THE EM ALGORITHM FOR IMAGING WITH GAPS IN THE DETECTOR PLANE

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Abstract. γ-ray telescopes based on coded-mask techniques does not give a direct image of the sky sources but its correlation with the mask response function. To obtain an image of the sky sources one has to treat mathematically the detected image, usually by correlating it with a reconstruction array (correlation methods). Unfortunately this kind of methods, although quite fast, have some limitations in the reconstructed image and in handling the real telescope response. To improve these results, several reconstruction methods have been developed and used. In the case of LEGRI (a γ-ray coded-mask telescope developed for the University of Valencia) we have used the EM algorithm, an iterative maximum likelihood algorithm with very good response and with good handling of the telescope response.

1. Introduction

One of the problems with γ-ray astronomy is the formation of images, due to the fact that this so energetic electromagnetic radiation is very difficult to focus in a plane because it easily passes through the matter; therefore the typical lenses and mirrors are useless with it and we cannot use classical telescopes. One of the solutions for γ-ray imaging is the use of a coded mask: a pattern of holes and opaque elements which is placed in front of a position-sensitive detector. It is in fact equivalent to a pin hole camera, but with more than one hole.

The main inconvenient with the coded-mask telescope is that we do not get a direct image of our sky sources, as in the case of using lenses (or a pin hole camera) but the correlation of the source with the mask. To reconstruct the original image we have to process mathematically the recorded data.

The usual reconstruction methods applied to coded mask telescopes are based on a correlation of the detector plane with a reconstruction array. These kind of methods are very popular due to its ease and speed, and usually are preferred when the coded mask telescope is very complex and has many components, but have problems in handling the data in difficult situations, as when the telescope moves for instance.

Nevertheless, a second class of reconstruction methods with a better data handling have been recently developed. They are based on the maximization of a certain

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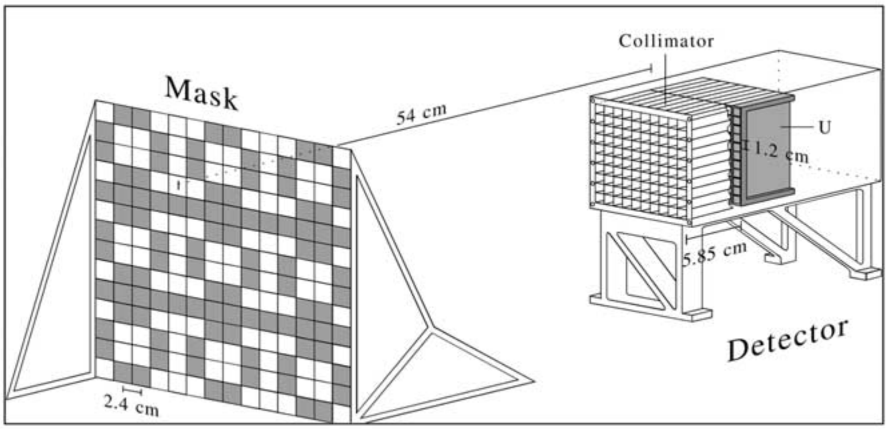

Figure 1. LEGRI system, showing the detector plane, the collimator and the mask pattern.

magnitude under the constraint of compatibility with the detected data and they are very popular due to its good results and flexibility, in spite of its slowness.

One of these methods is the EM algorithm (Lange and Carson, 1984), used by Lange and Carson initially for emission tomography. In this method the maximized magnitude is the likelihood. We have used this method in our instrument, the LEGRI γ-ray coded-mask telescope, and it showed a very good handling of the real telescope response (for example, movement of the telescope, unworkable detectors . . .), giving therefore a very good capability for image reconstruction. Although it is slower, it is suitable for not very complex telescopes as the case of LEGRI.

2. The LEGRI Telescope

LEGRI (Low Energy Gamma Ray Imager) is a soft γ-ray telescoped that uses the technology of coded masks. LEGRI is nowadays on board the spanish satellite Minisat-01, launched on April 1997 in a Low Earth Orbit (LEO) with a height of 550 km which passes across the South Atlantic anomaly. This fact induces great amounts of activation in the LEGRI structure material producing a strong background noise that dominates over the sky data.

LEGRI is made up (see Figure 1) basically by a 10×10 position-sensitive detector placed in front of a coded mask. The main characteristics of LEGRI are:

- Distance between mask elements centres: 2.4 cm
- Mask pattern: a 5×5 MURA (Gottesman and Fenimore, 1989) (see Figure 2) placed in a 2.8×2.8 mosaic (14×14 mask elements = 33.6×33.6 cm)
- Distance detector plane – mask: 54 cm
- Detector units: 80 HgI_2 and 20 Cd(Zn)Te room temperature solid state detectors.
- Energy range: 20–100 keV

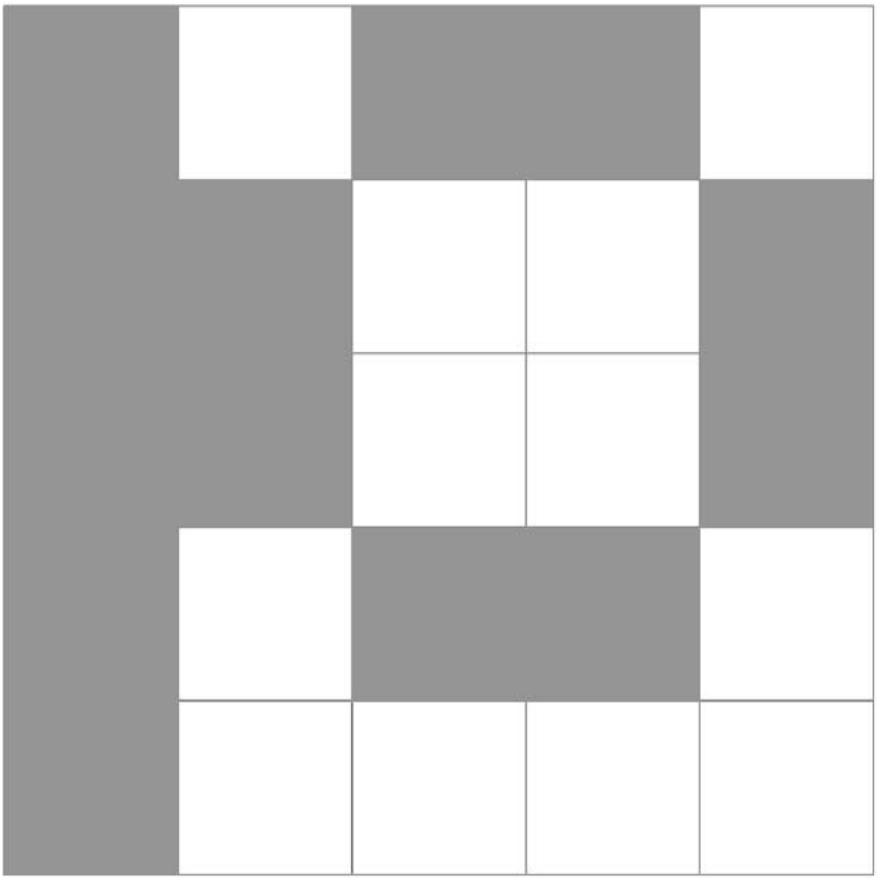

Figure 2. 5 × 5 MURA.

- Common electronics and thresholds for each row of 10 detectors
- Distance between detector unit centres: 1.2 cm (therefore 2 × 2 detector units have a size equivalent to a mask element). Detector plane size = 12 × 12 cm
- Collimator height: 5.85 cm (it limits the field of view of each detector to the mask)
- Field of view: ±10.5°
- Angular resolution: 1.27° (compare it with the Moon that has an angular size of ∼ 0.5°)

Unfortunately, due to problems in the launching and the effect of the strong radiation environment where LEGRI is orbiting, LEGRI has a severe damage in the detector plane; about the 80% has become useless. In fact, we have only 17 operative detectors (which are Cd(Zn)Te). As a result of this effect and the strong (dominating) background noise, LEGRI bas a low sensitivity and only the strongest sky sources can be seen, as the Crab nebula.

3. Movement and Pointing

One of the effects that should be well known and controlled in a telescope is its movement and pointing. When a telescope is on board a flying satellite, as our case, one needs to know if it is really pointing to the zone of the sky that one wants to study or not, and also has to be sure if it does not move from this region (or unless if it moves in a known way).

In fact, changing the pointing is often sought, and it is used as a way to get more information from the sky sources, improving therefore the quantity of different data we collect: in each different pointing, the telescope will look at the source from different angles. This implies that during each different pointing, the incoming radiation will be modulated by different mask elements, impinging on different detectors. This gives us more information than with a single pointing.

In our case this procedure is essential, as we can only count with 17 detector units instead of 100! Otherwise it would be impossible to get enough information to reconstruct any image.

To determine the pointing and movement, LEGRI includes a star sensor: it consist on a optical camera with a CCD, which pointing direction is parallel to the LEGRI's pointing direction. It takes each two seconds a picture of the sky and the software compares each one with a star catalogue in order to obtain the pointing direction and the rotation angle respect to the sky parallels (the *roll*). This gives a very good angular resolution of $\sim 1'$. Unfortunately the CCD of the star sensor saturates easily if a strong light source (as the Sun or the Earth or the Moon) illuminates it, giving corrupt data in that cases.

Fortunately we have a second way to control the pointing using data from the platform: it has a magnetic attitude control which calculates the pointing direction of the satellite comparing the local magnetic fields with the theoretical Earth's magnetic field. It has worse angular resolution and a bigger error pointing but on the other hand it gives data constantly, without lacks. We can choose between both methods to control the changes in the telescope pointing.

4. Mathematical Description

4.1. Detection and data integration

We can describe mathematically the detection process by the following equation:

$$D_{klp} = \left(\sum_{\alpha\beta} O_{\alpha\beta} \Phi^{\alpha\beta}_{klp} + B_{kl} \right) T_p \tag{1}$$

where:
kl = detector unit indexes

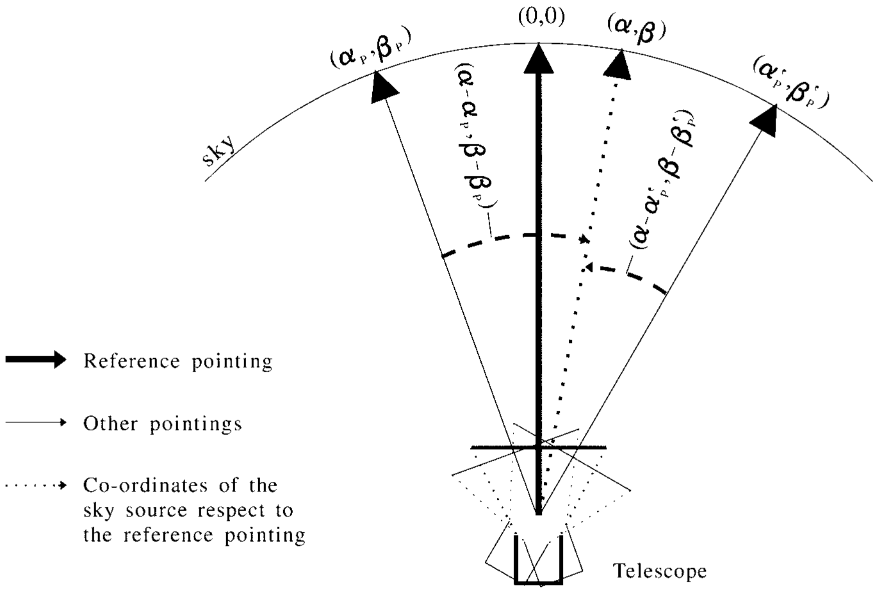

Figure 3. Different pointings at the same sky region. One of the is chosen as master reference system (MRS).

p = pointing direction index. The data from the detector plane are integrated in different sets for each different pointing direction

D_{kl} = detected counts in detector kl during pointing p

$\alpha\beta$ = sky pixel indexes

$O_{\alpha\beta}$ = emission intensities from the sky pixels per time unit and area unit

$\Phi_{klp}^{\alpha\beta}$ = sky flux es. It stands for how much sees detector kl coming from the sky pixel $\alpha\beta$ during pointing p ($\Phi = 1$ detects all the signal, $= 0$ does not detect anything; usually something between 1 and 0)

B_{kl} = background noise at detector kl in counts per detector and time unit,

T_p = integration time for pointing p

The function Φ takes into account all the effects that can affect to the radiation detection process as different efficiencies for each detector, different transparencies of the mask, strongbacks and other passive structures that can affect to the passage of radiation (collimators, shieldings ...). It takes also account of the changes of pointing. Fortunately this is quite easy to carry out just by considering a pointing (see Figure 3) as Master Reference System (MRS), relating all the other angular coordinates to this one. If $(\alpha_p \beta_p)$ are the coordinates of the pointing p, then:

$$\Phi_{klp}^{\alpha\beta} = \Phi_{kl}^{(\alpha-\alpha_p)(\beta-\beta_p)} \tag{2}$$

if we do not consider rotations among the different pointings.

If we do it, then Equation 2 turns in:

$$\Phi_{klp}^{\alpha\beta} = \Phi_{kl}^{(\alpha-\alpha_p)\cos(roll_p)+(\beta-\beta_p)\sin(roll_p)\ -\ (\alpha-\alpha_p)\sin(roll_p)+(\beta-\beta_p)\cos(roll_p)}. \tag{3}$$

4.2. The EM algorithm

Reconstructing an image consist in passing from our detected signal (see Equation 1) D_{kl} to our unknown sky intensities $O_{\alpha\beta}$. That is, in some sense it consist in invert Equation 1 (although the real inverse does not exist as the sky can be subdivided so finely as one desires; therefore we have more unknowns that equations and there exist infinite possible solutions). The reconstruction method we have used in this work is the EM algorithm ('*E*xpected value' and '*M*aximization') (Dempster *et al.*, 1977). It is an iterative by construction algorithm for computing maximum likelihood estimators from incomplete and noisy data. This image reconstruction technique has been successfully used in nuclear medicine (Lange and Carson, 1984), being applied for us to γ-ray astronomy for first time.

The philosophy of the method is the following: let us suppose that the data observer in an experiment is a vector D, with an associated conditional probability function $g(D|O)$, where O is a set of unknown parameters to be estimated (the sky pixels in our case); that is, g stands for the probability of obtaining the data D given the parameters O. Our aim is to find the set of parameters $O^{\max}$ than maximizes $g(D|O)$, which will our the best estimator for the real value of the parameters O.

The EM algorithm carries out this maximization via an iterative and indirect way: in general it is rather difticult to maximize $g(D|O)$ with respect to O. So we define a bigger (and fictitious) data space D_t where D will be a subset of D_t. That is, there is a (non-univocal) mapping $D_t \to D$ such that $D = s(D_t)$. We postulate for D_t also a conditioned probability function $f(D_t|O)$. Under these assumptions it is possible to obtain again $g(D|O)$ from $f(D_t|O)$ by means of the relation:

$$g(D|O) = \int_{D=s(D_t)} f(D_t|O)dD_t \tag{4}$$

where the integral is approximated to a discrete sum when we work with discrete variables. Now the two steps of the iterative EM algorithm at iteration n are:

– E step: form the conditioned expected value $E(\log f(D_t|O)|D, \tilde{O}^n)$ where $\tilde{O}^n$ is our current estimation of the parameters O.

– M step: maximize this expected value with respect to O, keeping $\tilde{O}^n$ constant. This gives us a new estimation $\tilde{O}^{n+1}$.

This method has the property that $g(D|\tilde{O}^{n+1}) \geq g(D|\tilde{O}^n)$ and therefore converges to the maximum likelihood estimator. Using this method and following steps analogous to Lange and Carson (1984) we obtain the final form of the EM algorithm for our case:

$$\tilde{O}^{n+1}_{\alpha\beta} = \tilde{O}^n_{\alpha\beta} \frac{\sum_{klp} \Phi^{\alpha\beta}_{klp} \left(\frac{D_{klp}}{\tilde{D}^n_{klp}} \right)}{\sum_{klp} \Phi^{\alpha\beta}_{klp}}. \tag{5}$$

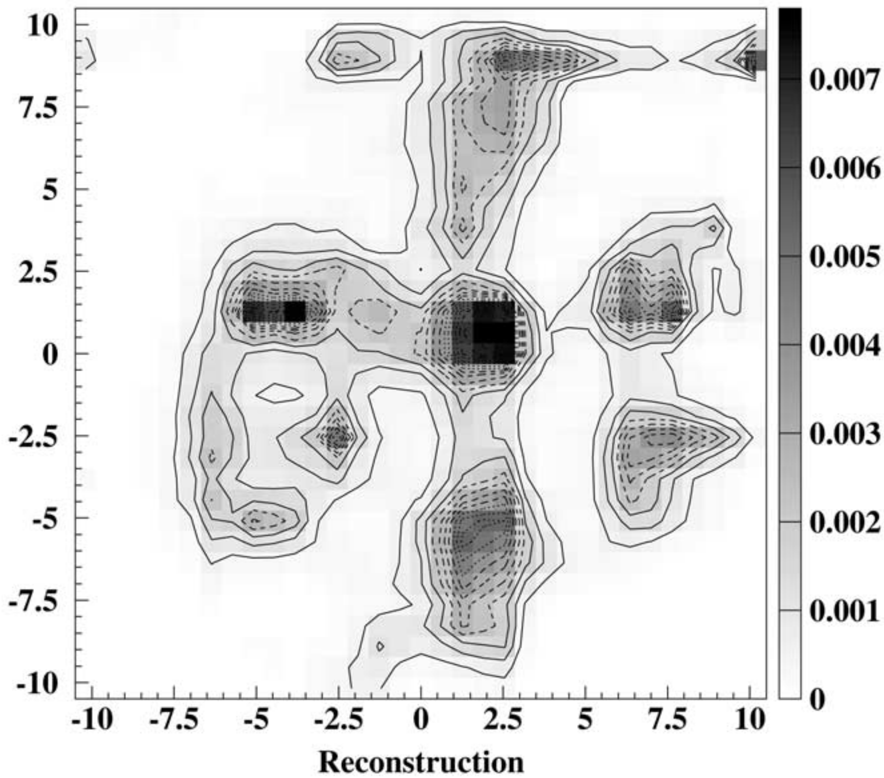

Figure 4. Reconstruction of the Crab Nebula from LEGRI real data.

But as we have several unworkable detectors we must avoid to make the sum over them, as for the broken detectors it will gives a 0/0 in the parenthesis. Therefore we will limit the sum over every kl to a sum only over the useful detectors.

5. Results

As we have already said, we have a high background level due to the South Atlantic anomaly activation and also several unworkable detectors. This limits us to study only the strongest sources, as is the case of the Crab Nebula:

The intensity we detect with the telescope LEGRI coming from the Crab Nebula has a signal value of about $S \sim 0.4$ counts s^{-1} cm^2, meanwhile the mean value of the background noise is of the order of $N \sim 0.6$ counts s^{-1} cm^2. This gives a significance level in ~ 40000 observation seconds of

$$f = \frac{S}{\sqrt{S+N}} \sim 80$$

bigger enough to get an image. The image we get can be seen in Figure 4, where we show the result of the EM algorithm applied to our experimental data. We can see at the central part of the figure the real signal from the Crab Nebula, and surrounding it, six ghosts or fake sources due to the cyclic mask pattern and the uncertainties produced for the damaged detector plane. Therefore, even with so severe limitations in our instrument (high background and only 17 detectors working), using the movement of the satellite we can compense this lack of information and perform imaging using the maximum likelihood method called EM algorithm.

6. Conclusions

We have applied the EM algorithm, an iterative method of maximum likelihood to the spanish LEGRI γ-ray coded-mask telescope on board Minisat-01 for image reconstruction, showing a very good adaptability to difticult conditions (i.e. high background level and an important damage in the detector plane) and handling easily the telescope movement which we have used as a way to compensate the lack of information from the source. Nevertheless, the difficulties in our instrument gives a poor sensitivity and we can only perform imaging of the strongest sky sources, as the Crab Nebula image showed in this paper.

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