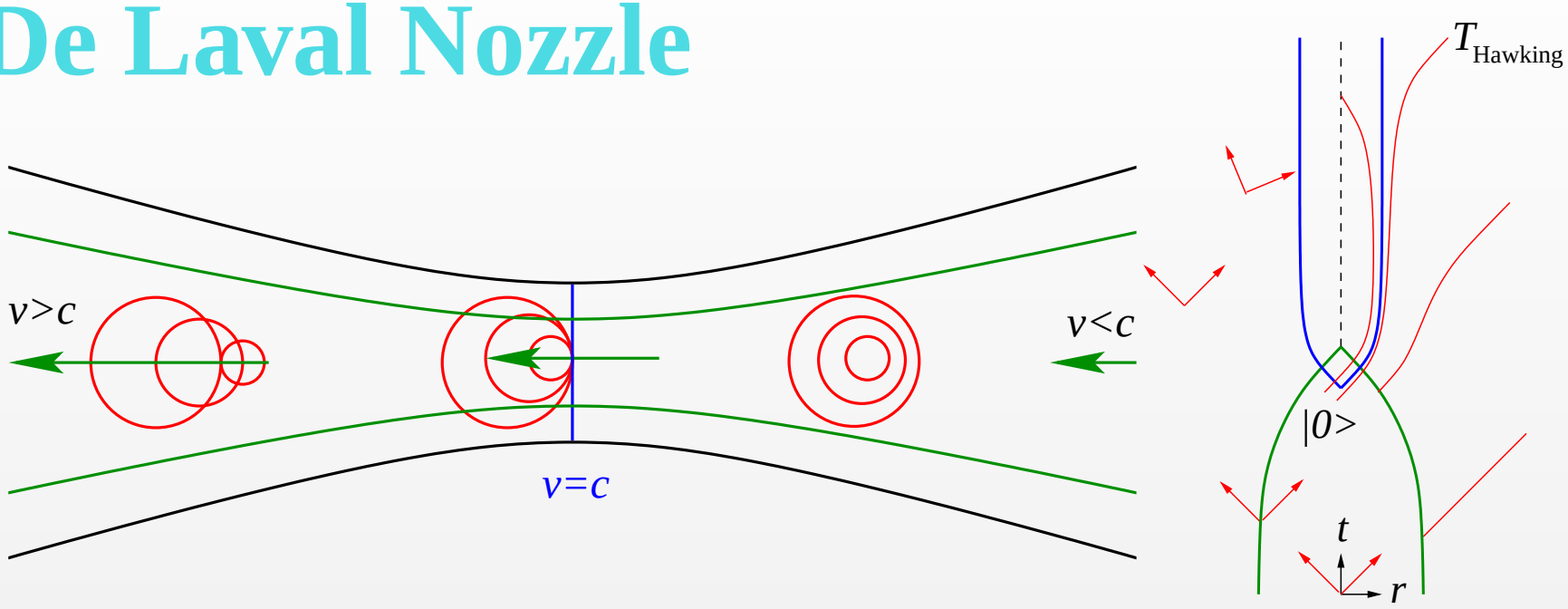
Problem: red-shift

$\longrightarrow$ Planck scale

$$E_{\text{Pl}} = \sqrt{\hbar c^5 / G_{\text{N}}} \approx 2 \times 10^9 J$$



De Laval Nozzle



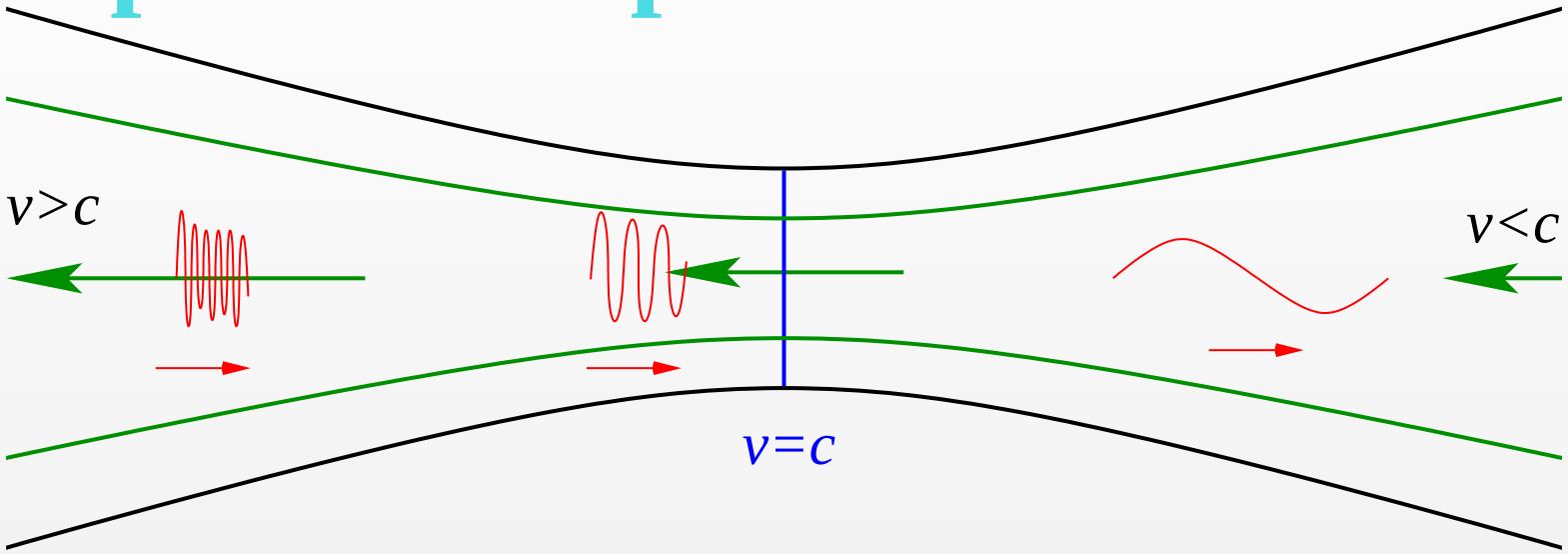
Fluid flow, Wall, Sound waves, Event horizon

$$T_{\text{Hawking}} = \frac{\hbar}{2\pi\, k_{\text{B}}} \left| \frac{\partial}{\partial r} \left(v_0 - c_{\text{s}} \right) \right| = \mathcal{O}(\text{nK} \ldots \text{K})$$

Toy model for underlying theory (quantum gravity?)
→ trans-Plancian problem

Experimentally measurable?!?

Impact of Dispersion Relation



W. G. Unruh and R. S.,

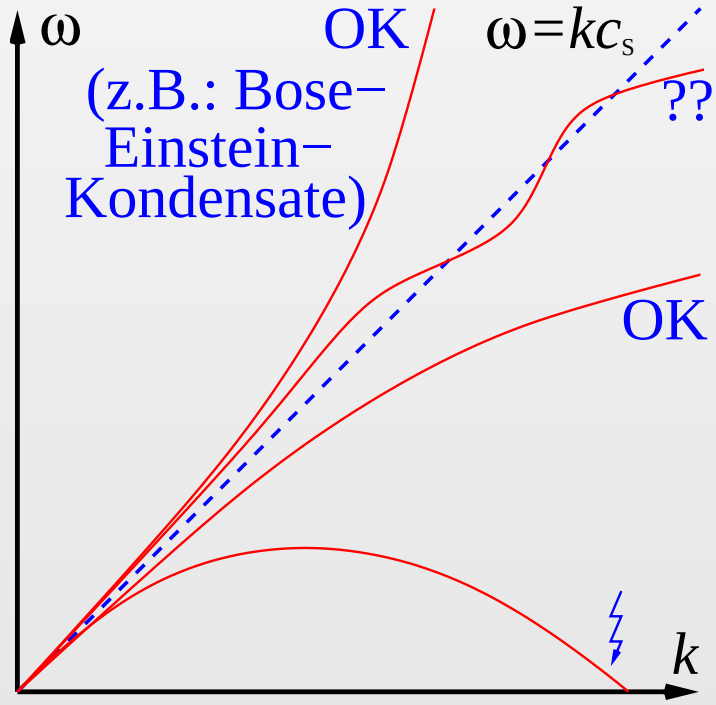
Phys. Rev. D **71**, 024028 (2005)

[Corley, Jacobson, Parentani etc.]

Stationary vs dynamical effects (initial burst), cf.

C. Barceló, L.J. Garay, G. Jannes,

Phys. Rev. D **79**, 024016 (2009)



Eddington-Finkelstein Metric

Metric $ds^2 = (1 - 2M/r)dV^2 - 2dV\,dr$

$$\left(2\partial_V\partial_r + \partial_r\left[1 - \frac{2M}{r} + f(\partial_r^2)\right]\partial_r\right)\Phi = 0\,,$$

Dispersion relation $f(\partial_r^2) \longrightarrow$ Hawking temperature

$$T_{\text{Hawking}}(\omega) = \frac{v_{\text{group}}(\omega)v_{\text{phase}}(\omega)}{8\pi M}$$

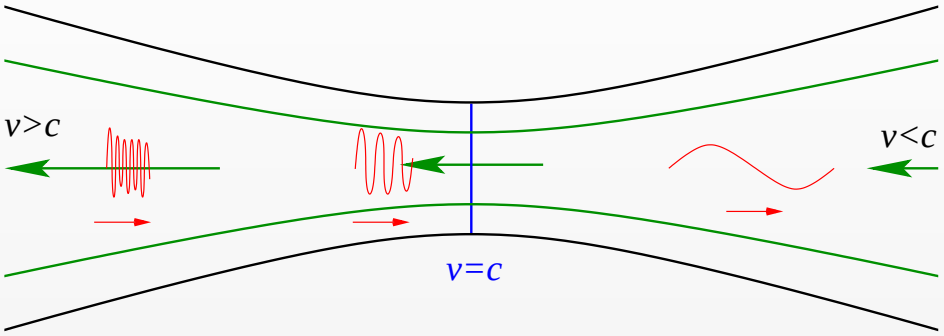
Ultra-violet catastrophe?

$$\omega^2 = c^2k^2(1 + \ell_{\text{P}}^2k^2) \quad \text{vs} \quad \omega = \frac{ck}{\sqrt{1 - \ell_{\text{P}}^2k^2}}$$

Cf. special relativity $E = mc^2/\sqrt{1 - v^2/c^2}$

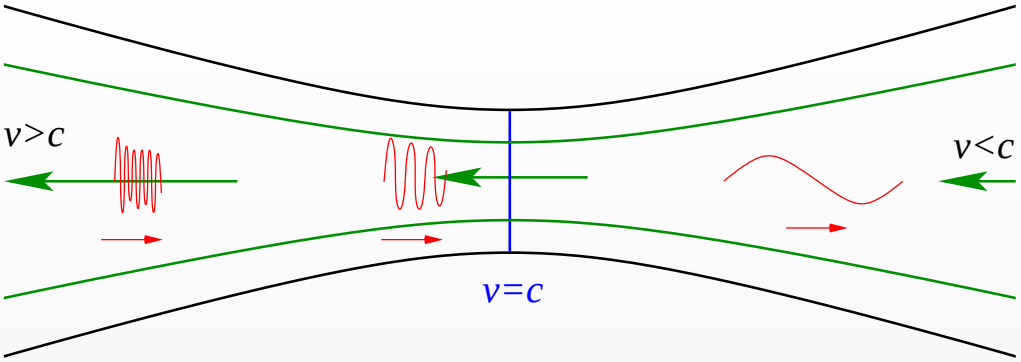
R. S. and W. G. Unruh, Phys. Rev. D rapid comm. (2008)

Deviations from Hawking



- No scale separation $dv/dr \not\ll c/\xi$
  (e.g., superradiance from single vortex???)
- Friction with walls $(\Omega + vk)^2 + i\gamma\Omega = \omega^2(k)$
  Lab system: Ω (observer at rest at $r \uparrow \infty$)
  Local fluid system: ω (freely falling observer)
- Non-adiabaticity away from horizon
  e.g., $\omega^2(k)$ and/or $v(r)$ too violent
- "Black-hole laser" instability
  white hole horizon (recycling???)

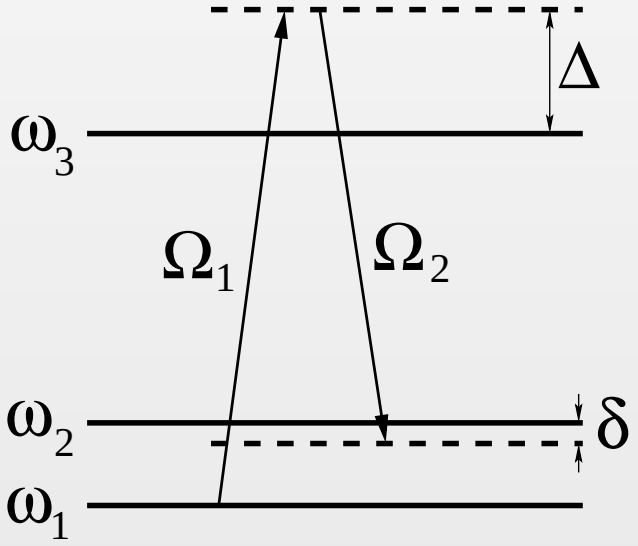
Detectability?



A few phonons
with small energies
$E = \mathcal{O}(10^{-13}\,\mathrm{eV})$
(broad spectrum)

Idea: doubly detuned optical Raman transitions

Transition only if missing
energy $\delta = \mathcal{O}(10^{-13}\,\mathrm{eV})$
is compensated by
simultaneous absorption
of a phonon with this
(or a higher) energy δ



Single phonons $\longrightarrow$ single atoms (countable)

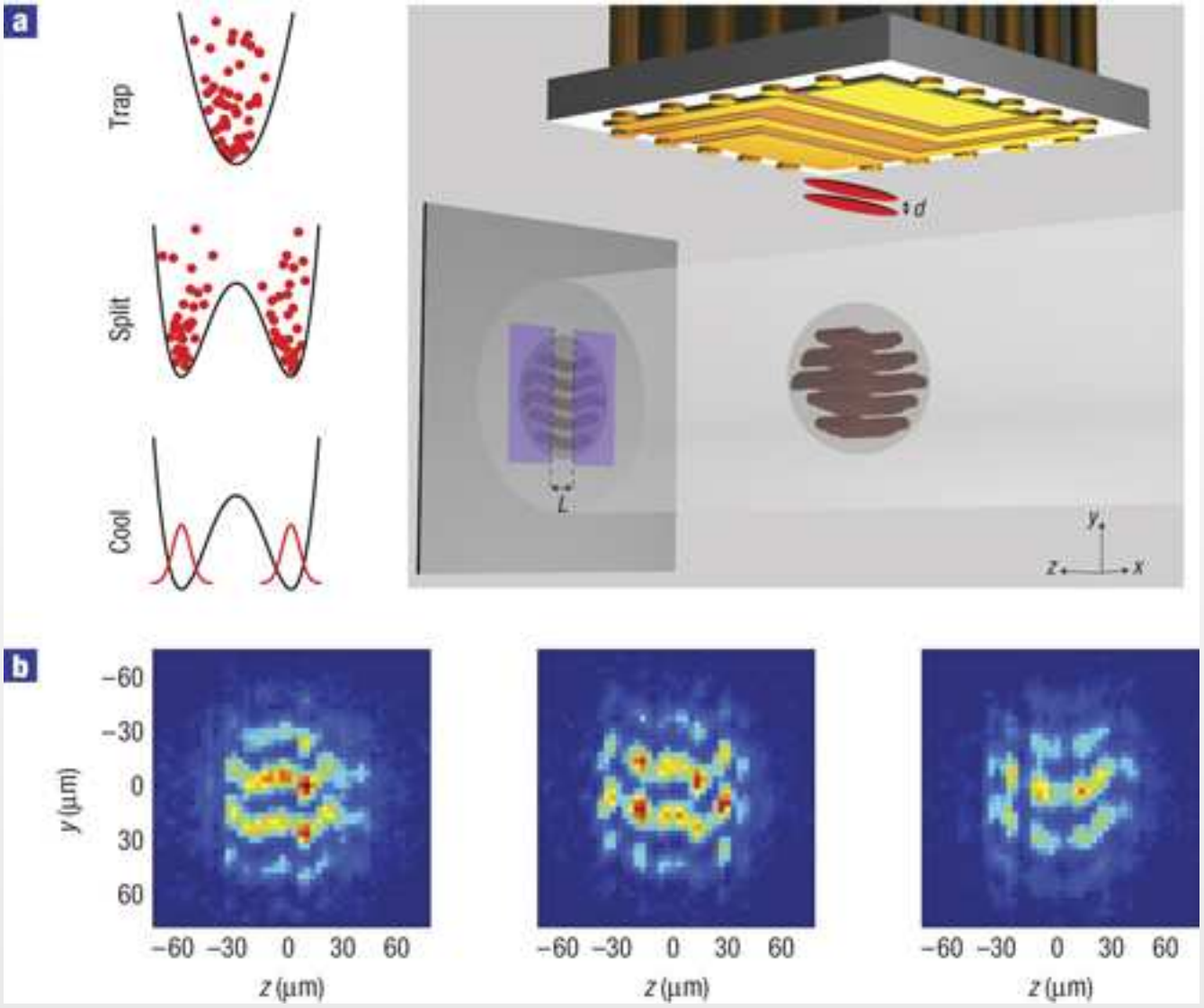
R. S., Phys. Rev. Lett. **97**, 190405 (2006) [idea together with Mark Raizen]

Detectability: Fluctuations

interference
$\longrightarrow$ phase
fluctuations
thermal &
quantum!

5000 atoms
$T = 30\text{nK}$
$\varrho = 60/\mu\text{m}$
$\xi = 0.3\mu\text{m}$

flowing
condensates?



S. Hofferberth *et al*, Nature Physics **4**, 489 (2008).

Advantages of BECs

- No walls $\longrightarrow$ low friction (superfluid)
- Slow speed of sound $c_s = \mathcal{O}(\mathrm{mm/s})$
- Tunable parameters $\varrho, c_s, V(\boldsymbol{r})$ etc.
- Quasi 1D-flows
- Ultra-low temperatures $T = \mathcal{O}(10\,\mathrm{nK})$

Major drawback: Meta-stable state

Three-Body Losses

Major drawback of BEC:

Meta-stable state $\longrightarrow$ three-body losses $\longrightarrow$ heating

J. Dziarmaga & K. Sacha, Phys. Rev. A **68**, 043607 (2003).

E.g., 10^7 particles with 1% quantum depletion and 1% three-body losses $\longrightarrow \mathcal{O}(10^3)$ phonons (noise)

R. S., Phil. Trans. Roy. Soc. (London) A **366**, 2895 (2008).

R. S., Class. Quant. Grav. **25**, 114027 (2008).

"Typical" parameters:
three-body heating $\gg$ Hawking effect

S. Wüster, Phys. Rev. A **78**, 021601 (2008).

New ideas are needed...

Apparent Horizon

Expanding condensate

Sound waves

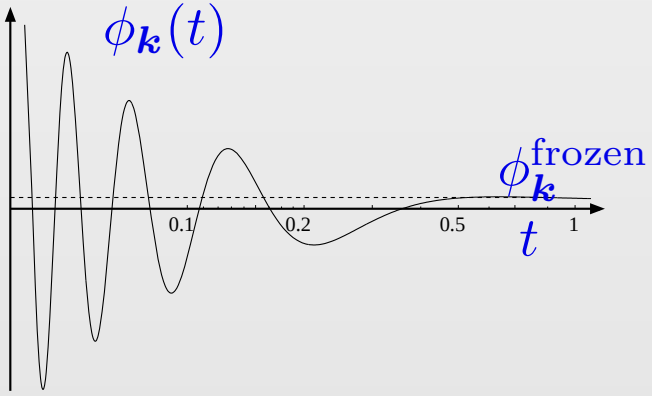
Apparent horizon at

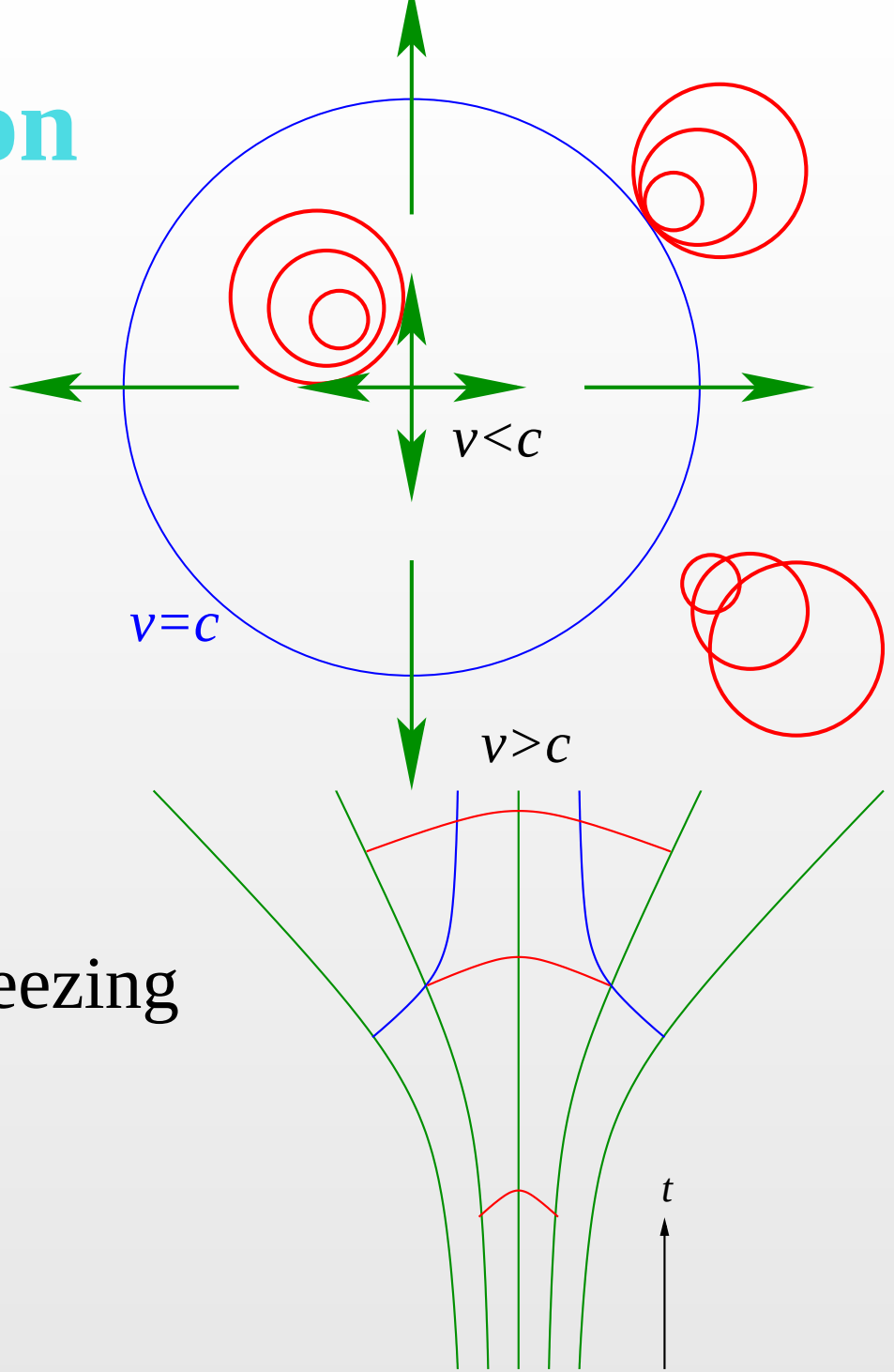
$v_{\text{Fluid}} = c_{\text{Sound}}$

Analogous to
expanding universe

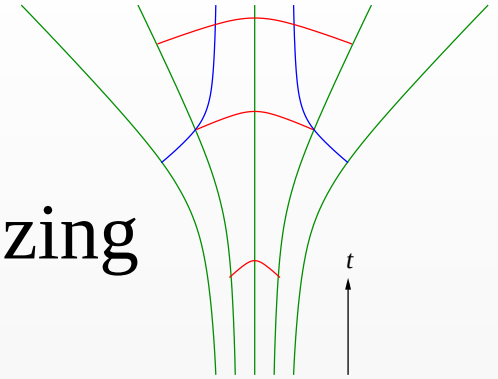
Phonon modes:

+ oscillation (initially)
+ horizon crossing $\longrightarrow$ freezing

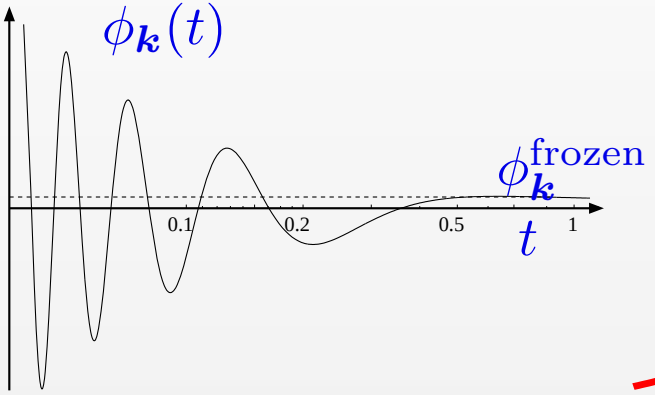


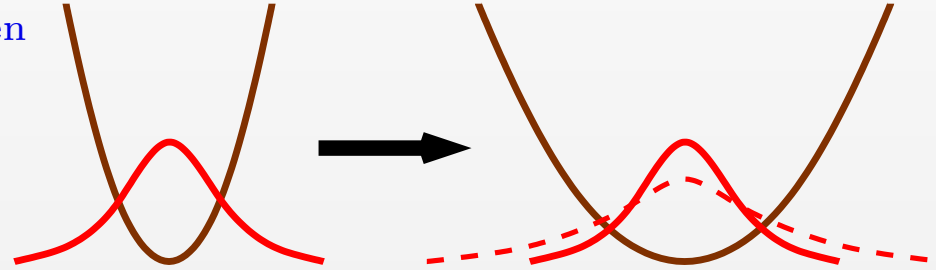


Quantum Fluctuations



Oscillation → horizon crossing → freezing





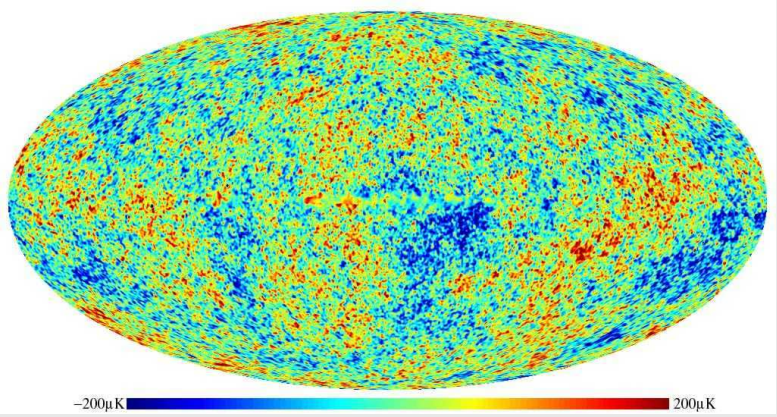
Amplification of quantum fluctuations (squeezing)
(in complete analogy to early universe, e.g., WMAP)

Quantum limit (accuracy)
of time-of-flight imaging

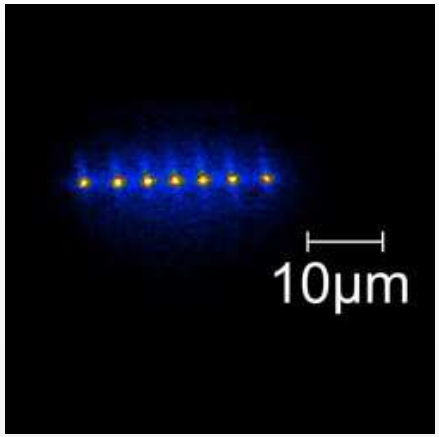
$$\langle \delta\hat{\varrho}(\boldsymbol{r})\delta\hat{\varrho}(\boldsymbol{r'})\rangle = \mathcal{O}(1\%)$$

M. Uhlmann, Y. Xu, and R. S.,

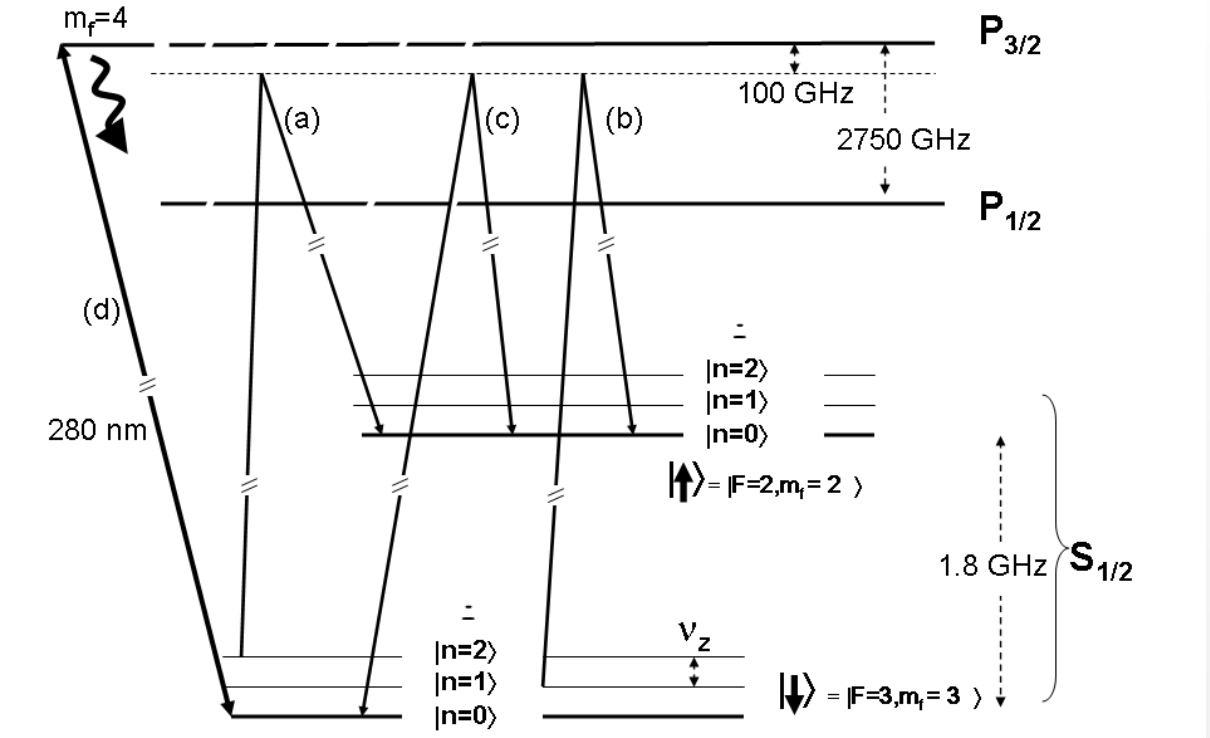
New J. Phys. **7**, 248 (2005)

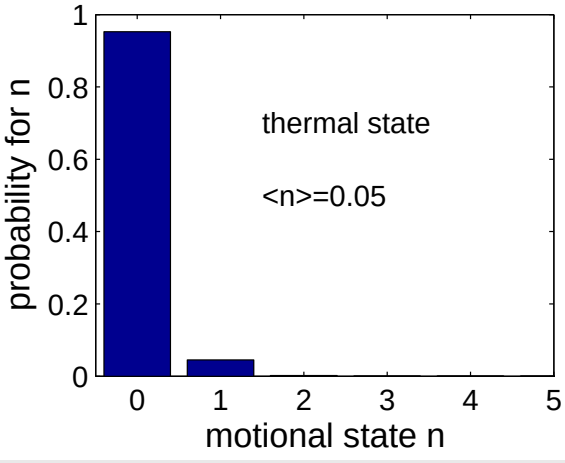


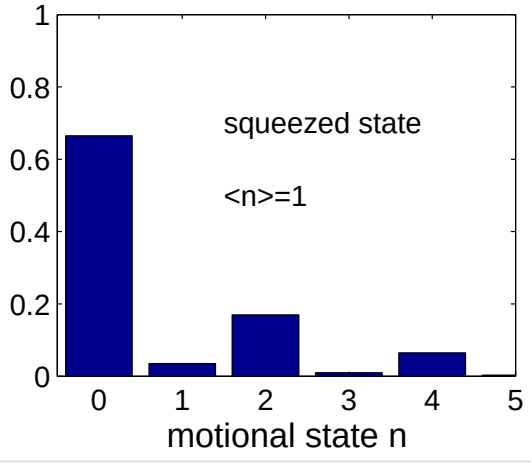
Ion Trap



Expansion or Contraction of Ion chain → Expanding Universe → Phonon pair creation Hawking???



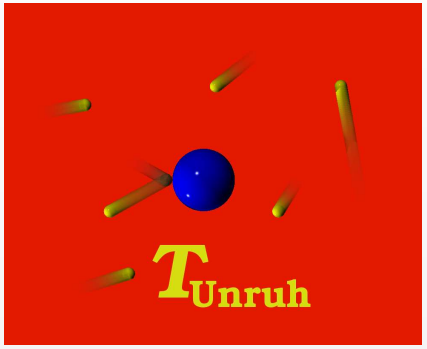


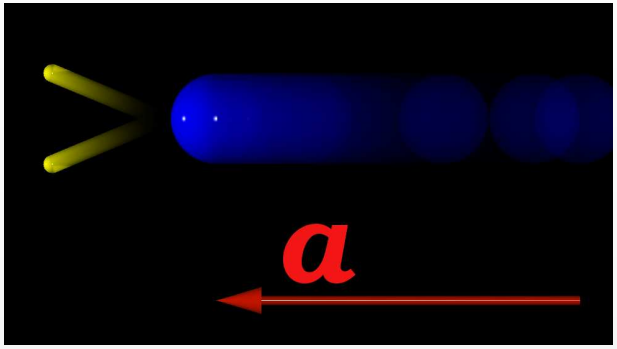


R. S. *et al.*, Phys. Rev. Lett. **99**, 201301 (2007).

Signatures of the Unruh Effect

Accelerated frame
$T_{\mathrm{Unruh}} = \hbar a/(2\pi c)$
Laboratory frame:
Pair creation





W. G. Unruh and R. M. Wald, Phys. Rev. D **29**, 1047 (1984).

Electrons $\gamma = 300$ in strong Laser fields $10^{18}\mathrm{W/cm}^2$

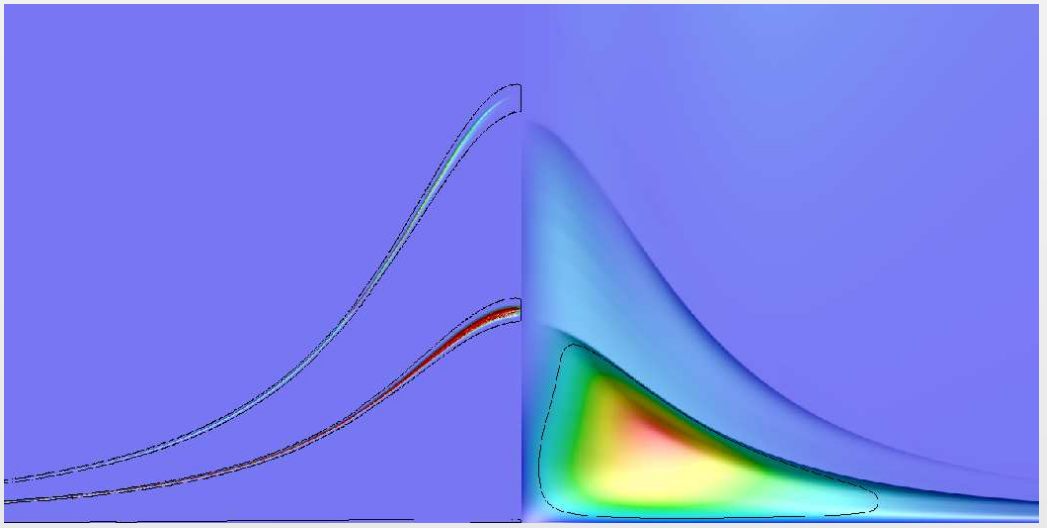
$0 < E < 2\mathrm{MeV}$

$0 < \vartheta < 1/100$

Larmor (left)

Unruh (right)

$P_{1e}^{\mathrm{Unruh}} = \mathcal{O}(10^{-10})$



R. S., G. Schaller, and D. Habs, Phys. Rev. Lett. **97**, 121302 (2006);

Announcement

12th MG Meeting

June 13th-18th 2009, Paris (France)

Session BHT 6 Analogue Gravity

June 1st: deadline for early registration

http://www.icra.it/MG/mg12/en/welcome.htm