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Revision of the Sachs-Wolfe effect

Joan Josep Ferrando, Juan Antonio Morales and Miquel Portilla
Departament de Física Teòrica, Universitat de València, 46100 Burjassot, Spain

The prediction of Sachs-Wolfe [1] is (excluding Doppler terms) the following: an observer pointing his detector in a direction $\vec{\nu}$ will measure a microwave background temperature given by

$$T(\vec{\nu}) = T_0(1 + \frac{1}{3}[\Phi(R\vec{\nu}) - \Phi(\vec{0})]) \tag{1}$$

where R is the radial comoving coordinate of the emitting point at the end of the recombination epoch. The scalar potential $\Phi(x^i)$ depends only on the spatial comoving coordinates and is a solution to the equation $\Delta\Phi = 6\delta(x^i)$. Inhomogeneities in the distribution of matter are then imprinted in the MWB temperature due to the inhomogeneities in the gravitational potential. Let us remember here the main steps in the derivation of this formula (1):

(i) After the recombination epoch, the space-time may be described as the growing mode of a $p = 0$ perturbation of an Einstein-de Sitter universe. Sachs-Wolfe [1] obtained this solution in comoving time-orthogonal coordinates $(\bar{\eta}, \bar{x}^i)$:

$$ds^2 = a^2(\bar{\eta})\{-d\bar{\eta}^2 + [(1 - \frac{10}{3}\Phi)\delta_{ij} - \frac{1}{3}\bar{\eta}^2 \frac{\partial^2\Phi}{\partial\bar{x}^i\partial\bar{x}^j}]d\bar{x}^i d\bar{x}^j\} \ , \quad a(\bar{\eta}) = \frac{2\bar{\eta}^2}{H_0} \tag{2}$$

where $\bar{\eta}$ runs from 0 to 1 at the present epoch, and Φ depends on the spatial coordinates only.

(ii) At some $\bar{\eta}_e$ in the above gauge the MWB as measured by observers moving with the matter was isothermal with temperature T_e independent of position.

The Liouville theorem for the radiation decoupled of the matter implies $T(\vec{\nu}) = T_e/(1+z)$. Computing the redshift z for a photon emitted by a source at rest with the matter on the last scattering surface one gets formula (1).

Hypothesis (ii) is gauge dependent; it may be true or not, but not necessarily the last scattering surface should be a surface of constant $\bar{\eta}$ time. The last scattering surface must be defined by physical arguments; this has not been done, and the Sachs-Wolfe prediction depends critically on the definition of that surface. This criticism can be found in the same Sachs-Wolfe paper [1] and has been elaborated recently by Stoeger et al. [2]. One could say that the result of this criticism is to expect a coefficient in equation (1) different from $\frac{1}{3}$, but in any case an effect proportional to Φ should exist. It is true that for a given observer (for example, the Earth) it is always possible to find a last scattering surface in order to neutralize the difference of redshift. For this election the MWB will be isotropic at the Earth, but most likely it will be anisotropic in general for other observers in the Cosmos; and if we want avoid to be at very special point in the Universe we must conclude that temperature anisotropies proportional to Φ should exist.

The scepticism about the Sachs-Wolfe prediction grows if one realizes that in the space-time (2) there exists a time-like vector field with the following property: if at some instant the observer represented by this vector field measures isotropic radiation, this one will remain isotropic in the future. This comes from a classic theorem by Ehlers et al. [3] [4] which states that this happens in and only in a conformally stationary space-time, and the observer measuring isotropic radiation is collinear with the conformal Killing vector. The metric (2) is not at first sight conformally stationary, but it can be proved [5] that introducing a new coordinate system (η, x^i) by the equations $\bar{\eta} = (1 + \frac{1}{3}\Phi)\eta$, $\bar{x}^i = x^i + \frac{1}{6}\eta^2\partial_i\Phi$, the metric (2) turns out to be:

$$ds^2 = a^2(\eta)\{-[1 + 2\Phi(x^i)]d\eta^2 + [1 - 2\Phi(x^i)]\delta_{ij}dx^i dx^j\} \tag{3}$$

which is obviously a conformally stationary metric. The matter is moving with velocity $V \propto d\Phi$ with respect to the observer measuring isotropic MWB temperature. Moreover, the temperature is given by $T(x^i) = b[1 - \Phi(x^i)]/a(\eta)$, with b a constant.

The main consequence of this is that we can have isotropic radiation in an inhomogeneous space-time like (3) without assuming some ad hoc form for the last scattering surface. So, if we change hypothesis (ii) by this one: "at the end of the recombination epoch, radiation was isotropic with respect to the observer $n = (-g_{00})^{-1/2}\partial_\eta$", one gets that MWB should be isotropic now [6].

But how understand that the observer measuring isotropic radiation at the recombination epoch is not at rest with matter? Before answer this question let us formulate the computation of MWB anisotropies in a more convenient way for metric forms (2), (3). Radiation decoupled from matter can be described by a distribution function $f(x, p)$ and, from Ehlers et al. results, isotropic blackbody radiation is given by

$$f_i = \frac{2h^{-3}}{exp[E/kT(x^i)] - 1}.$$

So, let us define the intrinsic fractional anisotropy $\xi(x, p)$ by the equation $f(x, p) = f_i(1 + \xi)$. Using the Liouville theorem for decoupled radiation, one get that ξ is constant along the null geodesics. Then all is reduced to compute ξ at the recombination age.

A simple model [7] for which we have got $\xi = 0$ is based in the following hypotheses:

(H1) After recombination the space-time is given by the Sachs-Wolfe metric (2).

(H2) Before recombination we have isotropic radiation in the reference frame at rest with matter.

(H3) We impose matching conditions to the metric on a space-like hypersurface (the last scattering surface).

In this model the velocity of matter with respect to the observer measuring isotropic radiation is discontinuous through the last scattering surface. That means that this velocity (that produces a dipolar anisotropy) has been originated during the process of recombination. This is just a model, but confirm us our scepticism about the Sachs-Wolfe prediction.

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