IMAGING IN X-RAY WITH CODED-APERTURE MASKS

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ABSTRACT

Imaging reconstruction methods for coded mask telescopes devoted to studying celestial X and γ ray sources, normally are based on correlation methods, using Fast Fourier Transforms to increase the computational speed. In complex telescopes using large arrays these methods are more suitable because they allow the images to be reconstructed quickly. In this paper we present alternative reconstruction methods (maximization methods) that can be used for not very complex coded mask systems. In the particular case of the LEGRI (Low Energy Gamma Ray Imager) which consists of a 10x10 pixeled detector plane together with a 14x14 coded mask (5x5 MURA basic pattern), we have found that the E-M algorithm (a maximum likelihood method) gives the best results, even when detector unit failures happen.

Keywords: LEGRI; X ray astronomy; coded mask telescopes; maximum entropy methods; E-M algorithm.

1. INTRODUCTION

When trying to obtain images from X or γ ray sources, it is not possible to use lenses or mirrors to focus an image, because of the very energetic nature of this radiation; the photons just pass through them. Therefore, we have to use another way to form images. One way is to use a coded mask (see Skinner 1988), which is a pattern of holes and opaque elements placed in front of a position sensitive detector. The image recorded is not a direct image of the source, as in the case of using lenses, but the correlation of the source with the mask. To obtain the original image, we have to process the recorded image. The usual way to do this is by correlating the recorded image with the mask or a modification of it. This method gives good results and is quite fast. Nevertheless, better results can be obtained with maximization methods, as maximum entropy or maximum likelihood for some particular applications. Such methods need a longer processing time, and for very complex telescopes they can be impracticable. However, for less complex telescopes, as the case of LEGRI, they are more suitable.

2. DETECTION AND RECONSTRUCTION

LEGRI is a coded mask telescope with a pixel detector (Reglero et al. 1996). The basic mask pattern is a 5x5 MURA (Gottesman & Fenimore 1989) placed in a mosaic of 14x14 elements, 2.4x2.4 cm each. The total size of the mask is 33.6x33.6 cm^2 (see figure 1).

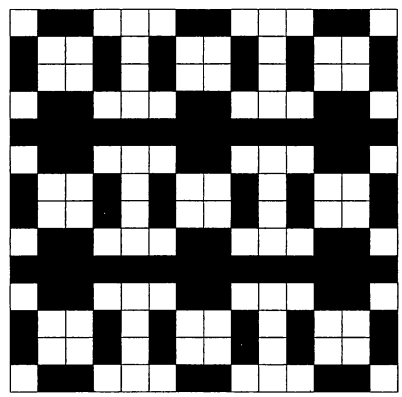

Figure 1: LEGRI mask pattern.

The detection of a source by our telescope can be described (Fenimore & Cannon 1978) by a correlation (*) of the source with the mask,

$$D = O * M + B \tag{1}$$

where D is an array representing the detector plane, O stands for the sky sources, M is the mask pattern (with a value of 1 for each hole in the mask and 0 for each opaque element), and B is a noise term. This can be written in a more explicit way as:

$$D_{kl} = \sum_{ij} O_{ij} M_{i+k\, j+l} + B_{kl} \tag{2}$$

which is the formal expression of the correlation.

In eq. 2 only the effect of the mask in the signal modulation is considered. More complete is the following equation:

$$D_{kl} = \sum_{ij} O_{ij} \Phi^{ij}_{kl} + B_{kl} \tag{3}$$

where Φ is a function that gives the fraction of the flux coming from the sky pixel *ij* seen by the detector *kl*. Φ includes all the factors that affects the signal, such as collimators, transparency, detector efficiency, etc...

Proceedings 2nd INTEGRAL Workshop 'The Transparent Universe', St. Malo, France, 16-20 September 1996, ESA SP-382 (March 1997)

2.1. Correlation Methods

Correlation methods for image reconstruction are based in eq. 2, and so one seeks an array G such that correlating it with the detected image D, will recover the original source O, or a good estimation of it ($\tilde{O}$):

$$D * G = \tilde{O}$$
$$D * G = O * M * G + B * G \qquad (4)$$

What we need then is that M*G be a delta function and B*G be as close as possible to 0. Both conditions cannot be totally fulfiled (Skinner & Ponman 1994), but it is possible a good compromise choosing a G array with a value of 1 for each hole in the mask and -1 for each opaque element (excepting the *(0,0)* element in the case of the MURA's that will be 1). This is called balanced correlation method.

When the detector plane is pixellated in elements smaller than the mask elements (as in LEGRI case), one can use some modifications of balanced correlation (see Fenimore & Cannon 1981), called *finely sampled balanced correlation* (FSBC) and *delta decoding* (δ-D) methods. In both, the G array is subdivided into smaller elements in order to have the same size as the detector elements. In the LEGRI case, where the mask element size correspond to 2x2 detector elements size, each G element is subdivided in 2x2 elements. This operation gives a more accurate location of the sky sources. In the case of FSBC, each 1 (-1) value of the G array is replaced by four (2x2) 1 (-1) values. In δ-D, only one of the subdivisions has the value 1 (-1); the other are 0. This procedure increases the contrast, but the reconstructed source position is shifted in a known way.

The angular resolution in both methods is given by the angle that a mask element subtends from the detector plane. Therefore, the separation power is defined by the mask and its distance to the detector plane.

2.2 Maximization Methods

These methods are based in eq. 3, where a more accurate description of the detection process is considered. However, they are slower and, depending on the complexity of the telescope, they could be totally useless.

The maximum entropy method has been succesfully used in astronomy, and it can be applied to any detection process formally described by the following expresion:

$$D_k = R_k(f) + B_k \qquad (5)$$

where D_k are the recorded data, B_k is the noise term, $\{f_i\}$ are the sources to be estimated (the sky map), and $R_k(f)$ is the response function of the detector; formally, eq. 5 is equivalent to eq. 3. The maximum entropy method looks for the most uniform sky map consistent with the data. This is obtained by defining an entropy function and maximising it, with restrictions imposed by the experimental data. There are some different entropy definitions. We use the definition given by Gull & Daniell (1978):

$$S = -\sum_{ij} O_{ij} \cdot \log O_{ij} \qquad (6)$$

To maximise it, with the restriction of consistency with the experimental data, we define the function Q, as

$$Q = -\sum_{ij} O_{ij} \log O_{ij} - \lambda \sum_{kl} \frac{\left(\tilde{D}_{kl} - D_{kl}\right)^2}{\sigma^2_{kl}} \qquad (7)$$

where the second term is the χ^2 function of the data (λ is a Lagrange multiplier). Maximizing Q respect to O_{ij} we obtain:

$$\tilde{O}_{ij} = e^{-1-2\lambda\sum_{kl}\Phi^{ij}_{kl}\frac{(\tilde{D}_{kl}-D_{kl})}{\sigma^2_{kl}}} \qquad (8)$$

This is a transcendental equation, because $\tilde{O}$ appears in both sides of the equation ($\tilde{D}$ is the *estimation* of the experimental data for the calculated $\tilde{O}$) and has to be solved by numerical methods. The direct iteration of eq. 8 does not converge to any solution, and one must promediate the solution of each iteration with the solution of previous iteration (a sort of "memory" of the process) to obtain convergence. Moreover, in each iteration λ must be selected such that the calculated value of χ^2 be close to the number of experimental data points.

The other maximization method we have considered is the E-M algorithm. It is an algorithm for computing maximum likelihood estimators iteratively. A good description can be seen in Dempster et al. (1977). The philosophy of the method is the following: let us suppose the observed data is a random vector D with an associated conditioned probability function g(D|O), where O are unknown parameters to be estimated (the sky pixels in our case). We look for the parameter set O^{max} that maximize g(D|O). In general it is rather difficult to maximize g respect to O. For that, we can define a larger space of theoretical data (D^t) where the optimization will be easier to achieve. We can only estimate the data of this theoretical space D^t indirectly by means of the real data D. Let us assume there is a mapping $D^t \rightarrow s(D^t)$ and the only D^t that we can know are those determined by $D=s(D^t)$. We postulate for the complete data also a conditioned probability function $f(D^t|O)$. We can recover g from f by:

$$g(D\,|\,O) = \int_{D'=s(D)} f(D'\,|\,O)\, dD' \qquad (9)$$

Each iteration of the E-M algorithm consist on two steps:

•E-step: To form the conditioned **E**xpected value

$$E(\log f(D'\,|\,O) \mid D, O^n) \qquad (10)$$

where O^n is the present estimation of the parameters.

•M-step: To **M**aximize this expected value with respect to O, keeping O^n constant. This give us a new array of parameters O^{n+1}.

The idea of the method is to obtain the O parameters that maximize log $f(D^t|O)$. As we *don't know* log $f(D^t|O)$, we maximize its expected value in the present iteration, given the *known* D data, and the present estimation of the parameters O^n. Let us consider the function:

$$H(O\,|\,O^n) = E(\log f(D'\,|\,O) \mid D, O^n) - \log g(D\,|\,O) \qquad (11)$$

This function has the characteristic (Dempster et al 1977):

$$H(O'\,|\,O) \leq H(O\,|\,O) \qquad (12)$$

and so,

$$\begin{aligned} &\log g(D\,|\,O^{n+1}) - \log g(D\,|\,O^n) = \\ &[E(\log f(D'\,|\,O^{n+1}) \mid D, O^n) - \\ &E(\log f(D'\,|\,O^n) \mid D, O^n)] \\ &+ [H(O^n\,|\,O^n) - H(O^{n+1}\,|\,O^n)] \end{aligned} \qquad (13)$$

As O^{n+1} maximizes eq. 10 keeping O^n constant, and according to eq. 12, we obtain:

$$\log g(D|O^{n+1}) \geq \log g(D|O^n) \leftrightarrow g(D|O^{n+1}) \geq g(D|O^n) \qquad (14)$$

The E-M algorithm, therefore, is designed to improve the likelihood in each iteration.

3. RESULTS

We have applied the four methods described to above to LEGRI in a simple case: an ideal 100 ph cm^{-2} s^{-1} sky source in the centre of the field of view, and a random background with a mean value of ~30 counts s $^{-1}$ per detector element. In figure 2 is shown the sky source, and in figure 3 the detected image assuming an integration time of one second. In order to perform a more easily comparison between the different methods, the simulated source is assumed to be in the centre of the pixel, so that shadows of mask elements conveniently align with detector pixels.

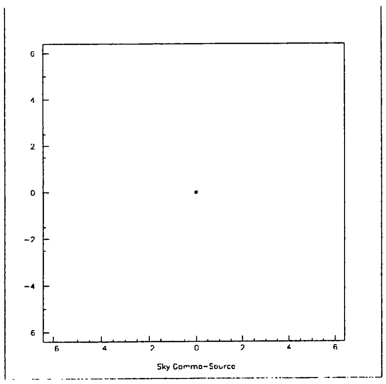

Figure 2: Sky gamma source.

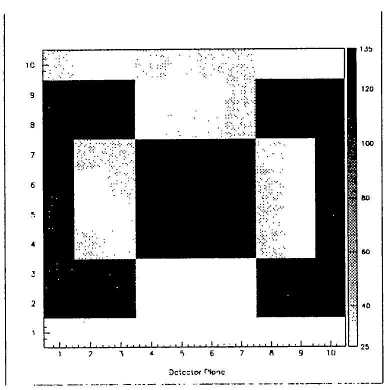

Figure 3: Detected image.

Figures 4, 5, 6 & 7 show, respectively, the reconstruction of the image by the methods of FSBC (0.2 sec of computing time), δ-D (0.2 sec.), maximum entropy (21 min, 100 iterations) and E-M algorithm (9 sec, 100 iterations). The reconstructions have been done at the LEGRI Science Operation Centre (S.O.C.) on a Sun Sparc 20 station.

As it can be seen in the figures, both FSBC and δ-D methods present the poorest quality contrast. Imaging performances are better in maximum entropy and E-M methods. A better peak reconstruction and lower noise is achieved with these methods.

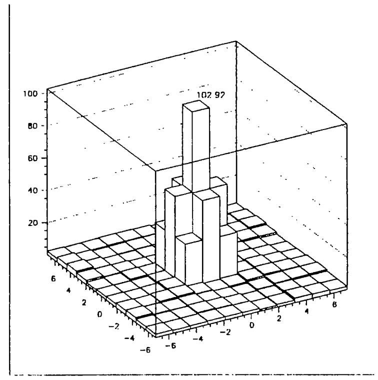

Figure 4: FSBC reconstruction.

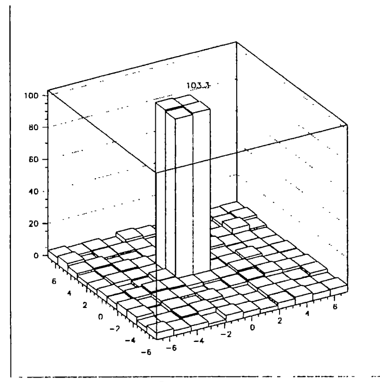

Figure 5: δ-D reconstruction.

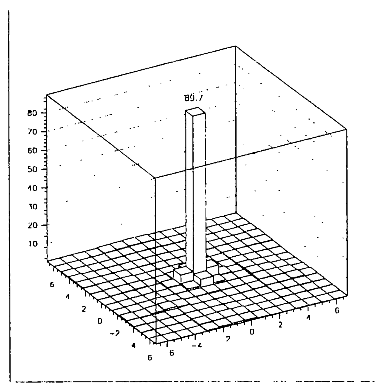

Figure 6: Maximum entropy reconstruction.

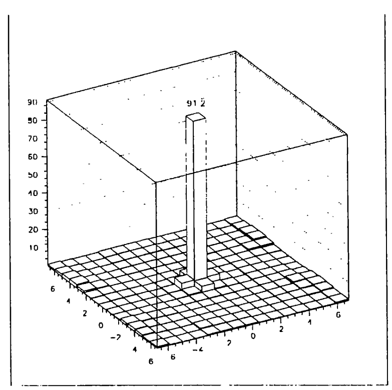

Figure 7: E-M reconstruction.

In order to compare the behaviour of the correlation and maximithation methods when a failure of some pixels happens, we have assumed the "loss" of a row of detectors and five more chosen at random (see fig. 8).

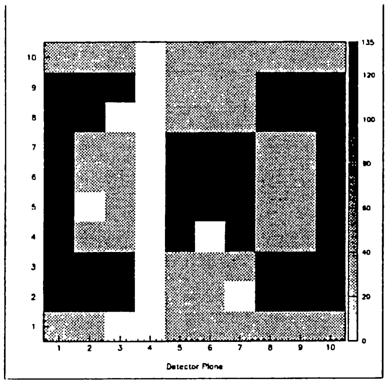

Figure 8: Detected image with 15 units disabled.

In the case of maximization methods, we have bypassed the damaged detector units (zero response). Concerning correlation methods, we have assumed a mean value (averaging the signal value of the surronding detectors) in the missing pixels. Figures 9, 10, 11 & 12 show, respectively, the source reconstructions for this case. In figure 9 & 10 we can clearly identify secondary "structures" that come up in the image for FSBC and δ-D methods. For maximum entropy and E-M algorithms the image is only a little worse than the previous one. without any secondary structure.

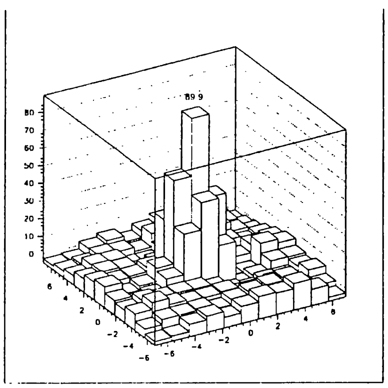

Figure 9: FSBC reconstruction of fig. 8 detector plane.

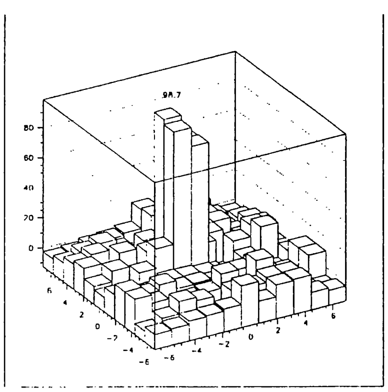

Figure 10: δ-D reconstruction of fig. 8 detector plane.

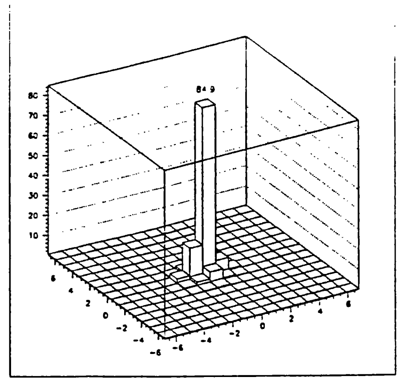

Figure 11: Maximum entropy reconstruction of fig. 8.

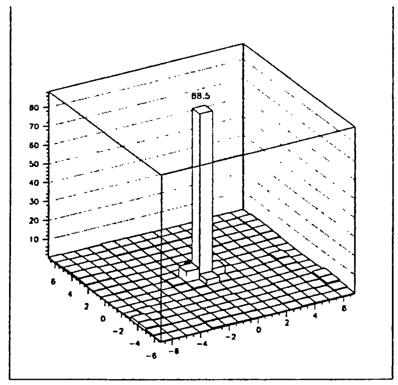

Figure 12: E-M reconstruction of fig. 8 detector plane.

5. CONCLUSIONS

The maximization methods, based on a more realistic description of the detection process, although more computer time consuming, give better quality images than the correlation methods for not very complex arrays. Their imaging capabilities are greater, and work better in adverse conditions, as when some detector units fail. Although the capabilities of the maximum entropy method and the E-M algorithm are similar, the first needs more computing time. Therefore, for LEGRI the use of the E-M algorithm for image reconstruction is expected to be more suitable.

REFERENCES

Dempster, A. P., Laird, N. M., Rubin, D. B. 1977, J. R. Statist. Soc. B 39, 1.

Fenimore, E. E., Cannon, T. M. 1978, Appl. Opt. 17, 337.

Fenimore, E. E., Cannon, T. M. 1981, Appl. Opt. 20, 1858.

Gottesman, S. R., Fenimore, E. E. 1989, Appl. Opt. 28, 4344.

Gull, S. F., Daniell, G. J. 1978, Nature 272, 686.

Reglero, V., et al. September 1996, International Conference on Small Satellites; Missions & Technology. Madrid.

Skinner, G. K. 1988, Scientific Amer. 256, 84.

Skinner, G. K., Ponman, T. J. 1994, Mon. Not. R. Astr. Soc. 267, 518.