

Hawking radiation in a BEC: problems & open questions

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Bill Unruh's Idea

Sound waves in irrotational flow $\delta \mathbf{v} = \nabla \phi$

$$\left(\frac{\partial}{\partial t} + \nabla \cdot \mathbf{v}_0 \right) \frac{\rho_0}{c_s^2} \left(\frac{\partial}{\partial t} + \mathbf{v}_0 \cdot \nabla \right) \phi = \nabla \cdot (\rho_0 \nabla \phi)$$

Scalar field ϕ in curved space-time

$$\square_{\text{eff}} \phi = \frac{1}{\sqrt{-g_{\text{eff}}}} \partial_\mu \left(\sqrt{-g_{\text{eff}}} g_{\text{eff}}^{\mu\nu} \partial_\nu \phi \right) = 0$$

Painlevé-Gullstrand-Lemaître metric

$$g_{\text{eff}}^{\mu\nu} = \frac{1}{\rho_0 c_s} \begin{pmatrix} 1 & \mathbf{v}_0 \\ \mathbf{v}_0 & \mathbf{v}_0 \otimes \mathbf{v}_0 - c_s^2 \mathbf{1} \end{pmatrix}$$

Phonons (quantized) $\leftrightarrow$ Quantum fields

Fluid flow (classical) $\leftrightarrow$ Gravitational field

Euler equation $\neq$ Einstein equations

Event Horizon

Collapsing matter

Singularity

Light cones, light rays

Event horizon

Distortion of quantum fluctuations

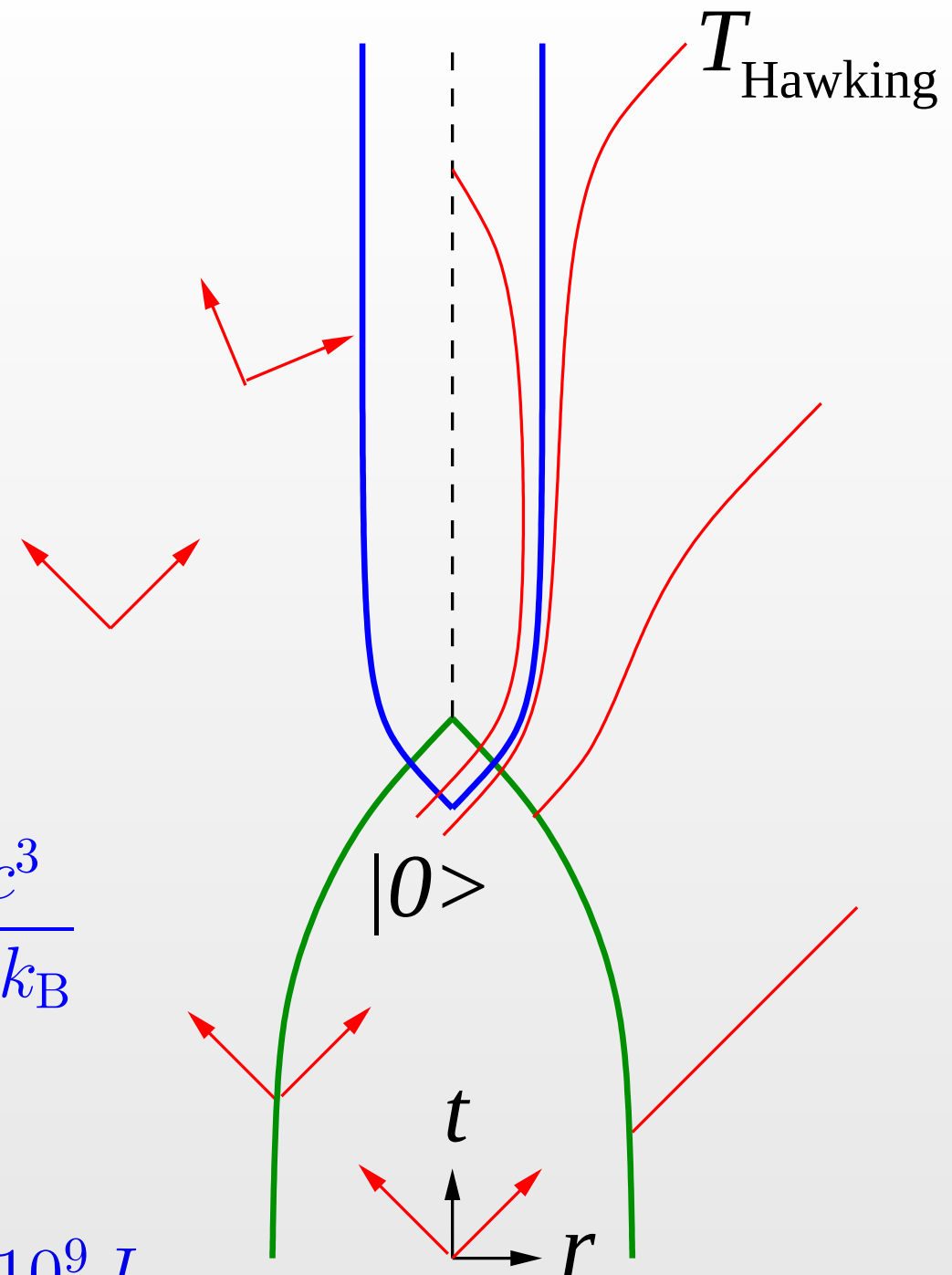
→ Hawking effect

$$T_{\text{Hawking}} = \frac{1}{8\pi M} \frac{\hbar c^3}{G_N k_B}$$

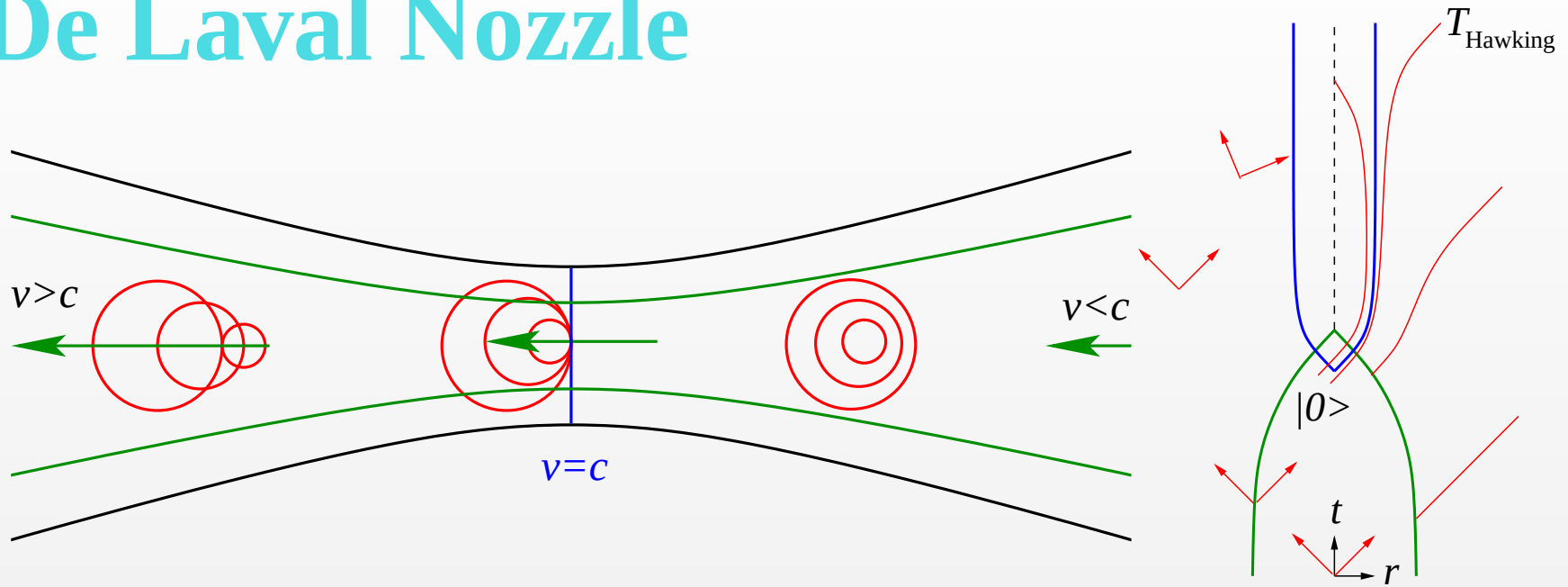
Problem: red-shift

→ Planck scale

$$E_{\text{Pl}} = \sqrt{\hbar c^5 / G_N} \approx 2 \times 10^9 J$$



De Laval Nozzle



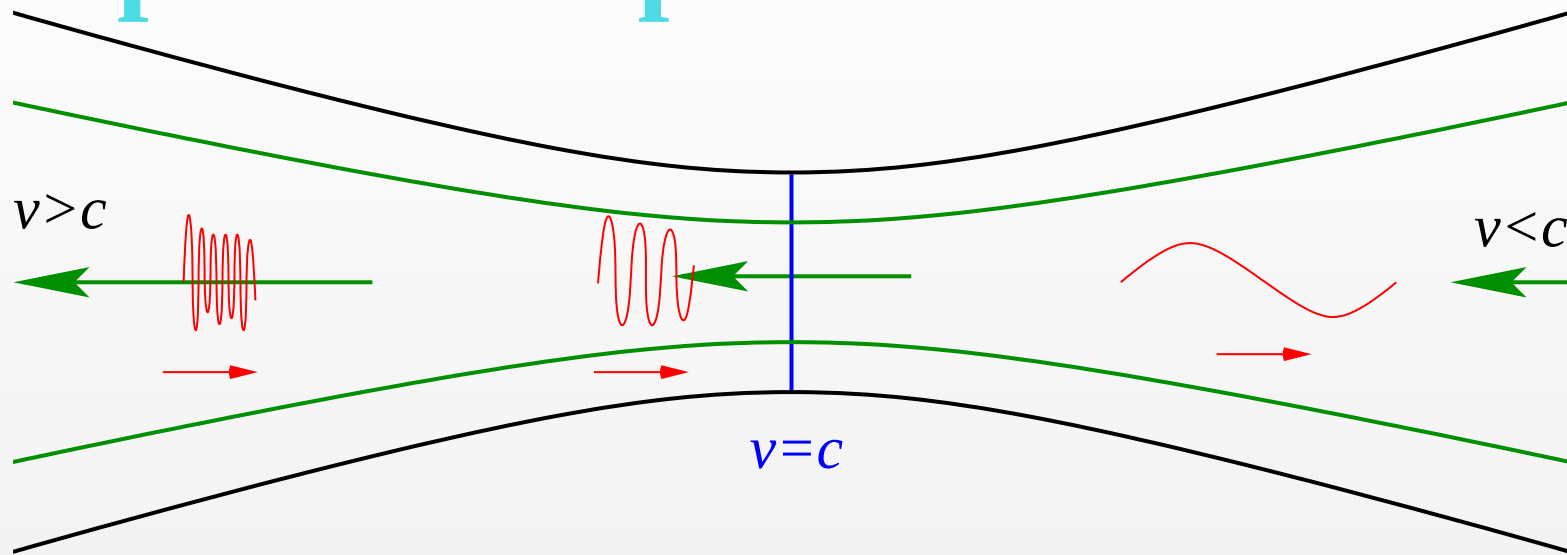
Fluid flow, Wall, Sound waves, Event horizon

$$T_{\text{Hawking}} = \frac{\hbar}{2\pi k_B} \left| \frac{\partial}{\partial r} (v_0 - c_s) \right| = \mathcal{O}(\text{nK} \dots \text{K})$$

Toy model for underlying theory (quantum gravity?)
 → trans-Planckian problem

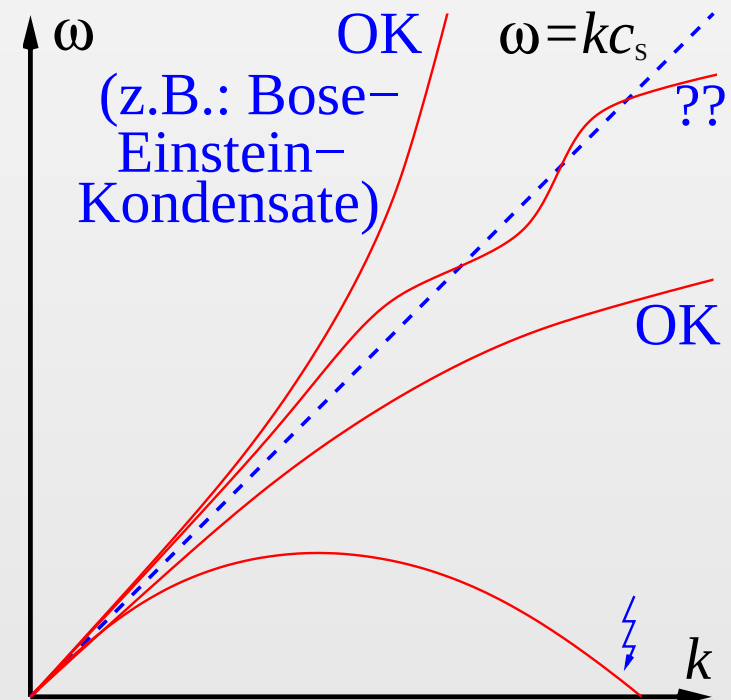
Experimentally measurable?!?

Impact of Dispersion Relation



W. G. Unruh and R. S.,
 Phys. Rev. D **71**, 024028 (2005)
 [Corley, Jacobson, Parentani etc.]

Stationary vs dynamical
 effects (initial burst), cf.
 C. Barceló, L.J. Garay, G. Jannes,
 Phys. Rev. D **79**, 024016 (2009)



Eddington-Finkelstein Metric

Metric $ds^2 = (1 - 2M/r)dV^2 - 2dV dr$

$$\left(2\partial_V \partial_r + \partial_r \left[1 - \frac{2M}{r} + f(\partial_r^2) \right] \partial_r \right) \Phi = 0 ,$$

Dispersion relation $f(\partial_r^2) \rightarrow$ Hawking temperature

$$T_{\text{Hawking}}(\omega) = \frac{v_{\text{group}}(\omega)v_{\text{phase}}(\omega)}{8\pi M}$$

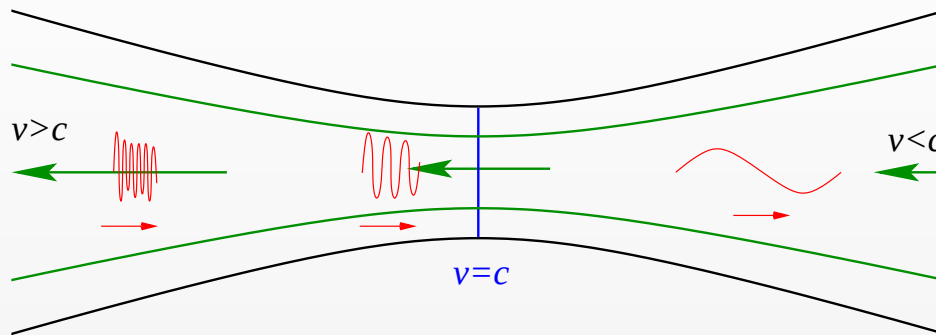
Ultra-violet catastrophe?

$$\omega^2 = c^2 k^2 (1 + \ell_P^2 k^2) \quad \text{vs} \quad \omega = \frac{ck}{\sqrt{1 - \ell_P^2 k^2}}$$

Cf. special relativity $E = mc^2 / \sqrt{1 - v^2/c^2}$

R. S. and W. G. Unruh, Phys. Rev. D rapid comm. (2008)

Deviations from Hawking



- No scale separation $dv/dr \not\ll c/\xi$
(e.g., superradiance from single vortex???)
- Friction with walls $(\Omega + vk)^2 + i\gamma\Omega = \omega^2(k)$
Lab system: Ω (observer at rest at $r \uparrow \infty$)
Local fluid system: ω (freely falling observer)
- Non-adiabaticity away from horizon
e.g., $\omega^2(k)$ and/or $v(r)$ too violent
- “Black-hole laser” instability
white hole horizon (recycling???)

Detectability?

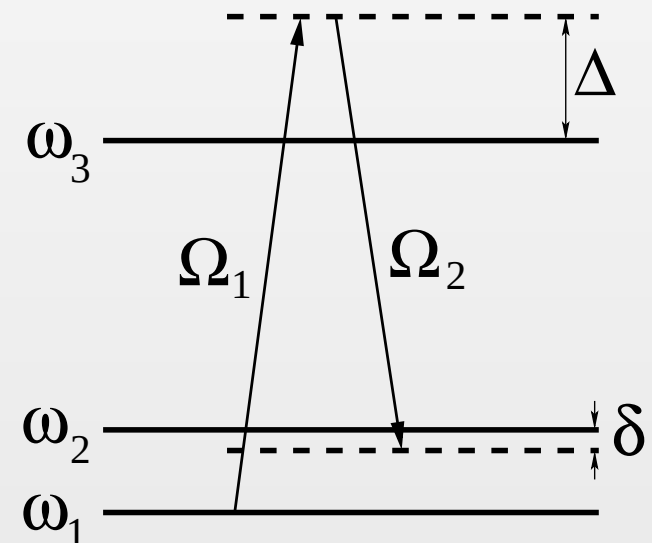
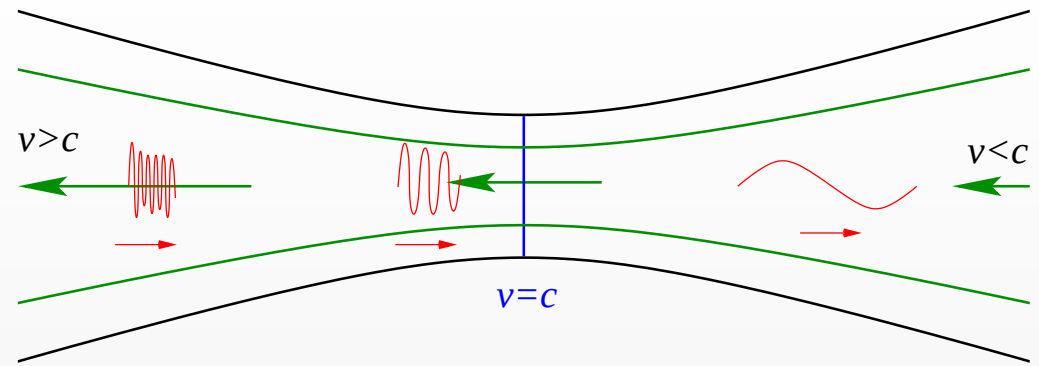
A few phonons
with small energies
 $E = \mathcal{O}(10^{-13} \text{ eV})$
(broad spectrum)

Idea: doubly detuned optical Raman transitions

Transition only if missing
energy $\delta = \mathcal{O}(10^{-13} \text{ eV})$
is compensated by
simultaneous absorption
of a phonon with this
(or a higher) energy δ

Single phonons $\rightarrow$ single atoms (countable)

R. S., Phys. Rev. Lett. **97**, 190405 (2006) [idea together with Mark Raizen]



Detectability: Fluctuations

interference
→ phase
fluctuations
thermal &
quantum!

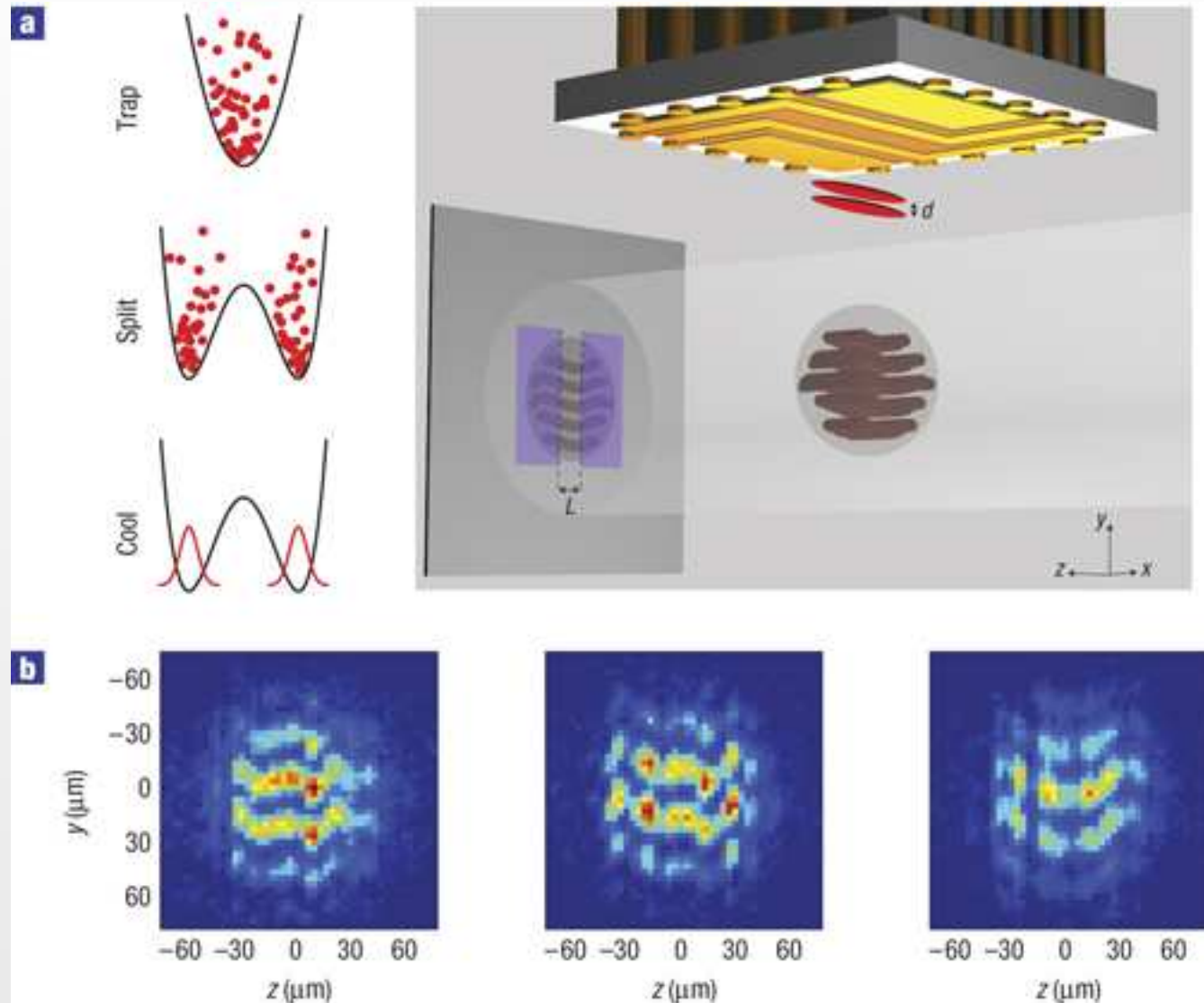
5000 atoms

$$T = 30\text{nK}$$

$$\varrho = 60/\mu\text{m}$$

$$\xi = 0.3\mu\text{m}$$

flowing
condensates?



S. Hofferberth *et al*, Nature Physics **4**, 489 (2008).

Advantages of BECs

- No walls $\rightarrow$ low friction (superfluid)
- Slow speed of sound $c_s = \mathcal{O}(\text{mm/s})$
- Tunable parameters $\rho, c_s, V(\mathbf{r})$ etc.
- Quasi 1D-flows
- Ultra-low temperatures $T = \mathcal{O}(10 \text{ nK})$

Major drawback: Meta-stable state

Three-Body Losses

Major drawback of BEC:

Meta-stable state $\rightarrow$ three-body losses $\rightarrow$ heating

J. Dziarmaga & K. Sacha, Phys. Rev. A **68**, 043607 (2003).

E.g., 10^7 particles with 1% quantum depletion and 1% three-body losses $\rightarrow \mathcal{O}(10^3)$ phonons (noise)

R. S., Phil. Trans. Roy. Soc. (London) A **366**, 2895 (2008).

R. S., Class. Quant. Grav. **25**, 114027 (2008).

“Typical” parameters:

three-body heating $\gg$ Hawking effect

S. Wüster, Phys. Rev. A **78**, 021601 (2008).

New ideas are needed...

Apparent Horizon

Expanding condensate

Sound waves

Apparent horizon at

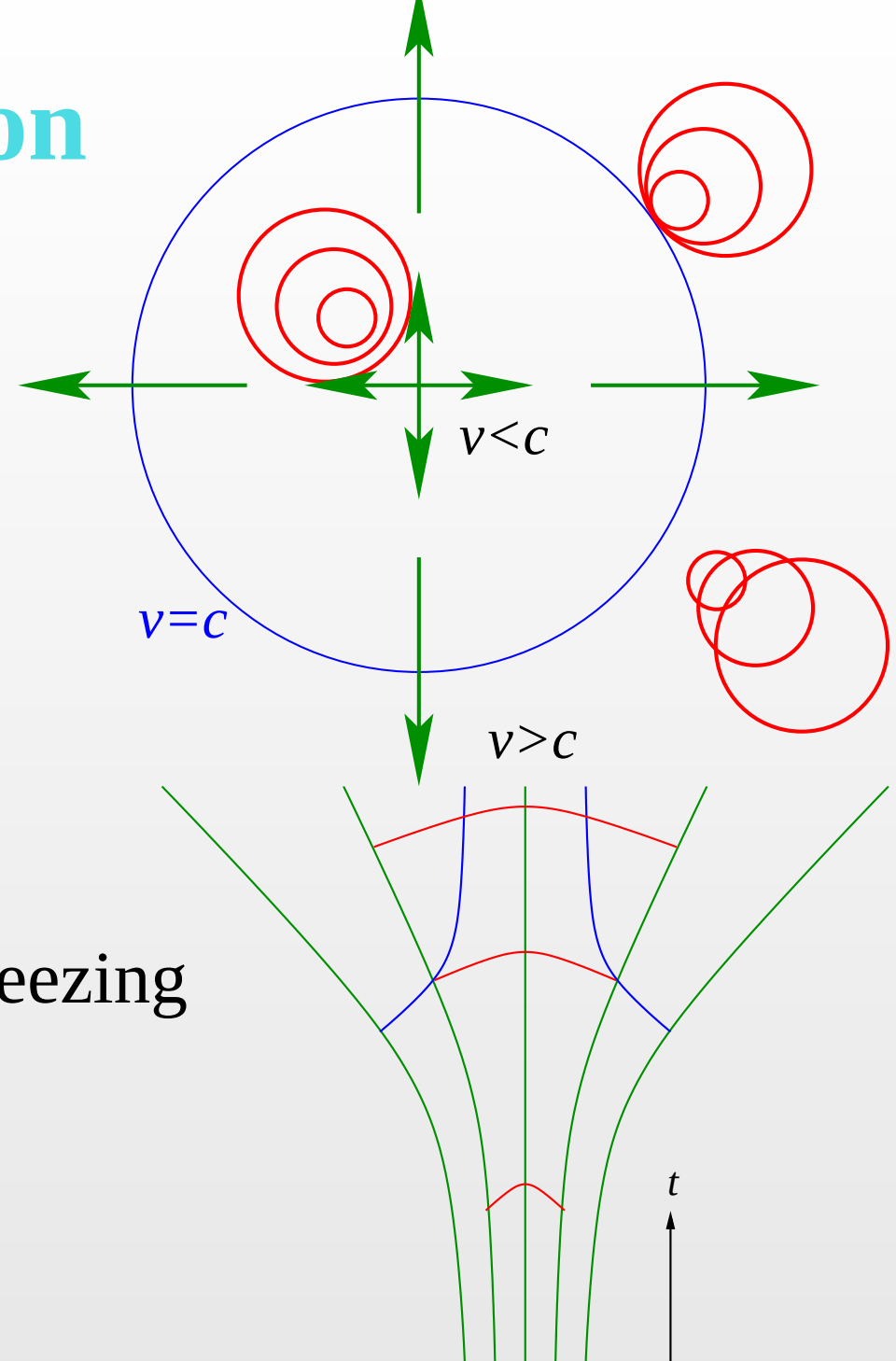
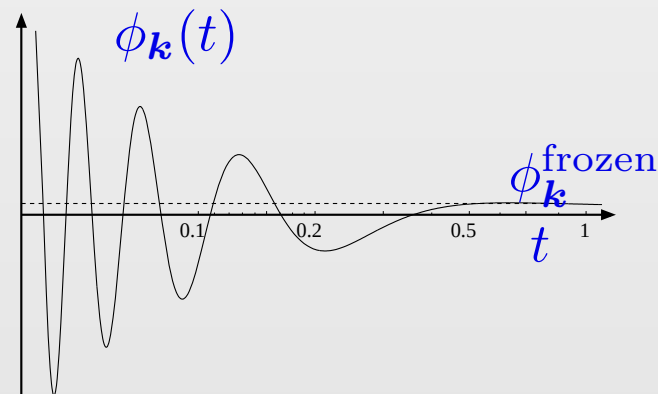
$$v_{\text{Fluid}} = c_{\text{Sound}}$$

Analogous to
expanding universe

Phonon modes:

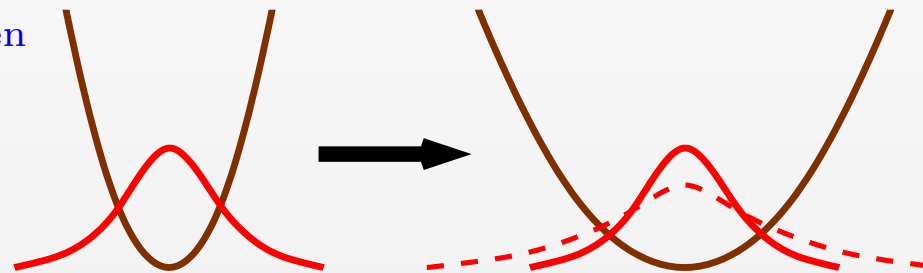
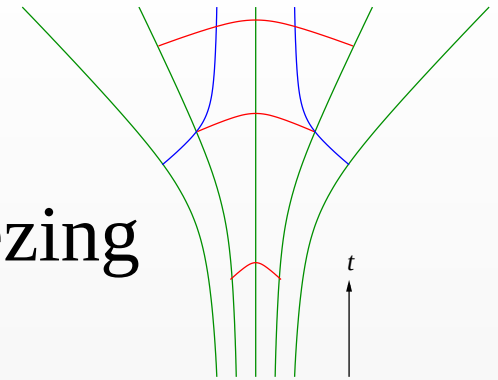
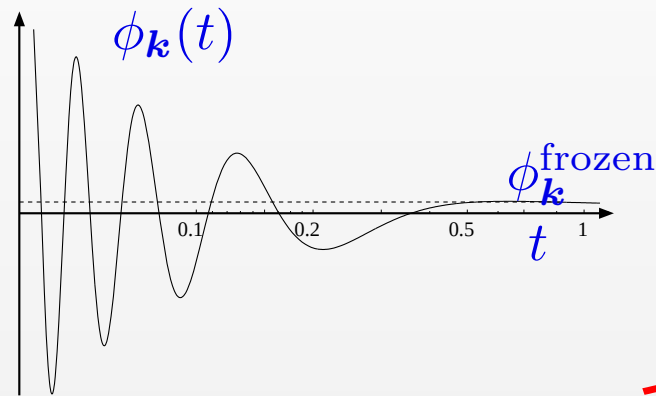
+ oscillation (initially)

+ horizon crossing $\rightarrow$ freezing



Quantum Fluctuations

Oscillation $\rightarrow$ horizon crossing $\rightarrow$ freezing



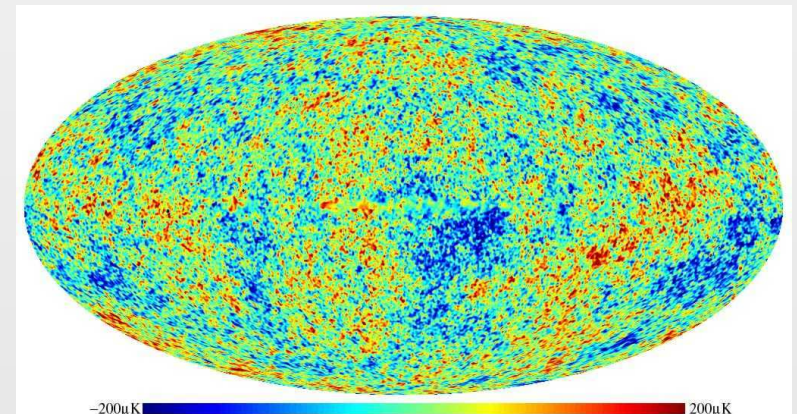
Amplification of quantum fluctuations (squeezing)
(in complete analogy to early universe, e.g., WMAP)

Quantum limit (accuracy)
of time-of-flight imaging

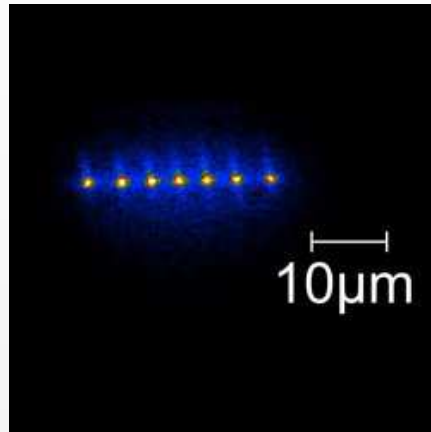
$$\langle \delta \hat{\varrho}(\mathbf{r}) \delta \hat{\varrho}(\mathbf{r}') \rangle = \mathcal{O}(1\%)$$

M. Uhlmann, Y. Xu, and R. S.,

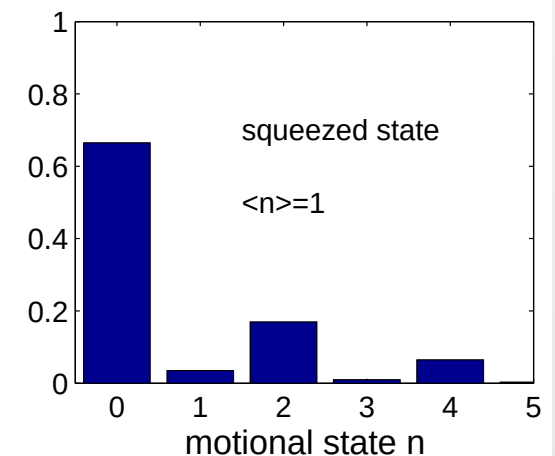
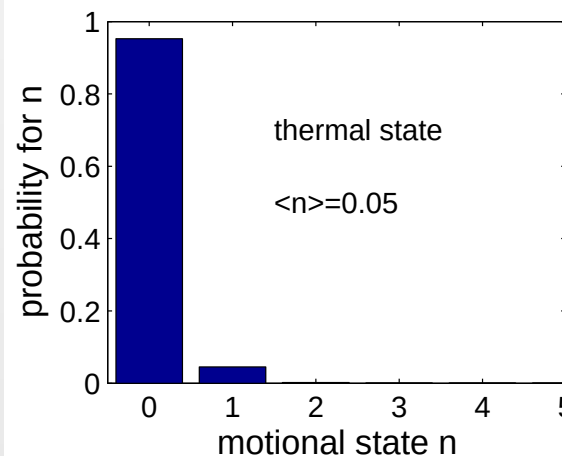
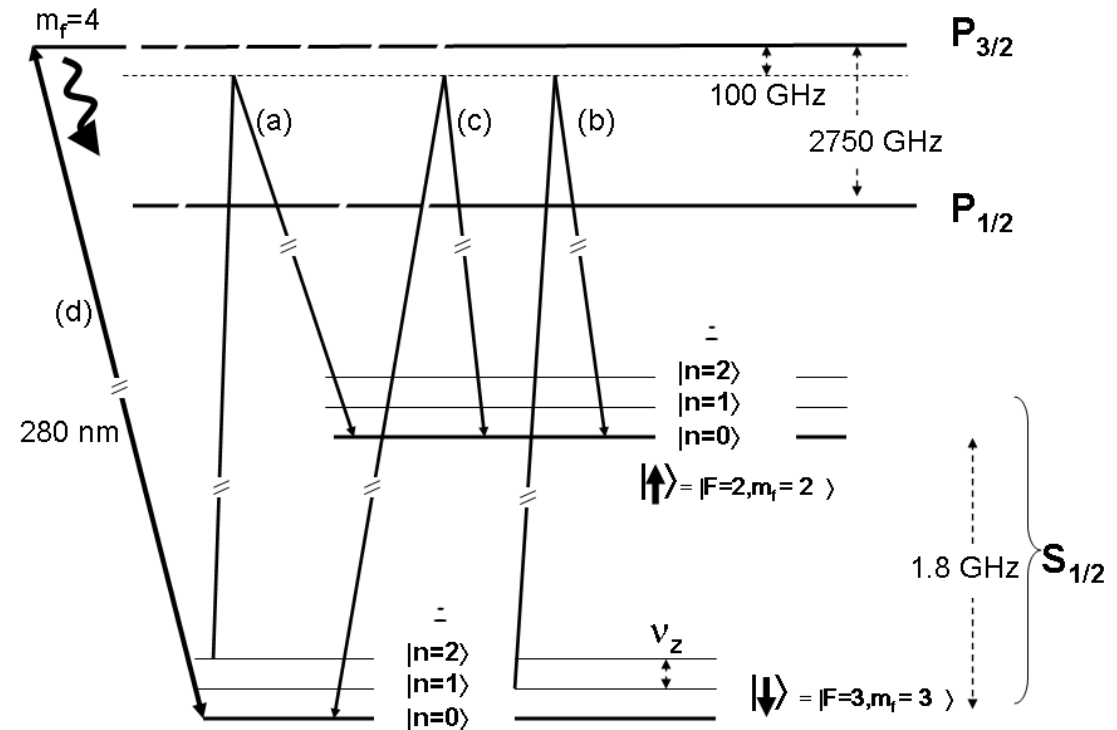
New J. Phys. 7, 248 (2005)



Ion Trap



Expansion or
Contraction of
Ion chain →
Expanding
Universe →
Phonon pair
creation
Hawking???



R. S. *et al.*, Phys. Rev. Lett. **99**, 201301 (2007).

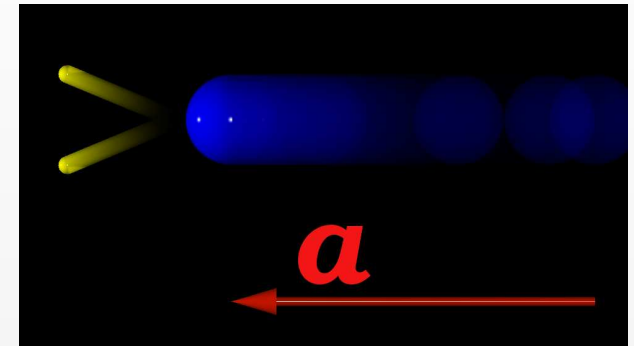
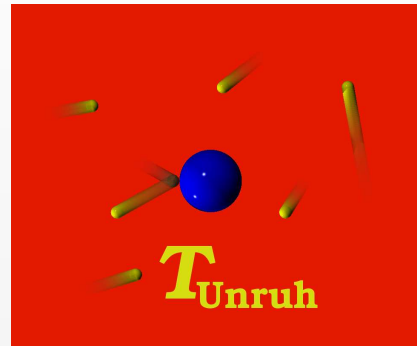
Signatures of the Unruh Effect

Accelerated frame

$$T_{\text{Unruh}} = \hbar a / (2\pi c)$$

Laboratory frame:

Pair creation



W. G. Unruh and R. M. Wald, Phys. Rev. D **29**, 1047 (1984).

Electrons $\gamma = 300$ in strong Laser fields 10^{18}W/cm^2

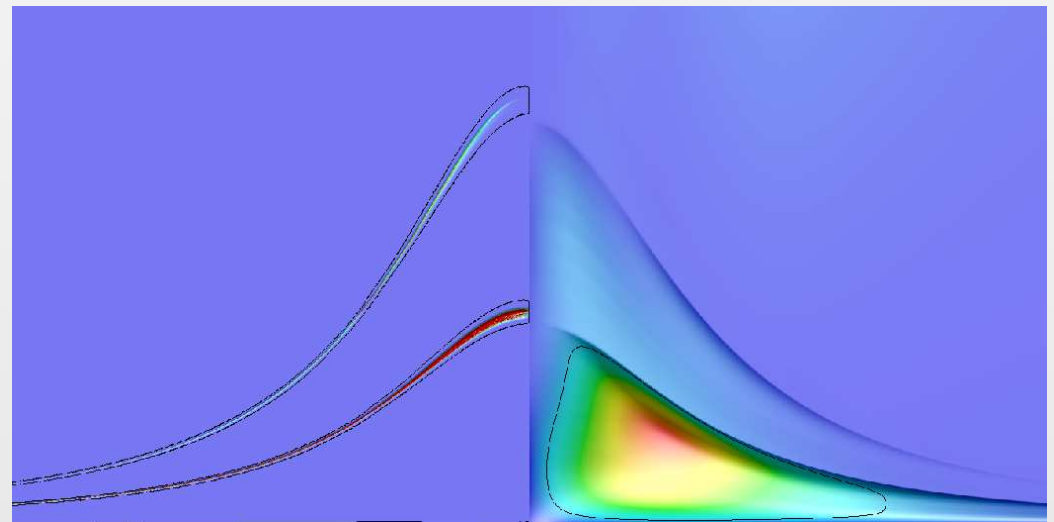
$$0 < E < 2\text{MeV}$$

$$0 < \vartheta < 1/100$$

Larmor (left)

Unruh (right)

$$P_{1e}^{\text{Unruh}} = \mathcal{O}(10^{-10})$$



R. S., G. Schaller, and D. Habs, Phys. Rev. Lett. **97**, 121302 (2006);

Announcement

12th MG Meeting

June 13th-18th 2009,
Paris (France)

Session BHT 6
Analogue Gravity

June 1st: deadline for
early registration

<http://www.icra.it/MG/mg12/en/welcome.htm>



The poster for the 12th Marcel Grossmann Meeting (MG12) in Paris, France, from July 12-18, 2009. It features a dark blue background with a starry space pattern. At the top, the title "MG12 PARIS 12-18 JULY 2009" is displayed in large white letters, with a stylized white satellite dish icon to the right. Below the title, the text "TWELFTH MARCEL GROSSMANN MEETING" is written in a large, gold, serif font. Underneath, a smaller line of text reads: "On recent developments in theoretical and experimental general relativity, astrophysics, and relativistic field theories". The poster is divided into several sections. On the left, the "INTERNATIONAL ORGANIZING COMMITTEE" lists names like David Grosse, Michel Bojowald, and others. Below this is the "INTERNATIONAL COORDINATING COMMITTEE" with a list of countries and their representatives, including Albania, Argentina, Australia, Austria, Belgium, Brazil, Canada, China, Colombia, Croatia, Czech Republic, Denmark, Estonia, Finland, France, Georgia, Germany, Greece, Hungary, Iceland, India, Iran, Israel, Italy, Japan, Korea, Kazakhstan, Kyrgyzstan, Latvia, Lithuania, Mexico, Netherlands, New Zealand, Norway, Poland, Portugal, Romania, Russia, Slovakia, Slovenia, South Africa, Spain, Sweden, Switzerland, Taiwan, Turkey, Ukraine, USA, Venezuela, and Vietnam. On the right, the "LOCAL ORGANIZING COMMITTEE" lists names like Pierre Binétruy, Jean-Pierre Luminet, and others. Below this is a section titled "The MG12 planetary sessions, as well as the satellite sessions, will take place at the UNESCO Headquarters in Paris and will be an official celebration of the United Nations 2009 Year of Astronomy." It describes the parallel sessions, the "L'été de la Gravité" festival, and the Marcel Grossmann Awards ceremony. At the bottom, there is a section for "Sponsored by (preliminary list)" with logos for various organizations like the French Republic, Paris Diderot University, and various scientific societies. The bottom right corner contains the website "http://www.icra.it/MG/mg12/en/welcome.htm" and a small photo of the conference venue.