Discussion Papers in Economic Behaviour

DPEB
02/10

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April, 2010

Delegated Agency in Multiproduct Oligopolies with Indivisible goods

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March 24, 2010

Abstract

This paper focuses on oligopolistic markets in which indivisible goods are sold by multiproduct firms to a continuum of homogeneous buyers, with measure normalized to one, who have preferences over bundles of products. Our analysis contributes to the literature on delegated agency games with direct externalities and complete information, extending the insights by Berheim and Whinston (1986, a, b) to markets with indivisibilities. By analyzing a kind of extended contract schedules -mixed bundling prices- that discriminate on exclusivity, the paper shows that efficient equilibria always exist in such settings. There may also exist inefficient equilibria in which the agent chooses a suboptimal bundle and no principal has a profitable deviation inducing the agent to buy the surplus-maximizing bundle because of a coordination problem among the principals. Inefficient equilibria can be ruled out by either assuming that *all* firms are pricing unsold bundles at the same profit margin as the bundle sold at equilibrium, or imposing the solution concept of subgame perfect *strong equilibrium*, which requires the absence of profitable deviations by any subset of principals and the agent. More specific results about the structure of equilibrium prices and payoffs for common agency outcomes are offered when the social surplus function is monotone and either submodular or supermodular.

Journal of Economic Literature Classification Numbers: C72, D21, D41, D43, L13.

Keywords: Multiproduct Price Competition, Delegated Agency Games, Mixed Bundling Prices, Subgame Perfect Nash Equilibrium, Strong Equilibrium.

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1 Introduction

This paper focuses on oligopolistic markets in which indivisible goods are sold by multiproduct firms to a continuum of homogeneous buyers, with measure normalized to one, who have preferences over bundles of products. In these settings linear pricing does not guarantee the existence of efficient Nash-equilibrium outcomes and, even worse, sometimes equilibrium (either efficient or inefficient) fails to exist. We wish to investigate whether a kind of non-additive prices, *mixed bundling prices*, restores equilibrium existence and efficiency. Mixed bundling refers to the practice of offering a consumer the option of buying goods separately or else packages of them (at a discount over the single good prices).

Our analysis contributes to the literature on agency games extending the insights by Berheim and Whinston (1986, a, b) to markets with indivisibilities. In our model, the principals are multi-product oligopolists offering a menu of prices for the different bundles of their own indivisible products and the agent is the consumer. We tackle the general question of whether the mixed-bundling equilibrium contracts (prices) offered by multiple principals (firms) to an agent (consumer) will be efficient and how the social surplus will be split among them. Obviously a single firm can achieve efficiency and extract all surplus in this setting when it can price non-additively. However, when prices charged by one firm impose some externality on other firms it is far from clear why equilibrium should be expected to be efficient.

When several firms sell (non-homogeneous goods) to the same consumer using some price scheme as a price discrimination strategy, the price schedule which arises can also be modeled as an equilibrium to a common agency game. It is natural to allow the consumer the option of purchasing exclusively from one firm, or from a set of firms, and so common agency is no longer intrinsic to the game but a choice variable that is delegated to the agent. Furthermore, in many economics contexts, several principals contract with a common agent and each of those principals is directly affected by the action selected by the common agent regarding other principals. In such a context, there are direct externalities among principals. For instance, Martimort and Stole (2003), analyze the equilibrium set of a simple common agency (or "bidding") game: the set of outcomes which results from non-cooperative behavior among the principals when they compete through non-linear prices. They analyze two variations of the common agency game: the intrinsic common agency game and the delegated common agency game. Our model directly applies to the analysis of delegated agency games with direct externalities and complete information.

We also contribute to the understanding of the interaction between competition and non-additive pricing schemes. Because most of the theoretical work in multi-principal contract games has restricted attention in large part to intrinsic settings, it remains unclear how competition affects the character of non-additive prices among oligopolists. Clear exceptions, among others, are Armstrong and Vickers (2001) and Rochet and Stole (2002), who analyze nonlinear pricing applications assuming exclusive purchasing in which the consumer must buy from only one firm in equilibrium, and competition can only matter through the outside option; Ivaldi and Martimont (1994), who allow for purchasing from multiple vendors in equilibrium but restrict preferences such that full coverage arises in equilibrium; and Martimont and Stole (2009), who study how competition in non-linear pricing between two principals (sellers) affects market participation by a privately-informed agent (consumer). In Martimont and Stole (2009), each firm produces a good (which can be a substitute or a complement of the rival) and the consumer's demand function for the two goods is symmetric and linear in prices (with the consumer's private information appearing only in the demand intercept). In contrast, in our complete information model each firm produces a set of goods, the consumer has a quasi-linear utility function over bundles of goods and both multiple vendors (common agency) and exclusivity are possible equilibrium outcomes. In our analysis, the strategy space of principals are the set of firms' strategies in the complete information game of firms and the consumer. Thus, we consider extended contract schedules instead of only the equilibrium offers. There are at least two reasons for focusing on the complete information case. First, the literature on common agency games deals mainly with principals selling one or two substitute goods under continuity assumptions so that the general existence theorems (when available) for such games cannot be easily extended to cover models with indivisible goods in delegated agency games with externalities, even under complete information[1]. Second, ruling out informational considerations allow us to isolate the effect on the outcome of the principals' competition game from the effects stemming from private information. In particular, this permits us, in calculating the equilibrium payoff of the agent, to know the magnitude of the rent she obtains as the result of competition among principals.

To the best of our knowledge, our paper is the first one dealing with multiproduct oligopolistic competition when goods are indivisible, of a very general nature and the agent has preferences over bundles of goods. For instance, in many common situations, agents have complementary preferences for objects in the marketplace. Consider an agent trying to construct a computer system by purchasing components. Among other things, the agent needs to buy a CPU, a keyboard and a monitor, and may have a choice over several models for each component. The agent's valuations of a package depends on the components in any particular combination, involving products of either only one firm or several firms. This example is a general instance of allocation problems characterized

[1] For existence results in common agency games under incomplete information see, Carmona and Fajardo (2009), who show that general menus games satisfying enough continuity properties have subgame perfect equilibrium. See also Page and Monteiro (2003).

by heterogeneous, discrete resources and complementarities in agents' preferences. These kinds of models are probably close to many circumstances in real world markets, but they are also more difficult to analyze. With indivisibilities, it is well-known that many familiar properties of the profit functions may fail to ensure the existence of Nash-Bertrand prices. The use of marginal calculus is precluded, and the applications of fixed point theorems based on continuity properties, while still possible in some cases, is certainly not straightforward.

As already pointed out, linear contracts do not guarantee in these settings, even under extended contract schedules, the existence of (subgame perfect) equilibrium outcomes, either in common agency or in exclusive dealing (see Liao and Urbano, 2002, for technical details). The intuition is clear: suppose a market for systems, where two firms produce two goods each and the agent has preferences over systems (all the bundles of two goods) and that common agency is the efficient allocation. Starting from an efficient allocation where the agent buys from the two principals, any of them may find it profitable to deviate and exclusively deal with the agent. However, these deviations need not lead to an exclusive dealing equilibrium because both principals will compete fiercely and a deviation from a principal may be followed by other deviations from the rival and the sequence of deviations need not converge. Alternatively, consider that an exclusive dealing bundle is the efficient allocation and that the common agency bundles are quite attractive to the agent as compared with the exclusive dealing ones. In this case, the other exclusive dealing allocation is not very attractive to the agent and then, because of competition, the agent may get most of the surplus. Then, the principal selling the efficient bundle may find it profitable to rise the price of one of the individual components of its own bundle and set the other one in such a way that the agent chooses now the common agency bundle thus giving more profits to this principal. Again, this deviation need not lead to a delegated common agency equilibrium since the other principal may find another profitable deviation and so on.

Given the above result, one question is whether another kind of extended contracts -mixed bundling prices- ensures the existence of equilibrium. The answer is affirmative. Under mixed bundling contracts the agent has the option of buying bundles of goods from a firm at a discount over the single good prices. Hence, in the above market for systems, mixed bundling contracts are conditional on exclusive dealing. In a more general model, with multi-product firms and an agent with preferences over each bundle of goods, mixed bundling contracts are conditional on exclusive dealing for each bundle of two or more goods (*quasi-exclusive dealing*). Therefore, mixed-bundling contracts can be seen as either an aggressive pricing policy for exclusive dealing outcomes or as out-of-equilibrium offers sustaining the equilibrium consumption sets of individual components in delegated common agency allocations (involving either several principals or all of them). The discrimination on exclusivity helps principals set incentive-compatible contracts by both facilitating collusion on common agency outcomes and by representing a credible threat that avoids deviations by the principals. Thus, mixed bundling contracts makes it easier to sustain equilibrium outcomes and are sufficient to guarantee the existence of equilibrium.

However, equilibrium efficiency may require additional restrictions. For instance, in a typical (not necessarily efficient) delegated common agency (subgame perfect) Nash-equilibrium, the prices of the equilibrium bundles are well-specified, but any vector of sufficiently high prices for the individual components (out-of-equilibrium prices) would support any given equilibrium outcome. In this way, if a principal reduces the price of any of its unsold individual components, then the equilibrium consumption set is not upset since rivals' prices for their individual components are also too high to induce the agent to choose an alternative bundle to that of the (not necessarily efficient) equilibrium. This leads to well-known inefficiencies, since efficiency may imply some coordination among firms (as stressed by Martimort, 2007), specially if the efficient consumption is a multi-firm bundle (common agency). Thus, we need a refinement of the (subgame perfect) Nash-equilibrium concept which would require the candidate equilibrium to remain so even if all firms reduced the prices of their unsold bundles to some degree, thus restoring coordination and efficiency.

Can we find a class of strategies which ensures that efficiency is always reached at equilibrium? When the efficient bundle is a one-firm bundle (exclusive dealing), the above reasoning translates to the condition that the equilibrium is not perturbed even if any subset of the non-selling firms sets unit cost prices for their products. Here then, familiar arguments[2], i.e. the logic of Bertrand competition with homogeneous products, are easily extended, with all non-selling principals setting prices equal to unit cost, provided the subgame perfect Nash-Equilibrium concept is refined to cover deviations by subsets of non-selling firms. However, when the efficient bundle is a common agency

[2] Our model is also related to the literature of bundling and mixed bundling. The literature on multi-product pricing focuses on the use of bundling to extract a surplus from heterogeneous buyers or to price-discriminate (Adam and Yellen, 1976; Schmalensee, 1984; McAfee *et al.*, 1989) or to apply monopoly leverage across markets (Whinston, 1990; Choi, 1996, 2004; Carbajo *et al.*, 1990) or to deter entry into the market (Whinston, 1990; Nalebuff, 1999, 2004). In all these models, demands are assumed to be continuous, there is private information and consumers are heterogeneous. In contrast, in our setting with indivisible good, mixed bundling is profitable when there is a representative consumer and no opportunity to apply leverage across markets, even under complete information. Mixed bundling also takes place with substitute goods, in contrast to the well-studied cases of bundling with independent or complementary goods. There are few general results for bundles of more than two goods. McAdams (1997) found that the existing analytical machinery for analyzing mixed bundling could not be easily generalized to even three goods, because of the interactions among sub-bundles. In general, price-setting for mixed bundling of many goods is an NP-complete problem requiring sellers to determine a number of prices and quantities that grows exponentially as the size of the bundle increases. Therefore, a general analysis is still lacking and, what is worse, it is not even known whether a Nash equilibrium may exist in such a general setting. In a related setup, Liao and Urbano (2002) and Liao and Tauman (2002) consider a duopoly and assume that each firm produces two complementary goods which are substitutes for the two corresponding goods produced by the other firm. Liao and Tauman (2002) find that mixed bundling strategies play a key role in stabilizing the market, although efficient and inefficient equilibria may exist. If the use of mixed bundling is not allowed, then Liao and Urbano (2002) show that subgame perfect linear pricing equilibria may fail to exist.

bundle, a new complication arises since the deviations belong to the selling principals; in particular, those deviations to their own exclusive dealing bundles. Here, if firms' exclusive dealing bundles are priced at unit costs, they become quite attractive to the agent and then the prices of the goods of the efficient multi-firm bundle have to be quite low, thus generating a low profit for the active firms. Therefore, there is an incentive for each principal to increase the prices of its own bundles above unit costs, making them less attractive, in order to increase the prices of their goods in the efficient bundle. Obviously, to set high prices for their exclusive dealing bundles would solve the problem while creating the above-mentioned coordination problem. What should be the prices of firms' own bundles? Each principal prices its unsold bundles at the same profit margin as the bundle sold at equilibrium, and then the equilibrium price vector has to be immune to this out-of-equilibrium prices. To avoid coordination problems, the above condition has to be satisfied by any subset of active firms pricing their own bundles. This means looking for an equilibrium concept which takes into account deviations by any set of firms and the player.

The Strong Equilibrium (Aumann, 1959) is the equilibrium concept capturing the above idea of stability against joint deviations that are mutually profitable for any subset of players. The extension to subgame perfection of such an equilibrium concept allows us to refine the equilibrium correspondence by selecting its efficient equilibria. In fact, we show that the set of subgame perfect strong equilibria of our game is the set of its efficient subgame perfect Nash-equilibria. Therefore inefficient equilibria can be ruled out by either assuming that all firms are pricing unsold bundles at the same profit margin as the bundle sold at equilibrium (thus inactive firms price all bundles at cost), or imposing the solution concept of strong equilibrium which requires the absence of profitable deviations by any subset of principals and the agent.

The question of equilibrium efficiency was already addressed by Bernheim and Whinston (1986,b), in the context of delegated common agency and where each principal can contract on the whole array of actions of the agent. Under complete information the so-called *truthful equilibrium* implements the outcome which maximizes the aggregated payoff of the grand coalition made of the principals and the agent. The rationale for truthful menus is that they are *coalition-proof*, i.e., immune to deviations by subsets of principals which are themselves immune to deviations by sub-coalitions, etc. Coalition-proof equilibrium payoffs can be implemented with truthful schedules in environments with quasi-linear utility functions. Unfortunately, in more complex settings, where the objective function need not be either concave or convex and what is more it may even fail to be continuous, the refinement of coalition-proof equilibrium may be too demanding to be satisfied. This is why our rationale for equilibrium menus which take into account deviations by any set of firms and the player is the weaker refinement of strong equilibrium. This refinement gives another (subgame perfect) noncooperative justification of *truthful equilibria* in the context of delegated agency games.

The paper offers a positive existence result: with mixed-bundling contracts and subgame perfect strong equilibria, *an efficient equilibrium outcome always exists*, no matter whether the efficient bundle is either exclusive dealing or common agency (involving either several principals or all of them). The paper extends the traditional wisdom of the delegated common agency literature to settings with multi-product principals producing indivisible goods and an agent with preferences over bundles of goods. By analyzing mixed-bundling contracts that discriminate on exclusivity and extend the space of contract schedules beyond equilibrium offers the paper provides 1) a proof of the existence of efficient subgame perfect Nash-equilibria and hence that of strong equilibria in delegated agency games, 2) a characterization of such equilibria, and 3) a characterization of the set of the principals' equilibrium rents by some projection of the core of such delegated agency games.

More specific results about the structure of equilibrium prices and payoffs for common agency outcomes can be offered when the social surplus function is monotone and either submodular or supermodular. In the former case, principals are substitutes and their equilibrium rents are equal to the principals' social marginal contributions with an agent's positive rent, thus reflecting market competition. In the latter case, the agent's rent is zero and then the core of the value function is always priced by the subgame perfect Nash-equilibrium rents. These findings extend to multi-product markets with indivisibilities those of Laussel and Lebreton (2001), who investigated the payoff structure of intrinsic common agency games through the properties of the agent's bidding participation constraints or characteristic function.

The paper is organized as follows. The model is presented in Section 2, while a motivating example is offered in Section 3. The characterization of efficient subgame perfect Nash-equilibrium in terms of all possible deviations and hence that of subgame perfect strong equilibrium are provided in Section 4. The existence and efficiency of such equilibria are proven in Section 5. Section 6 offers more specific results for common agency equilibrium outcomes when the social surplus function is monotone and either submodular or supermodular. Concluding remarks close the paper.

2 The model

Consider a set of the principals and a continuum of potential homogeneous buyers, with measure normalized to one (the agent). In our model the principals are n firms and each of them produces a set of goods. Moreover each firm's products can be different from or identical to those of any other firm. Let $N = \{1, 2, ..., n\}$ be the set of firms. Let Ω_i be firm i's set of products and $\Omega = \Omega_1 \times \ldots \times \Omega_n$ be the cartesian product of the sets of all firms. Let $c_i(w_i)$ be the (constant) unit cost of production of firm i for good $w_i \in \Omega_i$, where costs are additive, i.e. $c_i(T_i) = \sum_{w \in T_i} c_i(w), T_i \subseteq \Omega_i$.

A consumption set is a vector of subsets $S = (S_1, \ldots, S_n) \subseteq \Omega$, where $S_i \in 2^{\Omega_i}$ represents firm i selling set S_i in S (which can be the empty set if the agent does not buy anything from firm i) and let $c(S) = \sum_{i \in N} c_i(S_i)$ be the associated cost (where $c_i(\emptyset) = 0$).

A firm is said to be non-active in a given consumption set if none of its products is consumed and let $F(S)$ be the set of active firms in S, i.e. $F(S) = \{i \in N | S_i \neq \emptyset\}$.

The agent purchases either one or zero units of each one of the products and is characterized by her value function over any subset $S \subseteq \Omega$, $v(S)$, which represents her total willingness[3] to pay for consumption set S, with $v(\emptyset, \ldots, \emptyset) = 0$. It is assumed that the agent has no endowment of goods but she has enough money to buy any bundle of products $S \subseteq \Omega$.

Each firm i sets prices (offers contracts) for its products and can also offer subsets of them as bundles at a special price. Notice that this implies that we do no consider singleton contracts (direct mechanisms) in the delegated agency game. Such mechanisms do not allow for any offer to remain unchosen in equilibrium; in other words, out-of-equilibrium messages are possible, to use the language of mechanism design. Thus, a strategy of firm i, $i \in N$, is a 2^{Ω_i}-tuple specifying the price of each $w \in \Omega_i$ as well as the prices of each of any other subset of Ω_i. Let $p_i(T_i)$ be the price of $T_i \subseteq \Omega_i$, if $p_i(T_i) = \sum_{w \in T_i} p_i(w)$, then prices are linear and bundle T_i is not offered at a special price. If $p_i(T_i) < \sum_{w \in T_i} p_i(w)$, then subset T_i is offered as a bundle at a lower price. In this case, we say that firm i follows a *mixed bundling strategy.* Notice that when $T_i = \Omega_i$ then firm 1's mixed bundling strategies implies prices (contracts) conditioned on exclusive dealing, whereas when $T_i \subset \Omega_i$, they refer to exclusive dealing with respect to products on T_i. To avoid irrational off-equilibrium behavior, we restrict $p_i(T_i)$, $i \in N$, $T_i \subseteq \Omega_i$ to satisfy $p_i(T_i) \geq c_i(T_i)$ and set $p_i(\emptyset) = 0$. Let $\mathcal{P}_i = \mathcal{R}_+^{2^{\Omega_i}}$ be the set of functions $p_i : 2^{\Omega_i} \longrightarrow R_+$ such that $p_i \geq c_i$.

The sequence of events is as follows. First, each firm i chooses a price vector $p_i \in \mathcal{P}_i$ independently of and simultaneously to the other firms. Then, the agent observes the price vector $p = (p_1, ..., p_n)$ $\in \mathcal{P}_1 \times \cdots \times \mathcal{P}_n$, and selects a consumption set $S \subseteq \Omega$ as a function of p. Formally, we have a strategic game with $n+1$ players, n firms and an agent. Let $G^{MB}(n+1, v, c)$ (where MB stands for mixed bundling pricing) denote such a game. The set of strategies of each firm is set $\mathcal{P}_i$ and that of the agent is the set of functions from $\mathcal{P}_1 \times \cdots \times \mathcal{P}_n$ to $(2^{\Omega_1}, \ldots, 2^{\Omega_n})$. Finally, the payoff of each firm $i \in N$ is given by its profit function $\pi_i(S, p) = (p_i - c_i)(S_i(p))$ where $S(p)$ is the agent's consumption set corresponding to p and $(p_i - c_i)(S_i)$ means $p_i(S_i) - c_i(S_i)$ for all $i \in N$. The payoff for the agent

[3] The value function, v, can be derived by standard primitives. Suppose that the agent's utility function is quasi-linear in money, that is, it is given by $u(x_1, ..., x_n, m) = f(x_1, ..., x_n) + m$, where m is the monetary numerare and $(x_1, ..., x_n)$ is a consumption bundle. Then, $f(x_1, ..., x_n)$ measures the monetary value of the bundle $(x_1, ..., x_n)$. Let $S \subseteq N$ be a consumption set and let e^S be the corresponding quantities consumed, namely, $e_k^S = 1$ if $k \in S$ and $e_k^S = 0$ if $k \notin S$. The value function v is defined as $v(S) = f(e^S)$.

when purchasing S at prices p is her consumer surplus: $cs(S,p) = v(S(p)) - \sum_{k\in N} p_k(S_k(p))$.

Since the prices set by each firm affect the profits of the other firms and the agent chooses whether to purchase from either all the firms, a subset of them and not purchasing at all, our model can be understood as a delegated agency game with direct externalities, where n (multiproduct) principals offer contracts (price schedules) for any bundle of her own products and the agent (the consumer) chooses whether to accept all the contracts, a subset of them or none at all. Given the set of the agent's strategies, she is considered as another player and therefore game $G^{MB}(n+1, v, c)$ denotes such a delegated agency game with direct externalities, where the principals offer mixed-bundling contracts. Therefore, the strategy space of principals are the set of firms' strategies in the complete information game of firms and the consumer. Thus, we consider the extended space of contract schedules instead of only the equilibrium offers.

Let SPE be the set of pure strategy subgame perfect equilibria of $G^{MB}(n+1, v, c)$. If (S,p) is an element in SPE, p is called an SPE-price vector, S is an SPE-consumption set and (S,p) is denoted as an SPE-outcome. If bundle S has goods of only a subset of firms, then the SPE-consumption set is a *partial common agency* allocation and the SPE-outcome is a *partial common agency equilibrium*, being a *common agency equilibrium* if bundle S has goods of all the firms. Alternatively, when S has goods of only one firm, then the SPE-consumption set is an *exclusive dealing* allocation and the SPE-outcome is an *exclusive dealing equilibrium*.

The function $(v-c)(S) = v(S) - \sum_{i\in F(S)} c_i(S_i)$ is the *social surplus function*. Bundle $\widetilde{S}$ is *socially efficient* if $\widetilde{S} \in \arg\max_{(S)}\{(v-c)(S)\}$ and $V(N) = (v-c)(\widetilde{S})$ denotes the maximum social surplus. It is assumed that the social surplus function is always positive[4]. An SPE-outcome is *efficient* if its SPE-consumption set maximizes the social surplus.

3 Examples: Why mixed bundling prices?

The following simple example illustrates three main insights: first, linear prices contracts do not guarantee the existence of SPE-outcomes; second, mixed bundling prices are sufficient to restore such equilibrium existence. However, both efficient and inefficient equilibrium outcomes may co-exist. The final insight deals with the role of out-of-equilibrium prices to sustain equilibrium outcomes. More precisely, when the efficient consumption set is a common agency bundle, each firm's own bundles may have to be strategically set to avoid deviations to the (inefficient) exclusive dealing consumption sets. This is due to the existence of direct externalities between the principals, i.e., to the fact that

[4]Otherwise, if for every consumption set S, $v(S) < \sum_{i\in F(S)} c_i(S_i)$, then the model is degenerated. Hence, at every equilibrium point (S,p), $S = (\emptyset, \ldots, \emptyset)$ must hold and therefore no production will take place.

each principal's rent is directly affected by the other principals' price schedules.

3.1 Linear Prices

EXAMPLE 1. Let the set of firms (principals) be $N = \{1, 2\}$, each one producing $\Omega_1 = \{a, b\}$ and $\Omega_2 = \{c, d\}$, respectively. Assume for simplicity that $c_i(w) = 0$ for all $i \in N$ and $w \in \Omega_i$ and that firms set linear prices, i.e. the exclusive dealing bundles $\{a, b\}$ and $\{c, d\}$ are sold at prices $p_a + p_b$, and $p_c + p_d$, respectively. The agent's value function is[5]

$$v(S) = \begin{cases} 4 & S = \{a, b\} \\ 9 & S = \{a, d\} \\ 5 & S = \{b, c\} \\ \delta & S = \{c, d\} \\ 0 & \text{otherwise.} \end{cases}$$

with $0 < \delta < 9$. Think, for instance, of an agent trying to construct a computer system by purchasing two components, a CPU and a monitor, and having the choice of two models for each component. The agent's valuations of a package or bundle depends on the components in any particular combination, and then both common agency and exclusive dealing bundles are possible consumption sets.

The efficient consumption set is the common agency bundle $S = \{a, d\}$. Suppose that $\delta = 8$. Therefore, the agent values any of the common agency bundles more than those of exclusive dealing. Let us show that there are not linear prices (contracts) guaranteeing the existence of any equilibrium outcome. Traditional arguments suggest how to set prices for, say, the efficient consumption set $S = \{a, d\}$ (the proof of the remaining cases is similar). Recall that $cs(S, p)$ denotes the agent's surplus from purchasing S at prices p. Whether the agent's participation constraint is binding will help define the equilibrium prices:

1) Suppose that the agent's participation constraint is not binding, i.e. $cs(\{a, d\}, p) = 9 - p_a - p_d > 0$, or $p_a + p_d < 9$, then since S is a two-firm bundle firms' prices have to first guarantee the incentive compatibility constraints avoiding exclusive dealing, i.e., the agent's surplus associated to the consumption of set $S = \{a, d\}$, has to be bigger than or equal to the agent's surplus if either

[5] Notice that a more rigorous notation would be to denote by $\{\{a, b\}, \varnothing\}$ and $\{\varnothing, \{c, d\}\}$ the one-firm bundles and by $\{\{a\}, \{d\}\}$ and $\{\{b\}, \{c\}\}$ the two-firm bundles. To ease the notation and since no confusion will arise we follow the more simple notation of $\{a, b\}$, $\{c, d\}$, $\{a, d\}$ and $\{b, c\}$, to respectively denote such sets.

firm 2 or firm 1 were removed from the market,

$$
\begin{aligned}
cs(\{a,d\},p) &= 9-p_a-p_d \geq 4-p_a-p_b = cs(a,b), && (1)\\
cs(\{a,d\},p) &= 9-p_a-p_d \geq 8-p_c-p_d = cs(c,d), && (2)
\end{aligned}
$$

Notice, however, that both (1) and (2) have to be binding since otherwise either firm 1 could deviate by raising p_a or firm 2 by increasing p_d and be better off, without changing the agent's choice. Rewriting these equations we have that $p_d = 5 + p_b$ and $p_a = 1 + p_c$ and hence that $p_a + p_d = 6 + p_b + p_c$.

The multiproduct nature of the model includes another incentive constraint dealing with the agent's switching to the other common agency bundle, i.e., $cs(\{a,d\},p) = 9-p_a-p_d \geq 5-p_b-p_c = cs(\{b,c\},p)$, which implies that $p_a+p_d \leq 4+p_b+p_c$ that contradicts the previous result. Hence the agent's participation constraint has to be binding and thus $p_a + p_d = 9$.

2) Since, $p_a + p_d = 9$ the agent's surplus is zero and firms 1 and 2's profits are $9-p_d$ and $9-p_a$, respectively. However, firm 1 could deviate by setting $p_b = 0$ and $p_a = 4$ and trying to sell its own bundle $S' = \{a,b\}$. Therefore, at equilibrium it must be that $9 - p_d \geq 4$ or $p_d \leq 5$; similarly, to avoid firm 2' deviation by setting $p_c = 0$ and $p_d = 8$, and selling its own bundle $S" = \{c,d\}$, it must be that $p_a \leq 1$. Hence $p_a + p_d \leq 6$ which contradicts the assumption that $p_a + p_d = 9$.

Therefore, no linear prices support $S = \{a,d\}$ as the equilibrium consumption set. The principals' incentives to deviate and exclusively deal with the agent cannot be overcame by linear pricing. The intuition is that every time the price of a good in a bundle, say S, is changed, the prices of the other bundles which contain that good of S are also changed. Firms can use this property to design profitable deviations by either increasing or decreasing subsets of prices to discourage the consumption of the (efficient) common agency consumption set while encouraging that of exclusive dealing. The above reasoning can be applied to any other bundle to conclude that under linear pricing the equilibrium may fail to exist. Thus, when pricing a bundle S, all possible deviations (involving one firm and/or several firms) have to be taken into account in order to deter all of them. Sometimes it may be possible (when competition is either very tough or very weak) and very often not (for intermediate levels of competition, see Liao and Urbano, 2002).

3.2 Mixed bundling prices

Consider next that firms do not precommit to linear pricing. Let p_{ab} and p_{cd} be the prices of the exclusive dealing bundles $\{a,b\}$ and $\{c,d\}$, respectively, then $p_{ab} < p_a + p_b$, and $p_{cd} < p_c + p_d$.

Example 2. **The role of out-of-equilibrium exclusive dealing prices:** Consider example 1 where $\delta = 8$. Notice that in this case the common agency efficient bundle $S = v(a,d)$ is not highly valued by the agent, in the sense that $v(a,d) < v(a,b) + v(c,d)$. This will imply that the agent will get some surplus.

Since the efficient bundle S is one of common agency some particularly important deviations by principals are those to their exclusive dealing bundles. Although mixed-bundling contracts discriminate on exclusivity, if the out-of equilibrium prices p_{ab} and p_{cd} are priced at unit costs, they become quite attractive to the agent and then the prices of the goods of the efficient common agency bundle S have to be quite low to avoid the agent's choice of an exclusive dealing bundle. Therefore, there is an incentive for each firm to increase the prices of its own bundles above unit costs, making them less attractive. However, equilibrium subgame perfection imposes some restrictions on firms' exclusive dealing prices. In particular, each firm will set the price of its own bundle as to guarantee itself the same profit margin as under S, and then the equilibrium price vector will have to be immune to such an action.

Let us show that the efficient bundle $S = \{a,d\}$ is the unique SPE-consumption set, supported by equilibrium prices:

$$p_a = p_{ab} = v(a,d) - v(c,d) = 1,\ p_d = p_{cd} = v(a,d) - v(a,b) = 5,\ p_b + p_c \geq 2.$$

Thus, the out-of-equilibrium exclusive dealing prices p_{ab} and p_{cd} are set strategically by firms to be equal to the prices of their sold products. Notice also that $p_a = p_{ab} < v(a,b)$ and $p_d = p_{cd} < v(c,d)$ and that the consumer surplus is $cs(\{a,d\},p) = 9 - 6 = 3 > 0$. To following steps prove that the above outcome is the unique equilibrium:

1) Suppose that the agent's participation constraint is binding: $p_a + p_b = 9$. Since

$$\begin{aligned} cs(\{a,d\},p) &= 9 - p_a - p_d \geq 4 - p_{ab} = cs(\{a,b\},p), \\ cs(\{a,d\},p) &= 9 - p_a - p_d \geq 8 - p_{cd} = cs(\{c,d\},p), \end{aligned}$$

then $p_{ab} \geq 4$ and $p_{cd} \geq 8$ and since $v(a,d) < v(a,b) + v(c,d)$, either $p_a < 4$ or $p_d < 8$. If $p_a < 4$, then Firm 1 is better off first setting p_a and p_b high enough and then setting p_{ab} such that $4 - p'_{ab} > 0$ or $p'_{ab} = 4 - \varepsilon > p_a$, for ε sufficiently small and then the agent will switch to Firm 1's exclusive dealing bundle $\{a,b\}$. We can reproduce the same argument for $p_d < 8$.

2) Thus, the agent's participation constraint is not binding: $p_a + p_d < 9$, and the incentive

compatibility constraints avoiding exclusive dealing have to be so,

$$cs(\{a,d\},p) = 9 - p_a - p_d = 4 - p_{ab} = cs(\{a,b\},p), \quad (3)$$
$$cs(\{a,d\},p) = 9 - p_a - p_d = 8 - p_{cd} = cs(\{c,d\},p), \quad (4)$$

because if the left-hand side of either (3) or (4) were strictly bigger than its corresponding right-hand side, then each firm would have an incentive to profitably raise its price. More precisely, notice that if (3) were satisfied with strict inequality, then firm 1 will have an incentive to rise p_a and also since $8 - p_{cd} > 4 - p_{ab}$, then firm 2 will also have an incentive to rise both p_{cd} and p_d.

3) We prove first that at equilibrium p_{ab} and p_{cd} cannot be equal to unit costs. Assume, on the contrary that $p_{cd} = p_{ab} = 0$ (unit costs). These low prices make firms' exclusive dealing bundles very attractive to the consumer. To deter the consumer to choose them, p_a and p_d are quite low and so are firms' profits. This gives an incentive for firms to deviate. In particular, (3) and (4) are

$$cs(\{a,d\},p) = 9 - p_a - p_d = 4 = cs(\{a,b\},p),$$
$$cs(\{a,d\},p) = 9 - p_a - p_d = 8 = cs(\{c,d\},p)$$

therefore either $p_a + p_d = 5$ or $p_a + p_d = 1$. In the former case, the consumer will deviate to purchase the exclusive dealing bundle $\{c,d\}$ and in the later $9 - p_a - p_d = 8 > 4$ and either firm will have as above an incentive to profitably raise its price. For instance, Firm 2 can be better off raising sub-sets of prices: it may set p_{cd} and p_c high enough, say $p'_{cd} = p'_c = 4$ in order to discourage the consumption of bundles $\{c,d\}$ and $\{b,c\}$, and then set p'_d such that $9 - p_a - p'_d = 4$, i.e. $p'_d = 5 - p_a$. Notice that $p'_d > p_d = v(a,d) - v(c,d) - p_a = 1 - p_a$, and then Firm 2 obtains a higher profit ($\pi'_2 > \pi_2$), while the consumer still buys bundle $\{a,d\}$. Therefore $p_{ab} > 0$ y $p_{cd} > 0$.

4) Suppose now that $p_{ab} > p_a$. Then, by equation (3) firm 1 could be better off by decreasing the price of $\{a,b\}$ to p_{ab}-$\varepsilon > p_a$ and making the agent switch from $\{a,d\}$ to $\{a,b\}$. The same reasoning applies to $p_{cd} > p_d$. Therefore, at equilibrium $p_a \geq p_{ab}$ and $p_d \geq p_{cd}$.

5) Consider then that $p_a \geq p_{ab}$ and $p_d \geq p_{cd}$, by equations (3) and (4),

$$p_a + p_d = 5 + p_{ab} \text{ or } p_d = 5 + p_{ab} - p_a$$
$$p_a + p_d = 1 + p_{cd} \text{ or } p_a = 1 + p_{cd} - p_d$$

Clearly, $p_a = p_{ab}$ and $p_d = p_{cd}$, and then $p_d = 5 = p_{cd}$ and $p_a = 1 = p_{ab}$ is an equilibrium price vector, since no firm has an incentive to set either $p_a \neq p_{ab}$ or $p_d \neq p_{cd}$, respectively. For suppose that, say, firm 1 deviates and sets $p'_{ab} < p_a = 1$, then by equation (3), the consumer will switch to buy bundle $\{a, b\}$, but firm 1 will loose profits since $p'_{ab} < p_a$. Similarly, setting $p'_{ab} > p_a = 1$ will make equation 3 be satisfied with strict inequality and then applying the same reasoning than in step 2, firm 1 will have an incentive to rise p_a and also since $8 - p_{cd} > 4 - p'_{ab}$ by equations 3 and 4, then firm 2 will also have an incentive to rise both p_{cd} and p_d. We show next that $p_a = p_{ab}$ and $p_d = p_{cd}$ is the unique equilibrium price vector.

6) Suppose that at the equilibrium both $p_a > p_{ab}$ and $p_d > p_{cd}$ then $p_d < 5$ and $p_a < 1$. If say firm 1 increases it exclusive dealing price p_{ab} up to p_a, $p'_{ab} = p_a$, then equation (3) is satisfied with strict inequality thus creating again an incentive for both firms to increase prices and destroying the equilibrium: each firm has an incentive to increase p_{ab} up to p_a and p_{cd} up to p_d, respectively and then $p_a = p_{ab}$ and $p_d = p_{cd}$ is a profitable deviation.

7) Next suppose instead that at the equilibrium $p_a > p_{ab}$ but $p_d = p_{cd}$, then by the above inequalities $p_a = 1$ and $p_d < 5$. In this case, firm 2 increases p_{cd} up to some bound, i.e. $p'_{cd} = 5$, and then by equations 3 and 4, $4 - p_{ab} > 8 - p'_{cd} = 8 - 5 = 3$, and firm 1 has an incentive to increase p_{ab} up to 1, i.e. $p'_{ab} = 1 = p_a$. Similarly, consider instead that $p_a = p_{ab}$ but $p_d > p_{cd}$, then $p_a < 1$ and $p_d = 5$. Now is Firm 1 who increases p_{ab} up to some bound, i.e. $p'_{ab} = 1$ and again by equations 3 and 4 firm 2 has an incentive to increase p_{cd} up to $5 = p_d$.

Therefore the unique equilibrium price vector is $p_a = p_{ab}$ and $p_d = p_{cd}$ and the prices for products b and c are set high enough to make the other common agency bundle $\{b, c\}$ unprofitable for the agent

Example 3. **Efficient and inefficient equilibrium outcomes:** Consider again example 1, now with $0 < \delta < 1$. In this case both common agency bundles $\{a, d\}$ and $\{b, c\}$ are highly valuable to the agent in the sense that $v(a, d) > v(b, c) > v(a, b) + v(c, d)$. Now the consumer gets zero surplus and all the rent goes to principals. We prove that now both efficient and inefficient equilibrium outcomes exist.

By the same reasoning than in example 2 above, p_{ab} and p_{cd} cannot be equal to unit costs. However, since the two common agency bundles are highly valued with respect to the exclusive dealing ones, subgame perfection translates to setting $p_{ab} = M > v(a, b)$, $p_{cd} = M > v(c, d)$, M big enough[6], and then the binding constraint is $p_a + p_d = v(a, d) = 9 > v(a, b) + v(c, d)$, which implies $p_a \geq v(a, b)$ and $p_d \geq v(c, d)$, with at least one strict inequality. This additionally avoids principals' price deviation in order to sell its exclusive dealing bundle.

It is not difficult to show that a sub-game perfect equilibrium is the *efficient common agency*

[6] To see that assume on the contrary that $p_{ab} = p_a \leq v(a, b)$ and $p_{cd} = p_d \leq v(c, d)$, then the equilibrium price candidates are such that $p_a = p_{ab} = v(a, d) - v(c, d)$, $p_d = p_{cd} = v(a, d) - v(a, b)$, with $\pi_1 \leq v(a, b)$ and $\pi_2 \leq v(c, d)$.

outcome with consumption set $S = \{a, d\}$ and equilibrium prices satisfying:

$$\begin{aligned} p_a + p_d &= v(a,d) = 9,\ 9 - \delta \geq p_a \geq v(a,b) = 4,\ 5 \geq p_d \geq v(c,d) = \delta, \\ \text{and } p_k &= M, k \in \{b, c, \{a,b\}, \{c,d\}\} \end{aligned}$$

The high prices of products b and c and those of the exclusive dealing bundles $\{a, b\}$ and $\{c, d\}$ make it only attractive for the agent the efficient common agency bundle $S = \{a, d\}$. The agent's surplus is zero and firm 1's profits are $9 - p_d$. But principal 1 could sell consumption set $\{a, b\}$ instead by setting $p_a = p_b = M$ (M big enough) and $p_{ab} = 4$. Hence, at the equilibrium it must be that $9 - p_d \geq 4$ or $p_d \leq 5$; similarly, in order principal 2 does not deviate, $p_a \leq 9 - \delta$. Hence, at the equilibrium $p_a + p_d = 9$ with $4 \leq p_a \leq 9 - \delta$ and $\delta \leq p_d \leq 5$. The efficient common agency bundle is very valued by the agent compared with any of the exclusive dealing bundles. Hence, principals can obtain all the surplus. Notice that there are multiple sub-game perfect equilibrium prices sustaining the efficient common agency consumption set but in all of them the agent's surplus is zero and the sum of the firms' profits is a constant. Among them, let us highlight the following ones:

$$\begin{aligned} p_a &= 9 - \delta,\ p_d = \delta,\ p_b = p_c = p_{ab} = p_{cd} = M, \\ p_a &= 4,\ p_d = 5,\ p_b = p_c = p_{ab} = p_{cd} = M, \end{aligned}$$

where the first one gives the maximum possible profits to principal 1 and the second one does the same for principal 2. Any convex combination of these price vectors is also an equilibrium price vector.

Nevertheless, it is also easy to show that the *inefficient common agency outcome* $\{b, c\}$, supported by the price vector: $p_b + p_c = v(b,c) = 5$; $p_b \geq v(a,b) = 4$; $p_c \geq v(c,d) = \delta$; $p_k = M, \quad k \in \{a, d, \{a,b\}, \{c,d\}\}$, is also a sub-game perfect equilibrium. The reason parallels the one above. There are also multiple sub-game perfect equilibrium prices sustaining the inefficient consumption set. For instance,

$$\begin{aligned} p_b &= 5 - \delta, p_c = \delta, p_a = p_d = p_{ab} = p_{cd} = M, \\ p_b &= 4, p_c = 1, p_a = p_d = p_{ab} = p_{cd} = M, \end{aligned}$$

Notice that the inefficient outcome is not perturbed even if either principal 1 reduces the price of its remaining unsold goods to the price of the sold one, i.e. $p'_{ab} = p'_a = p_b$ or principal 2 sets $p'_{cd} = p'_d = p_c$. However, if both firms make that reduction simultaneously then the agent will have an incentive to buy a different bundle. The problem with these inefficient equilibria is that the two principals charge high prices for the goods of the efficient common agency bundle so that no individual firm can benefit from a price reduction of its part of the efficient bundle.

In conclusion, mixed bundling prices ensure the existence of equilibrium outcomes, but both efficient and inefficient outcomes belong to the Nash correspondence. However, we will show that if joint contract (price) reductions by principals were allowed, then only the efficient outcomes would remain.

4 Mixed bundling, efficiency and Strong Equilibria

In what follows, we characterize both the set of subgame perfect equilibria and the set of efficient subgame perfect equilibria of the delegated agency game $G^{MB}(n+1, v, c)$ where the principals (firms) might use mixed bundling contracts.

The subgame perfect Nash-equilibrium conditions preclude unilateral deviations from the agent (condition *BC* below) and from each principal (conditions *FC1-FC3* below). Namely, we need conditions that guarantee that each active principal does not have an incentive to either increase the equilibrium prices of its sold bundles (*FC1*) or reduce the prices of the unsold bundles to those of the sold ones in order to sell any of them profitably (*FC2*). In addition, the equilibrium conditions have to additionally guarantee that each non-active principals cannot benefit from price reductions to unit costs (*FC3*). The next Proposition characterizes the set of SPE-outcomes.

Proposition 1 *$(\widetilde{S}, \widetilde{p})$ is an SPE-outcome of G^{MB}, where $\widetilde{S} = (\widetilde{S}_1, \ldots, \widetilde{S}_n) \subseteq \Omega$ and $\widetilde{p} = (\widetilde{p}_1, ..., \widetilde{p}_n)$, $\widetilde{p}_i \in \mathcal{P}_i$ with $\widetilde{p} \geq c$, if and only if*

(BC) $v(\widetilde{S}) - \sum_{i \in F(\widetilde{S})} \widetilde{p}_i(\widetilde{S}_i) \geq v(S) - \sum_{i \in F(S)} \widetilde{p}_i(S_i)$, for all $S \subseteq \Omega$,

(FC1) For every $j \in F(\widetilde{S})$ there is $S^j \subseteq \Omega$ with $S^j_j = \emptyset$ such that

$$v(\widetilde{S}) - \sum_{i \in F(\widetilde{S})} \widetilde{p}_i(\widetilde{S}_i) = v(S^j) - \sum_{i \in F(S^j)} \widetilde{p}_i(S^j_i).$$

(FC2) For each $j \in F(\widetilde{S})$ and all $S \subseteq \Omega$ such that $j \in F(S)$

$$v(\widetilde{S}) - \sum_{i \in F(\widetilde{S})} \widetilde{p}_i(\widetilde{S}_i) \geq v(S) - [\widetilde{p}_j(\widetilde{S}_j) - c_j(\widetilde{S}_j) + c_j(S_j)] - \sum_{i \in F(S) \setminus j} \widetilde{p}_i(S_i).$$

(FC3) For each $j \notin F(\widetilde{S})$ and for all $S \subseteq \Omega$ such that $j \in F(S)$

$$v(\widetilde{S}) - \sum_{i \in F(\widetilde{S})} \widetilde{p}_i(\widetilde{S}_i) \geq v(S) - c_j(S_j) - \sum_{i \in F(S) \setminus j} \widetilde{p}_i(S_i).$$

Notice that (BC)-($FC3$) are implied by subgame perfection requirements: (BC) by the agent and ($FC1$)-($FC3$) by the firms' incentives. To see this, suppose that ($FC1$) does not hold, then by (BC) there is $j \in N$, such that for all $S^j \subseteq \Omega$ with $S^j_j = \emptyset$,

$$v(\widetilde{S}) - \sum_{i \in F(\widetilde{S})} \widetilde{p}_i(\widetilde{S}_i) > v(S^j) - \sum_{i \in F(S^j)} \widetilde{p}_i(S^j_i).$$

The above inequality implies that firm j is better off charging a price $\widetilde{p}_j(\widetilde{S}_j) + \varepsilon$, for a sufficiently small $\varepsilon > 0$, such that (BC) is still satisfied for all $S \subseteq \Omega$. Now, the agent observing the new price vector will again choose the consumption set $\widetilde{S}$, but firm j will obtain an extra profit of ε. Hence ($FC1$) must be verified if $(\widetilde{S}, \widetilde{p})$ is an SPE-outcome.

If ($FC2$) does not hold, then for some firm $j \in N$ there is a consumption set $S \subseteq \Omega$ such that

$$v(\widetilde{S}) - \sum_{i \in F(\widetilde{S})} \widetilde{p}_i(\widetilde{S}_i) < v(S) - [\widetilde{p}_j(\widetilde{S}_j) - c_j(\widetilde{S}_j) + c_j(S_j)] - \sum_{i \in F(S) \setminus j} \widetilde{p}_i(S_i).$$

hence firm j can set a price $p_j(S_j) = \widetilde{p}_j(\widetilde{S}_j) - c_j(\widetilde{S}_j) + c_j(S_j) + \varepsilon$, for a sufficiently small $\varepsilon > 0$, such that

$$v(\widetilde{S}) - \sum_{i \in F(\widetilde{S})} \widetilde{p}_i(\widetilde{S}_i) < v(S) - p_j(S_j) - \sum_{i \in F(S) \setminus j} \widetilde{p}_i(S_i),$$

which implies that the agent will now select the consumption set S and firm j will increase its profits.

If ($FC3$) is not verified, then for some firm $j \notin F(\widetilde{S})$ there is a consumption set $S \subseteq \Omega$ with $j \in F(S)$ such that

$$v(\widetilde{S}) - \sum_{i \in F(\widetilde{S})} \widetilde{p}_i(\widetilde{S}_i) < v(S) - c_j(S_j) - \sum_{i \in F(S) \setminus j} \widetilde{p}_i(S_i).$$

thus, similarly to the above, if firm j sets price $p_j(S_j) = c_j(S_j) + \varepsilon$, for a sufficiently small $\varepsilon > 0$, then the agent will select set S and firm j will increase its profits.

Conversely, if (BC), ($FC1$), ($FC2$) and ($FC3$) are satisfied, then $(\widetilde{S}, \widetilde{p})$ is an SPE-outcome since $\widetilde{S}$ is a best choice for the agent and no firm has an incentive to either reduce or increase its prices. Notice that the set S^j in ($FC1$) may be empty and in this case $v(\widetilde{S}) - \sum_{i \in F(\widetilde{S})} \widetilde{p}_i(\widetilde{S}_i) = 0$, and firms extract the entire consumer surplus.

Observe that Proposition 1 characterizes the set of all SPE-outcomes. It can be easily checked that all the SPE-outcomes of the above examples and, in particular, those of example 3 satisfy conditions (BC) to ($FC3$). To isolate efficient SPE-outcomes from inefficient ones, a new condition has to be imposed as illustrated in the following example.

EXAMPLE 4. Suppose that in example 3 above *both* active principals *simultaneously* reduce the prices of their unsold goods to those of the sold ones at a given inefficient SPE. For instance, consider the inefficient SPE-outcome, $S = \{b, c\}$, and prices: $\{p_b = 5 - \delta, p_c = \delta, p_a = p_d = p_{ab} = p_{cd} = M\}$. Set $p'_a = p'_{ab} = p_b = 5 - \delta$, and $p'_d = p'_{cd} = p_c = \delta$. At these new prices:

$$cs(\{a, d\}, p) = v(a, d) - (p_a + p_d) = 9 - (5 - \delta + \delta) = 4 > 0 = v(b, c) - (p_b + p_c) = cs(\{b, c\}, p)$$

and this inefficient SPE is ruled out. The same reasoning rules out all the remaining inefficient equilibria. Furthermore, it is also easily checked that the efficient SPE's are immune to these simultaneous price reductions.

With this idea in mind, we would like to consider only the subset of subgame perfect equilibrium outcomes which remains as equilibrium outcomes even if *all* non-active principals set unit cost prices and *all* active principals set prices for their unsold bundles equal to those of their sold ones adjusted by the cost-differential[7]. In other words, we want (*FC3*) to be satisfied for all subsets of non-active firms: for all $A \subseteq N \backslash F(\widetilde{S})$. Similarly, we want (*FC2*) to be satisfied for all subsets of active firms: for all $B \subseteq F(\widetilde{S})$. These conditions remove the set of SPE-outcomes in which some firms charge unreasonably high prices so that no individual firm can benefit from a price reduction of its products only. To define these restrictions on set SPE, consider the price vector $p = (p_1, ..., p_n) \in \mathcal{P}_1 \times \cdots \times \mathcal{P}_n$, and let $S \subseteq \Omega$. Define vector p^S for all $i \in N$, $T_i \subseteq \Omega_i$, as

$$p_i^S(T_i) = \begin{cases} p_i(S_i) & \text{if } i \in F(S),\ T_i = S_i \\ p_i(S_i) - c_i(S_i) + c_i(T_i) & \text{if } i \in F(S),\ T_i \neq S_i \\ c_i(T_i) & \text{if } i \notin F(S), \end{cases} \tag{1}$$

i.e. all the non-active firms set prices equal to the marginal cost, and all active firms set prices for unsold bundles equal to those of their sold bundles adjusted by the cost-differentials.

Definition 1 *For every triple (N, v, c), the subset SPE^* of SPE-outcomes of G^{MB} is defined as:*

$$SPE^* = \{(S, p) \in SPE | (S, p^S) \in SPE\}.$$

Equivalently, SPE^* is the set of equilibrium outcomes satisfying *BC*, *FC1*, and *FC4* (instead of *FC2* and *FC3*), where *FC4* is stated as,

[7] Notice that if unit costs were assumed to be zero, then these prices would amount to being equal to those of the sold bundles.

(FC4) For all $A \subseteq N \backslash F(\widetilde{S})$, all $B \subseteq F(\widetilde{S})$ and for all $S \subseteq \Omega$ such that $(A \cup B) \subseteq F(S)$,

$$v(\widetilde{S}) - \sum_{i \in F(\widetilde{S})} \widetilde{p}_i(\widetilde{S}_i) \geq v(S) - \sum_{i \in A} c_i(S_i) - \sum_{i \in B} [\widetilde{p}_i(\widetilde{S}_i) - c_i(\widetilde{S}_i) + c_i(S_i)] - \sum_{i \in F(S) \backslash (A \cup B)} \widetilde{p}_i(S_i).$$

Thus, we restrict the analysis to a certain subset SPE^* of SPE-outcomes. The next Proposition shows that *FC4* selects the set of efficient consumption bundles of G^{MB}.

Proposition 2 *For every value function v and unit cost vector c, if $(\widetilde{S}, \widetilde{p})$ is an SPE^*-outcome of G^{MB}, then $\widetilde{S}$ is socially efficient.*

Proof: If $(\widetilde{S}, \widetilde{p})$ is an SPE^*-outcome, then $(\widetilde{S}, \widetilde{p}^{\widetilde{S}}) \in SPE^*$. By *BC* in Proposition 1,

$$v(\widetilde{S}) - \sum_{i \in F(\widetilde{S})} \widetilde{p}_i^{\widetilde{S}}(\widetilde{S}_i) \geq v(S) - \sum_{i \in F(S)} \widetilde{p}_i^{\widetilde{S}}(S_i),$$

for all $S \subseteq \Omega$. Therefore,

$$\begin{aligned}
(v-c)(\widetilde{S}) - (v-c)(S) &\geq \sum_{i \in F(\widetilde{S})} (\widetilde{p}_i^{\widetilde{S}} - c_i)(\widetilde{S}_i) - \sum_{i \in F(S)} (\widetilde{p}_i^{\widetilde{S}} - c_i)(S_i) \\
&= \sum_{i \in F(\widetilde{S})} (\widetilde{p}_i^{\widetilde{S}} - c_i)(\widetilde{S}_i) - \sum_{i \in F(\widetilde{S}) \cap F(S)} (\widetilde{p}_i^{\widetilde{S}} - c_i)(S_i) \\
&= \sum_{i \in F(\widetilde{S}) \backslash F(S)} (\widetilde{p}_i - c_i)(\widetilde{S}_i) \geq 0,
\end{aligned}$$

given that $\widetilde{p}_i^{\widetilde{S}}(S_i) = c_i(S_i)$ for all $i \notin F(\widetilde{S})$ and $p_i^{\widetilde{S}}(S_i) - c_i(S_i) = p_i^{\widetilde{S}}(\widetilde{S}_i) - c_i(\widetilde{S}_i)$ for all $i \in F(\widetilde{S}) \cap F(S)$. Thus, $(v-c)(\widetilde{S}) \geq (v-c)(S)$ for every $S \subseteq \Omega$. ■

The definition of SPE^* captures the idea of stability against joint deviations that are mutually profitable to any subset of firms and the agent. We look for an equilibrium concept behind conditions *BC*, *FC1*, and *FC4*, or set SPE^*, refining the set of SPE-outcomes. Aumann (1959), proposed the concept of *Strong Equilibrium* as an equilibrium such that no subset of players has a *joint* deviation that strictly benefits *all* of them. The extension of that concept to a *Subgame Perfect Strong Equilibrium -SPSE-* of our game G^{MB} is as follows,

Definition 2 *$(\widehat{S}, \widehat{p})$ is an SPSE-outcome of G^{MB}, where $\widehat{S} = (\widehat{S}_1, \ldots, \widehat{S}_n) \subseteq \Omega$ and $\widehat{p} = (\widehat{p}_1, ..., \widehat{p}_n)$, $\widehat{p}_i \in \mathcal{P}_i$ with $\widehat{p} \geq c$ if*

i) $v(\widehat{S}) - \sum_{i \in F(\widehat{S})} \widehat{p}_i(\widehat{S}_i) \geq v(T) - \sum_{i \in F(T)} \widehat{p}_i(T_i)$, for all $T \subseteq \Omega$, *ii) For all $M \subseteq N$ there is no $(p'_j)_{j\in M}$ and $S \subseteq \Omega$ such that*

a) $(p'-c)_j(S_j) > (\widehat{p}-c)_j(\widehat{S}_j)$, *for each* $j \in M$, *and*

b) $v(S) - \sum_{j\in M} p'_j(S_j) - \sum_{i\in N/M} \widehat{p}_i(S_i) > v(\widehat{S}) - \sum_{i\in N} \widehat{p}_i(\widehat{S}_i)$

Since the deviating coalition can be either an individual firm or the agent, this implies that an $SPSE$-outcome is therefore an SPE. Moreover, it is shown next that conditions *BC*, *FC1*, and *FC4* are the only conditions characterizing $SPSE$-outcomes.

Proposition 3 *For every value function v, unit cost vector c, and game G^{MB}, the set of $SPSE$-outcomes coincides with the set of SPE^*-outcomes.*

Proof: Let us first prove that any $SPSE$-outcome is an SPE^*. Let $(\widehat{S}, \widehat{p})$ be an *SPSE* and suppose that it is not an SPE^*. By *FC4* of the definition of SPE^*, there are sets $A \subseteq N \backslash F(\widehat{S})$, $B \subseteq F(\widehat{S})$ and $S \subseteq \Omega$ such that

$$v(\widehat{S}) - \sum_{i\in F(\widehat{S})} \widehat{p}_i(\widehat{S}_i) < v(S) - \sum_{i\in A} c_i(S_i) - \sum_{i\in B}[\widehat{p}_i(\widehat{S}_i) - c_i(\widehat{S}_i) + c_i(S_i)] - \sum_{i\in F(S)\backslash(A\cup B)} \widehat{p}_i(S_i)$$

and the consumer switches to S. Thus, the coalition of firms in $A \cup B$ has an incentive to jointly deviate, obtaining higher profits than those under $(\widehat{S}, \widehat{p})$, which contradicts $(\widehat{S}, \widehat{p})$ being an $SPSE$-outcome.

Now let us assume that $(\widetilde{S}, \widetilde{p})$ is an SPE^*, but not an *SPSE*. Since both equilibrium concepts are subgame perfect, then $(\widetilde{S}, \widetilde{p})$ would not verify condition *ii)* above: $\exists M \subseteq N$, $(p'_j)_{j\in M}$ and $S \subseteq \Omega$ such that $\forall j \in M$, $(p'-c)_j(S_j) > (\widetilde{p}-c)_j(\widetilde{S}_j)$, and $v(S) - \sum_{j\in M} p'_j(S_j) - \sum_{i\in N/M} \widetilde{p}_i(S_i) > v(\widetilde{S}) - \sum_{i\in N} \widetilde{p}_i(\widetilde{S}_i)$. Define the sets $A = M \cap (N\backslash F(S))$ and $B = M \cap F(S)$, then condition *FC4* does not hold, which contradicts $(\widetilde{S}, \widetilde{p})$ being an SPE^*. ■

Therefore set $SPSE$ is characterized by set SPE^* and hence any $SPSE$-consumption set is socially efficient.

5 Existence, efficiency and characterization of Subgame Perfect Strong Equilibria.

5.1 Existence and efficiency

Our main result establishes that given any agent's value function v, and unit cost vector c, there is always a subgame perfect equilibrium of the delegated agency game G^{MB} verifying conditions BC, FC1 and FC4, i.e. set $SPSE$ is non-empty (Theorem 1). Moreover, not only is any $SPSE$-consumption set efficient (Proposition 2), but also any socially efficient consumption set belongs to an $SPSE$-outcome (Corollary 1).

Theorem 1 *For every value function, v, and unit cost vector, c, there is an equilibrium in $SPSE$.*

Proof: By Proposition 3, it suffices to prove that set SPE^* is non-empty.

Let $\widetilde{S} \in \arg\max_{(S)}\{(v-c)(S)\}$. Define the set

$$\Pi^f = \{\pi \in R^n_+ | \pi_i = 0, \forall i \notin F(\widetilde{S}) \text{ and } \sum_{i \in F(\widetilde{S}) \setminus F(S)} \pi_i \leq (v-c)(\widetilde{S}) - (v-c)(S), \forall S \subseteq \Omega\}.$$

Set Π^f is non-empty since $0 \in \Pi^f$ and it is bounded, since for every $\pi \in \Pi^f$ we have $\pi_i \leq (v-c)(\widetilde{S}) - (v-c)(\widetilde{S} \setminus \widetilde{S}_i)$ for all $i \in F(\widetilde{S})$ and $\pi_i = 0$ for all $i \in N \setminus F(\widetilde{S})$.

Also observe that Π^f is closed and hence compact. Thus Π^f contains an element $\widetilde{\pi}$ which is maximal with respect to the lexicographical order on Π^f. Let $\widetilde{p} = (\widetilde{p}_1, ..., \widetilde{p}_n) \in \mathcal{P}_1 \times \cdots \times \mathcal{P}_n$ be defined as $\widetilde{p}_i(T_i) = \widetilde{\pi}_i + c_i(T_i)$ for all $i \in N, T_i \subseteq \Omega_i$. Notice that $\widetilde{p}_i(T_i) = c_i(T_i)$ for all $i \notin F(\widetilde{S})$, and $\widetilde{p}_i(\widetilde{S}_i) - c_i(\widetilde{S}_i) = \widetilde{\pi}_i$ for all $i \in F(\widetilde{S})$. This leads to,

$$\widetilde{p}_i(T_i) = \widetilde{\pi}_i + c_i(T_i) = \widetilde{p}_i(\widetilde{S}_i) - c_i(\widetilde{S}_i) + c_i(T_i),$$

for all $i \in F(\widetilde{S})$ and $T_i \neq S_i$. Hence $\widetilde{p}_i(T_i) = \widetilde{p}_i^{\widetilde{S}}(T_i)$ as defined in (1).

We claim that $(\widetilde{S}, \widetilde{p})$ is an SPE^*-outcome. Since $\widetilde{p}_i = \widetilde{p}_i^{\widetilde{S}}$, it suffices to prove that $(\widetilde{S}, \widetilde{p})$ is an SPE-outcome. Let $S \subseteq \Omega$, since $\widetilde{\pi} \in \Pi^f$, then $\sum_{i \in F(\widetilde{S}) \setminus F(S)} \widetilde{\pi}_i \leq (v-c)(\widetilde{S}) - (v-c)(S)$ or equivalently

$$(v-c)(\widetilde{S}) - \sum_{i \in F(\widetilde{S})} \widetilde{\pi}_i \geq (v-c)(S) - \sum_{i \in F(S)} \widetilde{\pi}_i.$$

hence,

$$v(\widetilde{S}) - \sum_{i \in F(\widetilde{S})} \left[\widetilde{\pi}_i + c(\widetilde{S}_i)\right] \geq v(S) - \sum_{i \in F(S)} \left[\widetilde{\pi}_i + c(S_i)\right],$$

which by the definition of $\widetilde{p}_i$ is condition BC of Proposition 1.

Conditions FC2 and FC3 of Proposition 1 hold trivially, given that $\widetilde{p}_i(T_i) = \widetilde{p}_i^{\widetilde{S}}(T_i)$.

To prove condition FC1 of Proposition 1 suppose, on the contrary, that $j \in F(\widetilde{S})$ exists such that $v(\widetilde{S}) - \sum_{i \in F(\widetilde{S})} \widetilde{p}_i(\widetilde{S}_i) > v(S) - \sum_{i \in F(S)} \widetilde{p}_i(S_i)$ for all $S \subseteq \Omega$ with $S_j = \emptyset$. Let

$$\varepsilon = \min_{S \subseteq \Omega, S_j = \emptyset} \left\{ v(\widetilde{S}) - \sum_{i \in F(\widetilde{S})} \widetilde{p}_i(\widetilde{S}_i) - \left(v(S) - \sum_{i \in F(S)} \widetilde{p}_i(S_i) \right) \right\},$$

then $\varepsilon > 0$. Let $\widehat{\pi} \in R_+^n$ be defined as

$$\widehat{\pi}_i = \begin{cases} \widetilde{\pi}_i & i \neq j \\ \widetilde{\pi}_i + \varepsilon & i = j \end{cases}$$

and let $\widehat{p} = (\widehat{p}_1, ..., \widehat{p}_n)$ be defined as $\widehat{p}_i(T_i) = \widehat{\pi}_i + c_i(T_i)$ for all $i \in N, T_i \subseteq \Omega_i$. Notice that $\widehat{p}_i(T_i) = \widetilde{p}_i(T_i)$ for all $i \in N \backslash j$, $T_i \subseteq \Omega_i$ and $\widehat{p}_j(T_j) = \widetilde{p}_j(T_j) + \varepsilon$. Since $\widetilde{\pi}$ is a maximal element of Π^f, $\widehat{\pi} \notin \Pi^f$. However, we will show that $\sum_{i \in F(\widetilde{S}) \backslash F(S)} \widehat{\pi}_i \leq (v-c)(\widetilde{S}) - (v-c)(S)$ for all $S \subseteq \Omega$ and hence $\widehat{\pi} \in \Pi^f$, which is a contradiction. Given $S \subseteq \Omega$, if $j \in F(S)$, then $\sum_{i \in F(\widetilde{S}) \backslash F(S)} \widehat{\pi}_i \leq (v-c)(\widetilde{S}) - (v-c)(S)$ since $\widetilde{\pi}_i = \widehat{\pi}_i$ for $i \neq j$. Suppose next that $j \notin F(S)$, by the definition of ε

$$v(\widetilde{S}) - \sum_{i \in F(\widetilde{S})} \widetilde{p}_i(\widetilde{S}_i) - \varepsilon \geq v(S) - \sum_{i \in F(S)} \widetilde{p}_i(S_i)$$

but by the definition of $\widehat{p}$ and $\widetilde{p}$ the left hand side of the above inequality can be written as,

$$v(\widetilde{S}) - \sum_{i \in F(\widetilde{S})} \widetilde{p}_i(\widetilde{S}_i) - \varepsilon = v(\widetilde{S}) - \sum_{i \in F(\widetilde{S})} \left[\widetilde{\pi}_i + c_i(\widetilde{S}_i) \right] - \varepsilon = (v-c)(\widetilde{S}) - \sum_{i \in F(\widetilde{S})} \widehat{\pi}_i$$

and the right hand side as

$$v(S) - \sum_{i \in F(S)} \widetilde{p}_i(S_i) = v(S) - \sum_{i \in F(S)} [\widetilde{\pi}_i + c_i(S_i)] = (v-c)(S) - \sum_{i \in F(S)} \widehat{\pi}_i.$$

Hence,

$$\sum_{i \in F(\widetilde{S}) \backslash F(S)} \widehat{\pi}_i \leq (v-c)(\widetilde{S}) - (v-c)(S)$$

as claimed. Therefore $(\widetilde{S}, \widetilde{p})$ is an SPE^*-outcome. ■

The proof of Theorem 1 also shows that efficiency is a sufficient condition to belong to an $SPSE$-outcome, i.e. if $\widetilde{S} \in \arg\max_{(S)}\{(v-c)(S)\}$, then there is a price vector $\widetilde{p}$ such that $(\widetilde{S}, \widetilde{p})$ is an

$SPSE$-outcome. This, jointly with Proposition 2, allows us to assert that the *subgame perfect strong equilibrium* concept selects the set of efficient SPE of G^{MB} from the SPE correspondence.

Corollary 1 *For every value function v and unit cost vector c, $SPSE$ is the set of efficient SPE-outcomes of G^{MB}.*

5.2 Characterization of $SPSE$-profit vectors

$SPSE$-consumption sets have been characterized as the socially efficient ones. In this section, we characterize the set of firms' profits (the principals' rents) which comes from $SPSE$-outcomes in the delegated agency game G^{MB}. By Definition 1 and Proposition 3, notice that if (S, p) is an $SPSE$-outcome, then (S, p^S) is also an $SPSE$-outcome and the principals and the agent obtain the same payoffs under such outcomes: the two equilibria are payoff-equivalent. Thus, any pair (S, p^S) makes it possible to identify its payoff-equivalence class and for any $SPSE$-price vector, we can only consider (S, p^S) as the representative element of its equivalence class. We will show that the vector of principals' profits from $SPSE$-outcomes are their most preferred points in the core of the agency game G^{MB}. Let us define such a core as

$$core(v-c) = \{(\pi^b, (\pi_i)_{i\in N}) \in R_+^{n+1} | \ \pi^b + \sum_{i\in N} \pi_i = V(N) \text{ and } \pi^b + \sum_{i\in F(S)} \pi_i \geq (v-c)(S), \ \forall S \subseteq \Omega\}.$$

and let Π^{PF} be the Pareto frontier of the projection of $core(v-c)$ on the n last coordinates. Formally,

$$\Pi^{PF} = \{(\pi_i)_{i\in N} \in R_+^n | \text{ there is } \pi^b \geq 0 \text{ with } (\pi^b, (\pi_i)_{i\in N}) \in core(v-c) \text{ and there is no other } (\pi'^b, (\pi'_i)_{\in N}) \in core(v-c), \text{ such that } \pi'_i \geq \pi_i, \text{ for all } i \in N \text{ and } \pi'_j > \pi_j \text{ for at least some } j\}.$$

Notice that the core of the game is defined through linear inequalities, thus it is a polytope and so is Π^{PF}. We will prove that Π^{PF} is the set of the principals' equilibrium rents. Furthermore, set Π^{PF} can be expressed as the convex linear combination of adjacent vertices. First, consider the set of socially efficient bundles, $\arg\max_{T\subseteq\Omega}\{(v-c)(T)\}$. The following lemma states that if a principal is not selling in *all* the efficient bundles, then his coordinate or payoff in the core of the delegated agency game G^{MB} will be zero.

Lemma 1 *Let v be a value function and c an unit cost vector. If $(\pi^b, (\pi_i)_{i\in N}) \in core(v-c)$, then $\pi_j = 0$ for all $j \notin \cap_{S\in\arg\max_T\{(v-c)(T)\}} F(S)$.*

Proof: Let $S \in \arg\max_T\{(v-c)(T)\}$, then $\sum_{i\in N\setminus F(S)} \pi_i \leq 0$ since $V(N) = (v-c)(S)$ and $\pi^b + \sum_{i\in F(S)} \pi_i \geq (v-c)(S)$. Hence $\pi_j = 0$ for all $j \notin F(S)$, $S \in \arg\max_T\{(v-c)(T)\}$. ■

The following result shows that: first, each element $(\pi_i)_{i\in N} \in \Pi^{PF}$ has associated an equilibrium price vector (a contract vector) such that π_i is firm i's profit or rent; second, if p is an $SPSE$-price vector, then the corresponding firms' profit vector belongs to Π^{PF}. In the Appendix it is proven[8],

Proposition 4 *For every value function v and unit cost vector c it is verified that*

i) if $(\pi_i)_{i\in N} \in \Pi^{PF}$, then $(S,p) \in SPSE$, where S is any socially efficient consumption set and $p_i(T_i) = \pi_i + c_i(T_i)$ for all $i \in N, T_i \subseteq \Omega_i$,

ii) if (S,p) is an $SPSE$-outcome, then $(\pi_i)_{i\in N} \in \Pi^{PF}$, where $\pi_i = (p_i - c_i)(S_i)$, $i \in N$.

Given the above Proposition, Lemma 1 implies that a firm could only obtain a positive profit if it were selling at least a component of every efficient bundle. Alternatively, if none of the firms sell at least one component of every efficient bundle, then Lemma 1 asserts that every equilibrium consumption set is offered at unit cost prices and the consumer extracts the entire social surplus.

The intuition is clear, suppose two firms and the market for systems of the motivating example. If non principal sells at least one component of every efficient bundle, then there exist at least two efficient systems: the exclusive dealing bundles. At equilibrium $(v-c)(a,b) = (v-c)(c,d)$. By

[8] Alternatively *all* the $SPSE$-prices vectors can be characterized by the optimal solutions of a family of linear programming problems. To this end we define a family of objective functions which take into account the lexicographical order of the elements of a vector. Let μ be an ordered partition of N in the sense that the order of the elements in the partition is relevant. Thus, μ and μ' can give rise to the same partition, but with a different order in their elements. Let Γ denote the set of all the ordered partitions. Write $\mu = \{N_1, N_2, ..., N_L\} \in \Gamma$ to mean that under μ the first element of the partition is N_1, the second is N_2 and the last one is N_L. Note that L can differ from one partition to another, but $N_1 \cup \ldots \cup N_L = N$. We then define the linear programming problem, μ-**LPP** as

$$\begin{array}{ll} Max & \sum_{l=1}^{L} \left(\sum_{i\in N_l} \pi_i \right) 10^{d(L-l)} \\ s.t. & \pi^b + \sum_{S_i \in S} \pi^i_{S_i} \geq (v-c)(S) \qquad \forall S \subseteq \Omega \\ & \pi_i - \pi^i_{T_i} \geq 0 \qquad \forall i \in N,\ \forall T_i \subseteq \Omega_i \\ & \pi^b + \sum_{i\in N} \pi_i = V(N) \\ & \pi^b, \pi_i, \pi^i_{T_i} \geq 0, \end{array}$$

The set of solutions is a non-empty convex polytope, since $\pi^b = V(N)$, $\pi_i = \pi^i_{T_i} = 0$ defines a feasible solution. Denote this set by $sol(\mu$-$\mathbf{LPP})$. If $(\pi^b, (\pi_i), (\pi^i_{T_i})) \in sol(\mu$-$\mathbf{LPP})$ represents a generic solution of μ-**LPP**, then π^b can be interpreted as the consumer surplus, (π_i) as the firms' profits and $(\pi^i_{S_i} + c_i(S_i))$ as the price vector. Let us define Π as the set of solutions of the family of linear programming problems whose components (π_i) are non Pareto dominated by those components of other solutions.

$$\Pi = \{(\pi^b, (\pi_i), (\pi^i_{T_i})) \in sol(\mu - \mathbf{LPP}) | \mu \in \Gamma \text{ and there is no other } (\pi'^b, (\pi'_i), (\pi'^i_{T_i})) \in sol(\mu' - \mathbf{LPP}),$$
$$\mu' \in \Gamma, \text{ such that } \pi'_i \geq \pi_i, \text{ for all } i \text{ and } \pi'_j > \pi_j \text{ for at least some } j\}.$$

It is not difficult to show that i) if $(\pi^b, (\pi_i), (\pi^i_{T_i})) \in \Pi$, then $(\pi_i) \in \Pi^{PF}$; and vice versa ii) if $(\pi_i) \in \Pi^{PF}$, then $(\pi^b, (\pi_i), (\pi^i_{T_i})) \in \Pi$ where $\pi^i_{T_i} = \pi_i$ for all $i \in N, T_i \subseteq \Omega_i$ and $\pi^b = V(N) - \sum_{i\in N} \pi_i$. Then, by Proposition 4 every $SPSE$-price vector defines a solution of μ-**LPP** for some ordered partition μ of N and each element $(\pi^b, (\pi_i), (\pi^i_{T_i})) \in \Pi$ defines an $SPSE$-price vector. The proof of these results can be provided by the authors upon request.

Lemma 1, firms' profits are zero and then by Proposition 4, $p_{ab} = v(a,b) - \max\{(v-c)(c,d), 0\}$, $p_{cd} = v(c,d) - \max\{(v-c)(a,b), 0\}$ and therefore the agent obtains all the surplus. Next, suppose that only one firm, say Firm 1, sells at least one component of every efficient bundle. This can be only if $\{a,b\}$ is the efficient bundle but $\{c,d\}$ is not. Then, the profit of Firm 1 is $(v-c)(a,b) - \max\{(v-c)(c,d), 0\}$, which is positive since $(v-c)(a,b) > (v-c)(c,d)$. Finally, suppose that both Firm 1 and 2 sell at least one component of every efficient system. The efficient bundles must be either $\{a,d\}$ and $\{b,c\}$, or $\{a,d\}$, or $\{b,c\}$. In each of them, at equilibrium p_{ab} and p_{cd} are higher than unit costs and either $p_{ab} = p_a$, $p_{cd} = p_d$ or $p_{ab} = p_b$, $p_{cd} = p_c$ or both. Therefore, equilibrium prices are higher than unit costs and firms' profits are positive.

Additionally notice that when principals are identical (they produce the same substitute products with the same technology) the above results imply that all firms set unit cost prices for the equilibrium bundles, so that the agent chooses an efficient consumption set and obtains the entire surplus. From the agent's point of view, the principals are perfect substitutes so that there is strong competition among them. Nevertheless, our model goes beyond the classical Bertrand Theory in two respects. On the one hand, the principals' products need not be perfect substitutes but be their bundles; in this case, the principals are indistinguishable for the agent. On the other hand, complementarities give rise to new equilibrium patterns even with indistinguishable firms.

In conclusion, the above Proposition 4 shows that if all socially efficient consumption sets of the agency game G^{MB} are such that $\widetilde{S}_i \neq \emptyset$ and $\widetilde{S}_j = \emptyset$ for all firms $j \in N \backslash i$, i.e. the agent chooses only the products of a single firm (exclusive dealing), then $\widetilde{S}$ is sold as a bundle at price $p_i(\widetilde{S}_i) = v(\widetilde{S}) - \alpha$, where $\alpha = \max_{S \subseteq \Omega, S_i = \emptyset}\{(v-c)(S), 0\}$. Principal i obtains positive profits, the products of any other principal are offered at unit cost prices and the agent obtains a positive payoff equal to α. But if there are two or more socially efficient exclusive dealing bundles, then they are sold at unit cost prices and the agent obtains the entire surplus.

On the other hand, when the equilibrium consumption set is a common agency bundle (i.e., it contains products of two or more principals), then although the principals might offer their products as bundles at special prices, the agent selects a subset of products of each firm. Therefore, mixed bundling contracts that discriminate on exclusivity can be seen as either an aggressive pricing policy for exclusive dealing outcomes or as out-of-equilibrium offers sustaining the equilibrium consumption sets of individual components in delegated common agency allocations (involving either several principals or all of them). However, the precise form of equilibrium prices is difficult to obtain unless we know the specific value functions. The next section characterizes the $SPSE$-prices (contracts) and the rent-sharing between the agent and the principals of monotonic social surplus functions.

6 Monotonic social surplus functions: Delegated Common Agency.

In this section we study the surplus sharing between the agent and the principals in delegated common agency with externalities, when the social surplus function is monotonic. In this context, exclusive dealing is never an equilibrium consumption set, and the agent will contract with either a subset of principals (partial common agency) or with all of them (common agency). Our findings extend previous results by Laussel and Lebreton (2001) in the framework of intrinsic common agency and monotonic social surplus functions. We show that when the social surplus function is supermodular, then the agent's surplus is zero and the set of principals' rents is completely characterized by the convex hull of the vertex arising from their accumulative marginal contributions, which coincides with the Pareto frontier of $core(v-c)$. We also prove that when the principals are substitutes (a more general condition than strong subadditivity), then at any equilibrium the principals' rents are their marginal contributions and the agent obtains the difference between the social marginal contribution of her consumption set S and the sum of the social marginal contributions of the principals in S.

Let us introduce some convention in notations. Let $\Omega \backslash S$ be $(\Omega_1 \backslash S_1, \ldots, \Omega_n \backslash S_n)$ and let $v(S_i)$ be $v(\emptyset, \ldots, \cup S_i, \ldots \emptyset)$ i.e., a consumption set where the agent only buys to principal i. Given $w \in \Omega_i$ and $S \subseteq \Omega$, let $S + w$ be $(S_1, \ldots, S_i\{w\}, \ldots, S_n)$.

Definition 3 *i)* $(v-c)$ *is* monotonic *if and only if* $(v-c)(S) \leq (v-c)(T)$ *whenever* $S \subseteq T \subseteq \Omega$,

ii) $(v-c)$ *is* submodular *if and only if* $(v-c)(T+w) - (v-c)(T) \leq (v-c)(S+w) - (v-c)(S)$ *whenever* $S \subseteq T \subseteq \Omega \backslash w$*, and*

iii) $(v-c)$ *is* supermodular *if the opposite inequality holds.*

Thus, the monotonicity of $(v-c)$ implies that the social surplus increases for larger consumption sets. If $(v-c)$ is monotonic, then, by Theorem 1, there is always an $SPSE$-outcome with Ω as the equilibrium consumption set. Furthermore, if $(v-c)$ is strictly monotonic, then Ω (common agency) is the unique equilibrium consumption set. Thus under strict monotonicity of the social value function, G^{MB} is a delegated common agency game with externalities where the agent contracts with all the principals (common agency). The next lemma shows that principal i's equilibrium rent has an upper bound, which imposes a constraint on the maximum prices of the bundles.

Lemma 2 *If* $(\Omega, p) \in SPSE$*-outcome set, then* $p_i(\Omega_i) - c(\Omega_i) \leq smc^*(\Omega_i)$ *for all* $i \in N$

Proof: Let $\Omega \backslash \Omega_i = (\Omega_1, \ldots \Omega_{i-1}, \emptyset, \Omega_{i+1}, \ldots, \Omega_n)$ and let $i \in N$, by BC, $v(\Omega) - \sum_{k \in N} p_k(\Omega_k) \geq$

$v(\Omega\backslash\Omega_i) - \sum_{k\in N\backslash i} p_k(\Omega_k)$ or equivalently, $v(\Omega) - v(\Omega\backslash\Omega_i) \geq p_i(\Omega_i)$. Thus, $smc^*(\Omega_i) \geq (p_i - c_i)(\Omega_i)$. ■

Suppose now that the social surplus function is *submodular.* This is a condition that seems particularly attractive since it expresses, in the case of indivisible goods, the idea that the marginal utility of an item decreases when the bundle of goods to which it is added gets larger. The submodularity of $(v - c)$ reflects a kind of substitution among products or bundles of products so that there is competition among principals and the agent will obtain some surplus.

The *social marginal contribution* of consumption set $S \subseteq \Omega$, $smc^*(S)$, i.e. the increase in social surplus due to S, is

$$smc^*(S) = (v-c)(\Omega) - (v-c)(\Omega\backslash S) = v(\Omega) - v(\Omega\backslash S) - c(S), \qquad \forall S \subseteq \Omega.$$

Following Shapley (1962), we say that principals are *substitutes* if the social marginal contribution of consumption set S is bigger than or equal to the sum of the social marginal contributions of firms in S,

$$smc^*(S) \geq \sum_{i\in F(S)} smc^*(S_i) \qquad \forall S \subseteq \Omega. \tag{FS}$$

The next proposition states that the equilibrium prices of monotonic social surplus functions satisfying FS are equal to the social marginal contributions of the principals' efficient bundles plus their corresponding marginal costs. In the Appendix it is proven,

Proposition 5 *Let $(v - c)$ be a monotonic social surplus function and suppose that principals are substitutes (FS holds). Then $(S^*, p^*) \in SPSE$-outcome set, with $(v - c)(S^*) = (v - c)(\Omega)$ and $p_i^*(T_i) = smc^*(\Omega_i) + c(T_i)$, for all $T_i \subseteq \Omega_i$, and $i \in N$. The converse is also true.*

If $(v - c)$ is strictly monotonic, then the unique equilibrium consumption set is Ω.

Each principal i sells S_i^* as a bundle and obtains its marginal contribution as its profits, $(p_i^* - c_i)(S_i^*) = smc^*(\Omega_i)$. Moreover, the agent surplus is positive, reflecting market competition under FS:

$$\begin{aligned} cs(S^*) = v(S^*) - \sum_{i\in F(S^*)} p_i^*(S_i^*) = (v-c)(S^*) - \sum_{i\in F(S^*)} (p_i^* - c_i)(S_i^*) = \\ = (v-c)(\Omega) - \sum_{i\in F(S^*)} smc_i^*(\Omega_i) = smc^*(\Omega) - \sum_{i\in F(S^*)} smc_i^*(S_i^*) \geq 0. \end{aligned}$$

Therefore when the principals are substitutes, the efficient equilibrium of G^{MB} is characterized by each principal i, $i = 1, 2, ..., N$, offering contracts $p_i^*(T_i) = smc^*(\Omega_i) + c(T_i)$, for all $T_i \subseteq \Omega_i$, with profits equal to $(p_i^* - c_i)(S_i^*) = smc^*(\Omega_i)$ and the agent obtaining the difference $smc^*(\Omega) - \sum_{i\in F(S^*)} smc_i^*(S_i^*)$ as her rent.

Two straightforward results are the following: 1) if $(v-c)$ is a submodular social surplus function, then principals are substitutes, i.e. FS is satisfied; thus the FS condition is more general than the concavity(or strong subadditivity) condition in Laussel and Lebreton (2001); and 2) if $(v-c)$ is a submodular value function and $smc^*(w) \geq 0$ for all $i \in N$ and $w \in \Omega_i$, then $(v-c)$ is monotonic (where $smc^*(w) = smc^*(\emptyset, \ldots, w, \ldots, \emptyset)$). Then, trivially from Proposition 5 we obtain the next result that extends to delegated common agency with externalities that of the above authors.

Corollary 2 *Let $(v-c)$ be submodular and $smc^*(w) \geq 0$, for all $i \in N$ and $w \in \Omega_i$. Thus, principals are substitutes, and for all i, principal i's equilibrium rent in any $SPSE$-outcome is equal to the social marginal contribution of its Ω_i.*

Now suppose that the social surplus function is *supermodular.* The supermodularity of $(v-c)$ reflects complementarities among products or bundles of products and hence among principals. Therefore it induces only weak market competition so that principals can extract the entire agent surplus. It is straightforward to prove that if $(v-c)$ is nonnegative and supermodular, then $(v-c)$ is monotonic.

Let Σ be the set of permutations (orderings) of N and let $\sigma \in \Sigma$ be any of its elements. Let P_i^σ be the set of principals which precede principal i with respect to permutation σ, i.e., for all $i \in N$ and $\sigma \in \Sigma$, $P_i^\sigma = \{j \in N | \sigma(j) < \sigma(i)\}$

Define, following Shapley (1971), the marginal contribution vector $x^\sigma(v-c) \in \mathbb{R}^n$ of $(v-c)$ with respect to ordering σ by,

$$x_i^\sigma(v-c) = V(P_i^\sigma + i) - V(P_i^\sigma), \qquad \text{for all } i \in N$$

In the Appendix it is proven,

Proposition 6 *Let $(v-c)$ be a supermodular value function, such that $(v-c)(w) \geq 0$ for all $i \in N$ and $w \in \Omega_i$. Then the agent's surplus is zero and the principals' equilibrium profits in any $SPSE$-outcome are $conv\{x^\sigma(v-c) | \sigma \in \Sigma\}$*

By Proposition 4, the principals' equilibrium rents are characterized by set $conv\{x^\sigma(v-c)|\sigma \in \Sigma\}$, which turns out to be the Pareto frontier of $core(v-c)$. Thus, when $(v-c)$ is supermodular, then the core of the value function is always priced by subgame perfect strong equilibrium payoffs.

7 Concluding Remarks

This paper has contributed to the literature on delegated common agency with externalities and complete information by extending the insights by Berheim and Whinston (1986, a,b) to multi-product market with indivisibilities.

First, we have characterized the equilibrium outcomes in these settings and, by considering a kind of extended contracts -mixed bundling contracts- that stresses the role of out-of-equilibrium offers, we have shown the equilibrium existence. Under mixed bundling contracts the agent has the option of buying bundles of goods from a firm at a discount over the single good prices. In our model, with multi-product firms and an agent with preferences over each bundle of goods, mixed bundling contracts are conditional on exclusive dealing for each bundle of two or more goods. Therefore, these contracts can be seen as either an aggressive pricing policy for exclusive dealing outcomes or as out-of-equilibrium offers sustaining the equilibrium consumption sets of individual components in delegated common agency allocations. The discrimination on exclusivity both facilitates collusion on common agency outcomes and represents a credible threat that avoids deviations by the principals, thus helping them set incentive-compatible contracts. We have also found that equilibrium need not be unique in the sense that many equilibrium price vectors may sustain the same equilibrium allocation. This is due to the fact that each principal offers contracts for its products and also offers subsets of them as bundles at a special price. Notice that this implies that we have not considered singleton contracts (direct mechanisms) in the delegated agency game. Such mechanisms do not allow for any offer to remain unchosen in equilibrium; in other words, out-of-equilibrium messages are possible and they have a commitment value. Furthermore, efficient and inefficient equilibria may belong to the sub-game Nash correspondence. The lack of coordination among the principals is the reason behind inefficient equilibria, in which the agent chooses a suboptimal bundle and no principal has a profitable deviation inducing the agent to buy the surplus-maximizing bundle.

Second, we have ruled out inefficient equilibria by either assuming that all firms are pricing unsold bundles at the same profit margin as the bundle sold at equilibrium, or imposing the solution concept of subgame perfect strong equilibrium, which requires the absence of profitable deviations by any subset of principals and the agent. Our rationale for equilibrium menus which take into account deviations by any set of firms and the player is less demanding than that of coalition-proof equilibrium which requires the equilibrium immunity to deviations by subsets of principals which are themselves immune to deviations by sub-coalitions, etc. Therefore, the refinement of strong equilibrium gives a another (subgame perfect) noncooperative justification of *truthful equilibria* (Bernheim and Whinston,1986, a,b) in the context of delegated agency games.

Finally, we have analyzed the specific structure of equilibrium prices and payoffs for common agency outcomes when the social surplus function is monotone and either submodular or supermodular. In the former case, principals are substitutes and their equilibrium rents are equal to the principals' social marginal contributions with an agent's positive rent, thus reflecting market competition. In the latter case, the agent's rent is zero and then the core of the value function is always priced by the subgame perfect Nash-equilibrium rents. With these results we have extended to multiproduct markets with indivisibilities those of Laussel and Lebreton (2001), who investigated the payoff structure of intrinsic common agency games.

An interesting extension of our analysis is to consider both multiple principals and agents. Our intuition is that a new kind of extended contracts, *non-anonymous* and mixed bundling contracts, is needed to ensure the equilibrium existence. This is left for future research.

8 Appendix.

Proof of Proposition 4*:* The proof of i) closely follows the steps of the proof of Theorem 1. Let (S,p) be an SPE^*-outcome, then $(S,p^S) \in SPE^*$. We define $\pi_i = (p_i^S - c_i)(S_i)$, $i \in N$ and $\pi^b = (v-c)(S) - \sum_{i\in N} \pi_i$. First let us prove that $(\pi^b, (\pi_i)_{i\in N}) \in core(v-c)$. By the definition of p^S, if $j \notin F(S)$, then $\pi_j = (p_j^S - c_j)(S_j) = 0$, which implies that $\pi^b = (v-c)(S) - \sum_{i\in F(S)} \pi_i$.

Since $V(N) = (v-c)(S)$, we have that $\pi^b + \sum_{i\in N} \pi_i = V(N)$. Given $T \subseteq \Omega$ by BC

$$v(S) - \sum_{i\in F(S)} p_i^S(S_i) \geq v(T) - \sum_{i\in F(T)} p_i^S(T_i),$$

which implies that

$$(v-c)(S) - \sum_{i\in F(S)} (p_i^S - c)(S_i) \geq (v-c)(T) - \sum_{i\in F(T)} (p_i^S - c)(T_i)$$

and hence,

$$\pi^b + \sum_{i\in F(T)} (p_i^S - c)(S_i) \geq (v-c)(T)$$

and $(\pi^b, (\pi_i)_{i\in N}) \in core(v-c)$.

Now let us prove that $(\pi_i)_{i\in N}$ belongs to Π^{PF}. Suppose, on the contrary, that $(\pi_i)_{i\in N}$ is Pareto dominated by an element $(\pi'_i) \in \Pi^{PF}$, i.e. $\pi'_j \geq \pi_j$, $j \in N$ and j_0 is such that $\pi'_{j_0} > \pi_{j_0}$. W.l.o.g. we can assume that π' is in Π^{PF}. Recall $S \in \arg\max_{T\subseteq\Omega}\{(v-c)(T)$, then by Lemma 1 we have that $j_0 \in F(S)$, otherwise $\pi'_{j_0} = 0 > \pi_{j_o}$ which is a contradiction. Let p' be defined as $p'_i(T_i) = \pi'_i + c_i(T_i)$, for all $i \in N, T_i \subseteq \Omega_i$, so that $p'_{j_0}(S_{j_0}) = \pi'_{j_0} + c_{j_0}(S_{j_0}) > \pi_{j_0} + c_{j_0}(S_{j_0}) = p_{j_0}(S_{j_0})$. By i), already proven, we have that (S, p') is an SPE^*-outcome, then firm j_0 has incentives to raise its equilibrium price vector up to p, which contradicts that (S, p) is a SPE^*-outcome. This implies that $(\pi_i)_{i\in N}$ belongs to Π^{PF}, as claimed. ■

Proof of Proposition 5: Assume w.l.o.g. that unit costs are zero. Given that v is monotonic $v(\Omega) \geq v(S)$. Moreover, $p^*_i(T_i) \geq v(\Omega\backslash(\Omega_i\backslash T)) - v(\Omega\backslash\Omega_i)$. Thus, prices are all positive and $p^*_i(\Omega_i) = v(\Omega) - v(\Omega\backslash\Omega_i)$.

First we prove that $(\Omega, p^*) \in SPE^*$-outcome set. By the monotonicity of v, Corollary 1 and that $= p^*_i(T_i) = p^{*\Omega}_i(T_i)$ it suffices to prove that $(\Omega, p^*) \in SPE$-outcome set.

Step 1: Consumer surplus is non-negative. By ((*FS*)), $smc^*(\Omega) = v(\Omega) \geq \sum_{i\in N} smc^*(\Omega_i) = \sum_{i\in N} p^*_i(\Omega_i)$, thus $cs \geq 0$.

Step 2: Condition (BC) in Proposition 1 is verified. Given $S \subseteq \Omega$, by ((*FS*)), $smc^*(\Omega\backslash S) = v(\Omega) - v(S) \geq \sum_{i\in N} smc^*(\Omega_i\backslash S)$, but

$$\begin{aligned}\sum_{i\in N} smc^*(\Omega_i\backslash S) &= \sum_{i\in N}[v(\Omega) - v(\Omega\backslash(\Omega_i\backslash S))] \\ &= \sum_{i\in N}[v(\Omega) - v(\Omega\backslash\Omega_i) + v(\Omega\backslash\Omega_i) - v(\Omega\backslash(\Omega_i\backslash S))] \geq \sum_{i\in N}[p^*_i(\Omega_i) - p^*_i(S_i),\end{aligned}$$

then $v(\Omega) - \sum_{i\in N} p^*_i(\Omega_i) \geq v(S) - \sum_{i\in F(S)} p^*_i(S_i)$.

Step 3: Condition (FC1) in Proposition 1 is verified. Given $j \in N$, by definition $p^*(\Omega_j) = v(\Omega) - v(\Omega\backslash\Omega_j)$, then $v(\Omega) - p^*(\Omega_j) = v(\Omega\backslash\Omega_j)$ which implies that $(\Omega) - \sum_{i\in N} p^*(\Omega_i) = v(\Omega\backslash\Omega_j) - \sum_{i\in N\backslash j} p^*(\Omega_i)$. Thus, $S^j = \Omega\backslash\Omega_j$ is the one which verifies (FC1) for all $j \in N$.

Step 4: Condition (FC2) in Proposition 1 is verified. This condition holds trivially since $p^*(S_i) = p^*(\Omega_i)$ for all $i \in N, S_i \subseteq \Omega_i$.

Condition (FC3) in Proposition 1 does not apply given that $N = F(\Omega)$.

Thus $(\Omega, p^*) \in SPE^*$-outcome set.

Now, consider S^* such that $v(S^*) = v(\Omega)$. Then by (*FS*),

$$0 = v(\Omega) - v(S^*) \geq \sum_{i\in N} p^*_i(\Omega_i) - p^*_i(S^*_i) \geq 0.$$

Thus, $p^*_i(\Omega_i) = p^*_i(S^*_i)$ for all $i \in N$. Moreover, for all $i \in N\backslash F(S^*)$ and all $T_i \subseteq \Omega_i$, $p^*_i(T_i) = 0$, given that

$$0 \leq v(\Omega\backslash(\Omega_i\backslash T_i)) - v(\Omega\backslash\Omega_i) \leq p^*_i(T_i) \leq v(\Omega) - v(\Omega\backslash\Omega_i) \leq v(\Omega) - v(S^*) = 0.$$

Then $(\Omega, p^*) \in SPE^*$-outcome set, where $p_i^*(S_i) = p_i^*(\Omega_i$ for all $i \in N, S_i \subseteq \Omega_i$. ■

Proof of Proposition 6: Let Σ be the set of permutations (orderings) of $N = \{1, 2, ..., n\}$ and let $\sigma \in \Sigma$ be any of its elements. Let P_i^σ be the set of firms which precede firm i with respect to permutation σ, i.e. for all $i \in N$ and $\sigma \in \Sigma$, $P_i^\sigma = \{j \in N | \sigma(j) < \sigma(i)\}$

Define, following Shapley (1971), the marginal contribution vector $x^\sigma(v - c) \in R^n$ of $(v - c)$ with respect to ordering σ by, $x_i^\sigma(v - c) = V(P_i^\sigma + i) - V(P_i^\sigma)$, for all $i \in N$. If $(v - c)$ is convex, then the marginal contribution vector $x^\sigma(v - c)$ will be positive. The equality between the sets $conv\{x^\sigma(v - c) | \sigma \in \Sigma\}$ and $core(v - c)$ is given in Driessen (1993), but this equality implies that $\pi^b = 0$ and hence that $core(v - c) = \Pi^{PF}$. Thus, by Proposition 4 consumer surplus is zero and the equilibrium prices are the $core(v - c)$. ■

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