Taxation with unemployment and household production: a computational approach

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Abstract

This paper incorporates in a systematic way the design of the tax structure in a large-scale applied general equilibrium model with household production and equilibrium unemployment. In doing so this work ties two different strands of the economic literature: optimal taxation in the presence of self supply, on the one hand, and social accounting matrices for policy evaluation, on the other hand. The application illustrates the impact of a VAT reduction in Canada whithin a household production model in the presence of search unemployment.

1 Introduction

The motivation underlying this paper is to present a general framework to analize tax reforms using simulation by means of computable general equilibrium (CGE) models that include household production and unemployment. CGE models have been fruitfully used in the analysis of the implications of fiscal policies over the past decades, but it was not until very recently that household production was incorporated into this framework to throw light on very particular issues. However all the previous references fail to consider unemployment despite the fact that the relationship of this variable with different tax strategies is a matter of concern for policy makers. In Europe, for instance, in 1999 the Commission allowed those Member States who so desired to experiment with reduced VAT rates on labor intensive services with the objectives of increasing employment and reducing the black economy. Therefore the consideration of unemployment together with household production in CGE model can help to evaluate the extension of different tax experiments. Recently Balistreri (2002) has shown the way to fit an equilibrium unemployment model in a CGE model. In this paper we incorporate unemployment in a household production model and derive some results for Canada extending the household production model of Iorweth and Whalley (2002) to consider the role of unemployment. As a central result is shown that significant differences in welfare gains profiles arise depending on the parameters of the matching function.

2 Household production and the competitive equilibrium

Consider a very simple economy with two sectors (C) and (S), one labor factor (N) and one representtive consumer with a utility function V. A general equilibrium in this economy can be formulated as a primal problem of a nonlinear program (NLP) in quantities in the following way:

$$\max\ V = V(C, S)$$

$$s.t \qquad C = C(N_c) \tag{1}$$

$$S = S(N_s) \tag{2}$$

$$\overline{N} = N_c + N_s \tag{3}$$

$$I = w\overline{N} = p_c C + p_s S \tag{4}$$

being p_c and p_s the price of C and S respectively, w the wage rate, N_c and N_s the labor employed for production of C and S, and I the income available for consumption.The nonlinear program approach breaks down quickly as the model becomes more complicated. For example, imagine two different consumers with different preferences and different endowments. What would be in this case the objective function?. We can think in maximizing one consumer utility subject to an arbitrary fixed level of utility for the other consumer (exploiting the firt theorem of the welfare economics) but in this case is not guaranteed that each consumer expenditure level implied by the solution is equal to the consumer's income. Thus the solution would generally be inconsistent with the general equilibrium.

A more flexible approach is to formulate the problem as a system of equations using a dual representaton of the problem. This approach follows the Mathiesen's representation (Mathiesen, 1985) of an Arrow-Debreu general equilibrium model, and has the advantage of being fully compatibe with a consistent benchmark equilibrium dataset, frecuently called social accounting matrix. Calibration of the model heavily relies in this dataset that can be obtained using very different sources of information but should be consistent at an aggregate level with national accounts. In Mathiesen's formulation, a competitive equilibrium can be represented by means of three different set of ecuations: (a) zero profit conditions; (b) market clearence conditions; and (c) income balance. Complementarity slackness is a feature of the equilibrium allocation (see Rutherford, 1999) and so the problem in hand can be though as one of the class of mixed complementarity problem - MCP-(Rutherford, 1995). The three basic set of variables in the problem are: (a) the prices, (b) the activity levels and (c) the income of the agents involved.

As a starting point cost functions are obtained from the underlying cost minimization problem so individual optimizing behavior is embedded in the model. Then the Shephard's lemma is used to obtain the demands for each commodity. The Shephard's lemma states that the partial derivatives of cost function are quanties. Thus the MCP equivalent to the previous NLP can be written as:

$$cc(w) \geq p_c \qquad \perp C \tag{5}$$

$$cs\left(w\right) \geq p_s \qquad \perp S \tag{6}$$

$$cv\left(p_c, p_s\right) \geq p_v \qquad \perp V \tag{7}$$

$$C \geq cv_{p_c}\left(p_c, p_s\right) V \qquad \perp p_c \tag{8}$$

$$S \geq cv_{p_s}\left(p_c, p_s\right) V \qquad \perp p_s \tag{9}$$

$$V \geq \frac{I}{p_v} \qquad \perp p_v \tag{10}$$

$$\overline{N} \geq cc_w(w)\,C + cs_w(w)\,S \qquad \perp w \tag{11}$$

$$I = w\overline{N} \tag{12}$$

where $cc(w)$ is the cost function for the C sector and $cs(w)$ is the cost function for the S sector. In this approach is convenient to add a new sector that uses the commodities C and S as the inputs and produce utility. This sector stands for the utility function and the cost function associated with it (cv) can be interpreted as the expenditure function. Equations 5 to 7 above represent zero-profit conditions. The complementarity problem requires that each inequality is associated with a particular variable. So if profit conditions holds as an strict inequality then profits are negative and the activity level (the complementary variable) is zero. To obtain a non-zero activity level profits should be zero. Equations 8 to 11 represent market clearing conditions. The subscript in the cost function stands for the partial derivative, and if the supply is not filled by the demand is because the good is free (price is zero). The last equation is the income balance.

Table 1 represents the information requirements to calibrate this very simple model in terms of a squared matrix. Table 1 says that sector C buys labor for a value of 200 m.u. and that sector S buys labor for a value of 100 m.u. (first and secon column). The only agent A of this economy receives income from labor for a value of 300 m.u. (column N) and spends a total of 300 m.u. in buying 200 m.u of C and 100 m.u of S (row and column V). A disagregated version of this table is usually called social accounting matrix (SAM). Consistence of the data with a general equilibrium is guaranteed because the sum of each row equals the sum of the correspondent column. The same information can be represented as well by means of a rectangular matrix[1] that we will call micro-consistency matrix (MCM) which offers a nice representation of the three sets of restrictions innvolved by the MCP above. Results are found in table 2. In this representation the first three columns stands for the three sectors and the last column stands for the consumer. On the left column we find the prices of all commodities involved in the problem.

[1] Although for this particular model this matrix is also a squared matrix this fact breaks down as soon as we include more than one factor

	C	S	V	N	A
C			200		
S			100		
V					300
N	200	100			
A				300	

Table 1: SAM for calibrating the model with one consumer, one factor and two sectors

	C	S	V	A
p_c	200		-200	
p_s		100	-100	
p_v			300	-300
w	-200	-100		300

Table 2: MCM for calibrating the model with one consumer, one factor and two sectors

A positive entry means a receipt or sale in a particular market. A negative entry signifies an expenditure or purchase in a particular market. Consistence is guaranteed because the sum of each column is zero (zero-profit conditions) and the sum of each row is zero (market clearing conditions). Notice that the numbers in the MCM are values, prices times quantities. The modeler is free as to how interpret these as prices versus quantities. A good deal is to choose units so that as many things as possible are initially equal to one. Prices can be chosen as one, and quantities for activities can be chosen such that the correspondent activity levels are also equal to one. For instance, activity C running at level one produces 200 units of good C. However, in the case of taxes or externalities, prices can not be all one.

To give now an intuition of how to incorporate the household production in this general equilibrium framework let consider first a very stilized model widely used in the literature of optimal taxation (Jacobsen et al., 2000; Piggott and Whalley, 2001; Iorweth and Whalley, 2002). In this model there exist three cathegories of products. The firts consists in all goods and services provided exclusively by the market (C). The second group is made up of services that can be produced both at the market and at home. We will call this group S and distinguish between M (services provided by the market) and R (services self supplied at home). A representative consumer also obtain utility from leisure L. Without loss of generality we will assume that the utility function is weakly separable in three blocks and that the labor time is the only relevant input for production. Let H be the time devoted to household production, N the time used for market production (C and M) and $\overline{L}$ the total labor endowment. Self-supplied services are produced by means of a household production function $R = R(H)$. Let us borrow the specific structure of Jacobsen et al. (2000) and write the model in the following way:

$$\max \ V = V(C, S(M, R), L) \tag{13}$$

$$s.t. \qquad C = C(N_c) \tag{14}$$

$$M = M(N_m) \tag{15}$$

$$R = R(H) \tag{16}$$

$$L = \overline{L} - H - N \tag{17}$$

$$N = N_c + N_m \tag{18}$$

$$wN = p_c C + p_m M \tag{19}$$

Incorporating 17 into 19 and adding $p_r R$ to each side of the equation the budget constraint may be rewritten as:

$$p_c C + p_s M + p_r R + wL = w\overline{L} + p_r R(H) - wH \tag{20}$$

where p_r stands for the shadow price of domestic production.This problem defines an utility function over a set of goods and services, including self-supplied services, and a budget constraint in which total income is given by the value of the total endowment of time augmented by the shadow profits derived from the household production activity. The consumers take that income and "buy" goods and services provided by the market and services produced at home. So in order to define the competitive equilibrium we need to add to the system of equations an additional activity R acting in the same way as the other but whose profits flow directly to the consumer income. In the constant returns case these profits are zero.A labor supply activity (TL) that transforms leisure time in labor time is also introduced to distinguish between net of tax wage $(p_l)$and gross of tax wage (w) when taxes are introced The MCP for this case is defined by the following expresions:

$$cc(w) \geq p_c \qquad \perp C \tag{21}$$

$$cm\left(w\right) \geq p_m \qquad \perp M \tag{22}$$

$$cr\left(p_l\right) \geq p_r \qquad \perp R \tag{23}$$

$$ctl\left(pl\right) \geq w \qquad \perp TL \tag{24}$$

$$cv\left(p_c, p_s, p_m, p_l\right) \geq p_v \qquad \perp V \tag{25}$$

$$C \geq cv_{p_c}\left(p_c, p_m, p_r, p_l\right) V \qquad \perp p_c \tag{26}$$

$$M \geq cv_{p_m}\left(p_c, p_m, p_r, p_l\right) V \qquad \perp p_s \tag{27}$$

$$R \geq cv_{p_r}\left(p_c, p_m, p_r, p_l\right) V \qquad \perp p_r \tag{28}$$

$$TL \geq cc_w\left(w\right) C + cm_w\left(w\right) M \qquad \perp w \tag{29}$$

$$\overline{L} \geq cv_{p_l}\left(p_c, p_m, p_r, p_l\right) U + ctl_{p_l}\left(p_l\right) + cr_{p_l}\left(p_l\right) \qquad \perp p_l \tag{30}$$

$$V \geq \frac{I}{p_v} \qquad \perp p_v \tag{31}$$

$$I = p_l \overline{L} \tag{32}$$

And the MCM for calibration takes the form of table 3. With respect to table 2 we need additional information on the time employed at home in

	C	M	R	TL	V	A
p_c	200				-200	
p_m		100			-100	
p_r			100		-100	
w	-200	-100		300		
p_l			-100	-200	-100	500
p_v					500	-500

Table 3: SAM for calibrating the model with household production with one consumer and one factor

producing services (meals, child care, clothing) and also information on time consumed as leisure.As Kleven et al (2000) point out the main difference between time devoted to home production and leisure for optimal tax analyisis is that the former competes directly with market production of services (but not with market production of other goods) an so can distort the pattern of demand for market produced goods and services.

Let's include now three different types of taxes: a tax on labor income (t_w); a pay-roll tax on the use of labor inputs (t_{ss}) and a value added tax (t_v). Table 4 offers the modification in the information matrix to accomodate for taxes in the model. Some aspects of this table desserve a few considerations. First, some taxes can be applied both on sector inputs or outputs. Consider the case of the tax on income that appears in the TL column in table 4. The calibrated output tax rate on income is $t_w^{'} = 0.2$. This tax rate is equivalent to a calibrated taxe rate on leisure use of $t_w = 0.25 = \frac{t_w^{'}}{1-t_w^{'}}$. From here onwards we will follow this second approach. Second, should be noted that in this model there is only one agent A that pays all the taxes and also collects them.

The model in a MCP format including taxes can now be written as:

$$cc\left(w\left(1+t_{ss}\right)\right) \geq p_c \qquad \perp C \tag{33}$$

$$cm\left(w\left(1+t_{ss}\right)\right) \geq p_m \qquad \perp M \tag{34}$$

$$cr\left(p_l\right) \geq p_r \qquad \perp R \tag{35}$$

$$ctl\left(pl\left(1+t_w\right)\right) \geq w \qquad \perp TL \tag{36}$$

	C	M	R	TL	V	A
p_c	220				-220	
p_m		110			-110	
p_r			100		-100	
w	-200	-100		300		
p_l			-100	-240	-100	440
p_v					563	-563
t_{ss}	-20	-10				30
t_v					-33	33
t_w				-60		60

Table 4: SAM for calibrating the model with household production with one consumer, one factor and taxes

$$cv\left(p_c\left(1+t_v\right), p_m\left(1+t_v\right), p_r, p_l\right) \geq p_v \qquad \perp V \tag{37}$$

$$C \geq cv_{p_c(1+t_v)}\left(p_c\left(1+t_v\right), p_m\left(1+t_v\right), p_r, p_l\right) V \qquad \perp p_c \tag{38}$$

$$M \geq cv_{p_m(1+t_v)}\left(p_c\left(1+t_v\right), p_m\left(1+t_v\right), p_r, p_l\right) V \qquad \perp p_s \tag{39}$$

$$R \geq cv_{p_r}\left(p_c\left(1+t_v\right), p_m\left(1+t_v\right), p_r, p_l\right) V \qquad \perp p_r \tag{40}$$

$$TL \geq cc_{w(1+t_{ss})}\left(w\left(1+t_{ss}\right)\right) C + cm_{w(1+t_{ss})}\left(w\left(1+t_{ss}\right)\right) M \qquad \perp w \tag{41}$$

$$\overline{L} \geq cv_{p_l}\left(p_c\left(1+t_v\right), p_m\left(1+t_v\right), p_r, p_l\right) U + ctl_{p_l(1+t_w)}\left(p_l\left(1+t_w\right)\right) TL + cr_{p_l}\left(p_l\right) R \qquad \perp p_l \tag{42}$$

$$V \geq \frac{I}{p_v} \qquad \perp p_v \tag{43}$$

$$I = p_l \overline{L} + t_{ss} w \left(cc_{w(1+t_{ss})} C + cm_{w(1+t_{ss})} M\right) + t_v \left(p_c cv_{p_c(1+t_v)} + p_m cv_{p_m(1+t_v)}\right) V + t_w ctl_{p_l(1+t_w)} TL \tag{44}$$

3 Equilibrium unemployment

This section incorporates the unemployment in the general equilibrium framework with household production. The concept of unemployment used, which is inspired in job search models, has been recently made operational for large-scale general equilibrium models by Balistreri (2002). The basic idea is to think in unemployment as the result of a process by which willing individuals are transformed into employed workers. This process involves an external effect through which the economy-wide vacancy rate and the unemployment rate affect the job matching probability of individual workers. The main features of the Pissarides' model (see Pissarides, 1990) are preserved in the model by means of a reduced form of the job matching opportunities determining individual labor supply. To give an intuition of the way in which equilibrium unemployment is introduced in the model, let first begin with a general formulation of external economies in one sector X, as characterized by Markusen (1990). This formulation allows the use of competitive general equilibrium tools despite the violation of constant returns in aggregate production.

Continue with the one factor case and define the production of an individual firm X_i in the sector X as:

$$X_i = X^{\beta} F(L_i) \tag{45}$$

where X^{β} captures the external effect. If all the firms are simetric we arrive to:

$$X = X^{\beta} F(L) \tag{46}$$

and hence:

$$X = F(L)^{1/(1-\beta)} \tag{47}$$

so there exist increasing returns to scale at the industry level. The individual firm perceives the total output as constant and maximizes as if it produces with constant returns to scale:

$$\pi_i = pX_i - wL_i = pX^{\beta} F(L_i) - wL_i \tag{48}$$

At aggregate level:

$$\pi = pX - wL = pX^{\beta} F(L) - wL \tag{49}$$

The first order condition gives:

$$pX^{\beta} \frac{\partial F(L)}{\partial L} = w \rightarrow p = X^{-\beta} \left(w \frac{\partial L}{\partial F(L)} \right) = X^{-\beta} cx(w) \tag{50}$$

where $cx(w)$ is the unit cost function associated with the constant returns to scale function $F(L)$ which gives the cost needed for producing $F(L) = 1$. This would be now the zero-profit condition for the X sector in a MCP. The labor factor required to produce $F(L)$ is obtained from the cost function and from the expresion 46:

$$cx_w(w) F(L) = cx_w(w) X^{1-\beta} \tag{51}$$

Expresion 51 should be taken into account in specifying the market clearing condition for the labor factor as a mixed complementarity problem.

Consider now a matching process that transforms labor supplied by an individual into employed labor. Assume that the process is characterized by external economies. Similarly to 46 an individual's employed labor is given by:

$$E_i = G(E) F(L_i) \tag{52}$$

where E is the total number of workers matched and $F(L_i)$ is the trivial function $F(L_i) = L_i$. Furthermore, the externality $G(E)$ can be decomposed to take into account the direct employment and the indirect unemployment:

$$E_i = H(E, U(E))L_i = H(E, u)L_i \tag{53}$$

where $U(E)$ is the unemployment that in equilibrium is a function of the employment.Let denote $\overline{w}$ the price of L (the reservation wage), while we will call p_l to the net of tax market wage and w to the gross of tax market wage as before. Now proceeding as in 50 we can writte:

$$p_l = \frac{\overline{w}}{H(E, u)} \rightarrow p_l H(E, u) = \overline{w} \tag{54}$$

Also adapting 51 to the problem in hand we get the total demanded labor supply L for the production of E:

$$ce_{\overline{w}}\left(\overline{w}\right)\frac{E}{H\left(E,u\right)}=\frac{E}{H\left(E,u\right)} \tag{55}$$

Introducing expresions 54 and 55 in the MCP we obtain:

$$cc\left(w\left(1+t_{ss}\right)\right)\geq p_c \qquad \perp C \tag{56}$$

$$cm\left(w\left(1+t_{ss}\right)\right)\geq p_m \qquad \perp M \tag{57}$$

$$cr\left(\overline{w}\right)\geq p_r \qquad \perp R \tag{58}$$

$$pl\left(1+t_w\right)\geq w \qquad \perp TL \tag{59}$$

$$\overline{w}\geq p_l H\left(E,u\right) \qquad \perp E \tag{60}$$

$$cv\left(p_c\left(1+t_v\right),p_m\left(1+t_v\right),p_r,\overline{w}\right)\geq p_v \qquad \perp V \tag{61}$$

$$C\geq cv_{p_c(1+t_v)}\left(p_c\left(1+t_v\right),p_m\left(1+t_v\right),p_r,\overline{w}\right)V \qquad \perp p_c \tag{62}$$

$$M\geq cv_{p_m(1+t_v)}\left(p_c\left(1+t_v\right),p_m\left(1+t_v\right),p_r,\overline{w}\right)V \qquad \perp p_s \tag{63}$$

$$R\geq cv_{p_r}\left(p_c\left(1+t_v\right),p_m\left(1+t_v\right),p_r,\overline{w}\right)V \qquad \perp p_r \tag{64}$$

$$TL\geq cc_{w(1+t_{ss})}\left(w\left(1+t_{ss}\right)\right)C+cm_{w(1+t_{ss})}\left(w\left(1+t_{ss}\right)\right)M \qquad \perp w \tag{65}$$

$$E\geq TL \qquad \perp p_l \tag{66}$$

$$\overline{L}\geq cv_{\overline{w}}\left(p_c\left(1+t_v\right),p_m\left(1+t_v\right),p_r,\overline{w}\right)V+cr_{\overline{w}}\left(\overline{w}\right)R+\frac{E}{H\left(E,u\right)} \qquad \perp\overline{w} \tag{67}$$

$$V \geq \frac{I}{p_v} \qquad \perp p_v \tag{68}$$

$$I = \overline{w}\overline{L} + t_{ss}w\left(cc_{w(1+t_{ss})}C + cm_{w(1+t_{ss})}M\right) + \tag{69}$$

$$t_v\left(p_c cv_{p_c(1+t_v)} + p_m cv_{p_m(1+t_v)}\right)V + t_w p_l TL \tag{70}$$

$$u = 1 - \frac{E}{\overline{L} - L - R} \tag{71}$$

An special case for $H(E, u)$ which guarantees a positive unemployment rate is given by:

$$H(E, u) = (1 - u_0)(E/E_0)^{\sigma}(u/u_0)^{\eta} \tag{72}$$

where u_0 is the benchmark unemployment rate and E_0 the benchmark employment, so in the base case $H(E, u) = (1 - u_0)$.

An example of a micro-consistent matrix for calibrating the previous model is given by table 5 where pl (and w) incorporates the wage premium according to 60. Column E transforms labor force into employed workers (labor force minus uenmployed workers) so the equality in column E in value terms holds because $pl = \frac{\overline{w}}{1-u_0}$. In addition to figures in table 5 to calibrate the model we need also information on $u_{0.}$

4 The two-factor model

This section extends the model to incorporate an additional factor for production that can be called at this stage capital input although can also be adapted to other taxable inputs. This model will represent our starting point for tax analysis applications. The extended MCM can be found in table 6 in which two new rows appear with respect to table 5: r which can be thought as the rental price of capital and t_k that stands for taxes on the use of capital that we will assume to be 30%. Denote by $\overline{K}$ to the endowment of capital and by Q to the capital used in household production so the new budget constraint when the new input is included in the model can be written as:

$$w\left(\overline{L} - H - L\right) + r\left(\overline{K} - Q\right) = p_c C + p_m M \tag{73}$$

	C	M	R	TL	E	V	A
p_c	220					-220	
p_m		110				-110	
p_r			100			-100	
w	-200	-100		300			
p_l				-240	240		
$\overline{w}$			-100		-240	-100	440
p_v						563	-563
t_{ss}	-20	-10					30
t_v						-33	33
t_w				-60			60

Table 5: SAM for calibrating the model with household production with one consumer, one factor, taxes and equilibrium unemployment

rearrenging this expresion and adding $p_r R$ to both sides of the equation results:

$$w\overline{L} + r\overline{K} + (p_r R - wH - rQ) = p_c C + p_m M + p_r R + wL \tag{74}$$

where the left hand side term in parenthesis can be unsderstood as the shadow profits associated to household production. For the constant returns case this term will be equal to zero.

The MCP can be inmediately extended to incorporate both the new commodity and tax:

$$cc\left(w\left(1+t_{ss}\right), r\left(1+t_k\right)\right) \geq p_c \qquad \perp C \tag{75}$$

$$cm\left(w\left(1+t_{ss}\right), r\left(1+t_k\right)\right) \geq p_m \qquad \perp M \tag{76}$$

$$cr\left(\overline{w}, r\right) \geq p_r \qquad \perp R \tag{77}$$

$$pl\left(1+t_w\right) \geq w \qquad \perp TL \tag{78}$$

$$\overline{w} \geq p_l H\left(E, u\right) \qquad \perp E \tag{79}$$

$$cv\left(p_c\left(1+t_v\right), p_m\left(1+t_v\right), p_r, \overline{w}\right) \geq p_v \qquad \perp V \tag{80}$$

	C	M	R	TL	E	V	A
p_c	350					-350	
p_m		370				-370	
p_r			200			-200	
w	-200	-100		300			
p_l				-240	240		
$\overline{w}$			-100		-240	-100	440
r	-100	-200	-100				400
p_v						1092	-1092
t_{ss}	-20	-10					30
t_v						-72	72
t_w				-60			60
t_k	-30	-60					90

Table 6: SAM for calibrating the model with household production with one consumer, two factors, taxes and equilibrium unemployment

$$C \geq cv_1 V \qquad \perp p_c \tag{81}$$

$$M \geq cv_2 V \qquad \perp p_s \tag{82}$$

$$R \geq cv_3 V \qquad \perp p_r \tag{83}$$

$$TL \geq cc_1 C + cm_1 M \qquad \perp w \tag{84}$$

$$E \geq TL \qquad \perp p_l \tag{85}$$

$$\overline{L} \geq cv_4 V + cr_1 R + \frac{E}{H(E,u)} \qquad \perp \overline{w} \tag{86}$$

$$V \geq \frac{I}{p_v} \qquad \perp p_v \tag{87}$$

$$I = \overline{w}\overline{L} + r\overline{K} + t_{ss} w \left(cc_1 C + cm_1 M\right) + \tag{88}$$

$$t_v\left(p_c cv_1 + p_m cv_2\right)V + t_w p_l TL + t_k r\left(cc_2 C + cm_2 M\right) \tag{89}$$

$$u = 1 - \frac{E}{\overline{L} - L - R} \tag{90}$$

In all the above problem the subscript j in the cost functions represents the partial derivative of the function with respect to argument j.

5 Intermediate inputs

In this section we introduce an additional commodity (F) which is produced by primary factors and is also used in producing other comodities (call it food). The table 7 exemplifies the information in terms of a micro-consistent matrix used to calibrate this model. A new sector and commodity has been introduced to account for this intermediate input which in this example is produced by using only labor. Notice also that there is a tax on the use of food in the household sector that does not exist for the rest of the sectors, because the market sectors can deduce the value added tax on the use of intermediate inputs.

Th MCP now is extended to incorporate equations 94 and 101 for taking account of food as a produced intermediate input.

$$cc\left(w\left(1+t_{ss}\right), r\left(1+t_k\right), p_f\right) \geq p_c \qquad \perp C \tag{91}$$

$$cm\left(w\left(1+t_{ss}\right), r\left(1+t_k\right), p_f\right) \geq p_m \qquad \perp M \tag{92}$$

$$cr\left(\overline{w}, r, p_f\left(1+t_v\right)\right) \geq p_r \qquad \perp R \tag{93}$$

$$cf\left(w\left(1+t_{ss}\right)\right) \geq p_f \qquad \perp F \tag{94}$$

$$pl\left(1+t_w\right) \geq w \qquad \perp TL \tag{95}$$

$$\overline{w} \geq p_l H\left(E, u\right) \qquad \perp E \tag{96}$$

	C	M	R	F	TL	E	V	A
p_c	360						-360	
p_m		430					-430	
p_r			244				-244	
p_f	-10	-60	-40	110				
w	-200	-100		-100	400			
p_l					-320	320		
$\overline{w}$			-100			-320	-100	520
r	-100	-200	-100					400
p_v							1213	-1213
t_{ss}	-20	-10		-10				40
t_v			-4				-79	83
t_w					-80			80
t_k	-30	-60						90

Table 7: SAM for calibrating the model with household production with one consumer, two factors, intermediate inputs, taxes and equilibrium unemployment

$$cv\left(p_c\left(1+t_v\right), p_m\left(1+t_v\right), p_r, \overline{w}\right) \geq p_v \qquad \perp V \tag{97}$$

$$C \geq cv_1 V \qquad \perp p_c \tag{98}$$

$$M \geq cv_2 V \qquad \perp p_s \tag{99}$$

$$R \geq cv_3 V \qquad \perp p_r \tag{100}$$

$$F \geq cc_3 C + cm_3 M + cr_2 R \qquad \perp p_f \tag{101}$$

$$TL \geq cc_1 C + cm_1 M + cf_1 F \qquad \perp w \tag{102}$$

$$E \geq TL \qquad \perp p_l \tag{103}$$

$$\overline{L} \geq cv_4 V + cr_1 R + \frac{E}{H\left(E,u\right)} \qquad \perp \overline{w} \tag{104}$$

	C	M	R	F	N	K	t_{ss}	t_v	t_w	t_k	A
C											396
M											473
R											244
F	10	60	40								
N	200	100	100	100							100
K	100	200	100								
t_{ss}	20	10		10							
t_v	36	43	4								
t_w					80						
t_k	30	60									
A					520	400	40	83	80	90	

Table 8: Alternative SAM for calibrating the model with household production with one consumer, two factors, intermediate inputs, taxes and equilibrium unemployment

$$V \geq \frac{I}{p_v} \qquad \perp p_v \tag{105}$$

$$I = \overline{w}\overline{L} + r\overline{K} + t_{ss}w\left(cc_1C + cm_1M + cf_1F\right) + \tag{106}$$

$$t_v\left(p_c cv_1 + p_m cv_2\right)V + t_v cr_3 R + t_w p_l TL + t_k r\left(cc_2C + cm_2M\right) \tag{107}$$

$$u = 1 - \frac{E}{\overline{L} - L - R} \tag{108}$$

The MCM can also be written as a Social Accounting Matrix. The table 8 shows the transformation.

6 The Iorweth and Whalley model with equilibrium unemployment

In this section we adapt the previous approach to the model developed by Iorweth and Whalley (2002) extended to include unemployment. This work considers both the case of constant returns to scale and the increasing returns to scale. In this section we will only deal with the constant returns to scale case. The pursued objective by these authors is to analize the efficiency gains derived from the exemption of food from sales and value added taxes, concluding that there is a presumption for more than fully taxing food used in household production to compensate for the exemption for the use of non-food inputs in household production. To understand the model of Iorweth and Whalley (IW from here onwards) in the mixed complementarity framework outlined, is useful to think in S as a sector that produces meals. These meals can be offered by restaurants (M) or produced at home (R). Unlike the model in the previous section, the IW model uses a transformation frontier that transforms the household's resource availability $\overline{F}$ (with a price $p_{\overline{f}}$) in different combinations of food and effective labor units. Let Q be the basket composed of food and efective labor (that can be offered to the market or consumed as leisure). The household transforms $\overline{F}$ into Q by means of the trivial function $Q = Q\left(\overline{F}\right)$ so the cost function is equal to $cq\left(p_{\overline{f}}\right) = p_{\overline{f}}$. The technology Q in turns uses total resources $\overline{F}$ to produce two outputs by means of a constant elasticity of transformation:

$$\overline{F} = \xi\left(\tau L^{\frac{\varepsilon+1}{\varepsilon}} + (1-\tau)\,F^{\frac{\varepsilon+1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon+1}} \tag{109}$$

where ε represents the elasticity of transformation. The function 109 is concave in L and F allowing to obtain a supply curve with positive slope for food and efective labor. The resulting price for the composed basket is:

$$p_q = \xi^{-1}\left(\tau^{-\varepsilon}\overline{w}^{1+\varepsilon} + (1-\tau)^{-\varepsilon}\,p_f^{1+\varepsilon}\right)^{\frac{1}{(\varepsilon+1)}} \tag{110}$$

There is not a capital factor, and while non-meals market sector (C) produces using labor as the only factor, food in addition to labor is required for production of meals. Initially VAT rate for food is set to zero, so this case corresponds to a situation in which food is exempt from any tax. However, according to IW the benchmark VAT rate on the other market goods is set to 15%. There are not social security contributions nor income tax in the benchmark, but we explicitly include them in the model to account for different fiscal experiments. The table 9 replicates in terms of a micro-consistent matrix the information provided by IW to calibrate their model.

The mixed complentarity program corresponding to the model in hand is the following:

	C	M	R	Q	TL	E	V	A
p_c	335.88						-335.88	
p_m		14.75					-14.75	
p_r			124.89				-124.89	
p_f		-4.84	-39.28	44.12				
w	-335.88	-9.91			345.79			
p_l					-345.79	345.79		
$\overline{w}$			-85.61	1056.17		-345.79	-624.77	
$p_{\overline{f}}$				-1100.29				1100.29
p_v							1152.8845	-1152.8845
t_{ss}	0	0						
t_v			0				-52.5945	52.5945
t_w					0			

Table 9: SAM that repicates the data used by Iorweth and Whalley (2002) for calibrating their model

$$cc\left(w\left(1+t_{ss}\right)\right) \geq p_c \qquad \perp C \tag{111}$$

$$cm\left(w\left(1+t_{ss}\right), p_f\right) \geq p_m \qquad \perp M \tag{112}$$

$$cr\left(\overline{w}, p_f\left(1+t_v\right)\right) \geq p_r \qquad \perp R \tag{113}$$

$$p_{\overline{f}} \geq p_q = \xi^{-1}\left(\tau^{-\varepsilon}\overline{w}^{1+\varepsilon} + (1-\tau)^{-\varepsilon} p_f^{1+\varepsilon}\right)^{\frac{1}{(\varepsilon+1)}} \qquad \perp Q \tag{114}$$

$$pl\left(1+t_w\right) \geq w \qquad \perp TL \tag{115}$$

$$\overline{w} \geq p_l H\left(E, u\right) \qquad \perp E \tag{116}$$

$$cv\left(p_c\left(1+t_v\right), p_m\left(1+t_v\right), p_r, \overline{w}\right) \geq p_v \qquad \perp V \tag{117}$$

$$C \geq cv_1 V \qquad \perp p_c \tag{118}$$

$$M \geq cv_2 V \qquad \perp p_s \tag{119}$$

$$R \geq cv_3 V \qquad \perp p_r \tag{120}$$

$$F \geq cm_2 M + cr_2 R \qquad \perp p_f \tag{121}$$

$$TL \geq cc_1 C + cm_1 M + cf_1 F \qquad \perp w \tag{122}$$

$$E \geq TL \qquad \perp p_l \tag{123}$$

$$\overline{L} \geq cv_4 V + cr_1 R + \frac{E}{H(E, u)} \qquad \perp \overline{w} \tag{124}$$

$$\overline{F} \geq Q \qquad \perp p_{\overline{f}} \tag{125}$$

$$V \geq \frac{I}{p_v} \qquad \perp p_v \tag{126}$$

$$I = p_{\overline{f}} \overline{F} + t_{ss} w \left(cc_1 C + cm_1 M \right) + \tag{127}$$

$$t_v \left(p_c cv_1 + p_m cv_2 \right) V + t_v cr_2 R + t_w p_l TL \tag{128}$$

$$u = 1 - \frac{E}{\overline{L} - L - R} \tag{129}$$

7 Fiscal experiments for Canada

It is widely believed that a targeted reduction in social security contributions is likely to be a more effective instrument for job creation that a reduction in VAT rates. For instance, in Europe, empirical studies of the effects of reducing VAT rates has shown that such measures are never the most effective (see Commission of the European Communities (2003)). In this section we aim two objectives. First we show how the consideration of equilibrium unemployment in the model alter importnatly the efficiency results of the tax experiments, and second we quantify the impact on unemployment itself of changes in VAT and payroll taxes in order to assess the most effective fiscal policy to fight unemployment.

According to Lin (2001) and Bédard (1998) the effective payroll tax paid by employers have been estimated to be 5.76% in 1992. The unemployment rate in turn is set at 11%.

(Figures 1 to 4 at the end)

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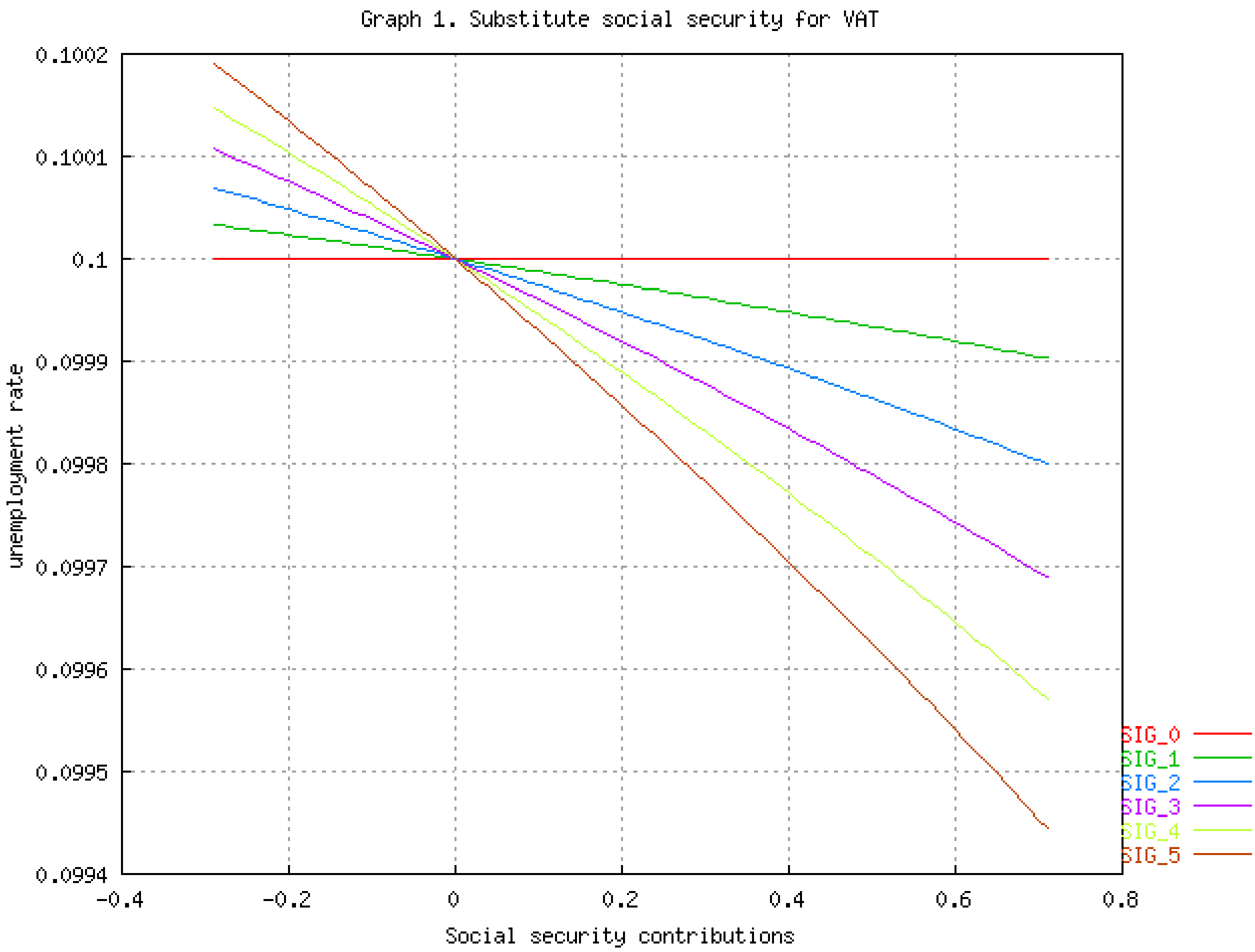
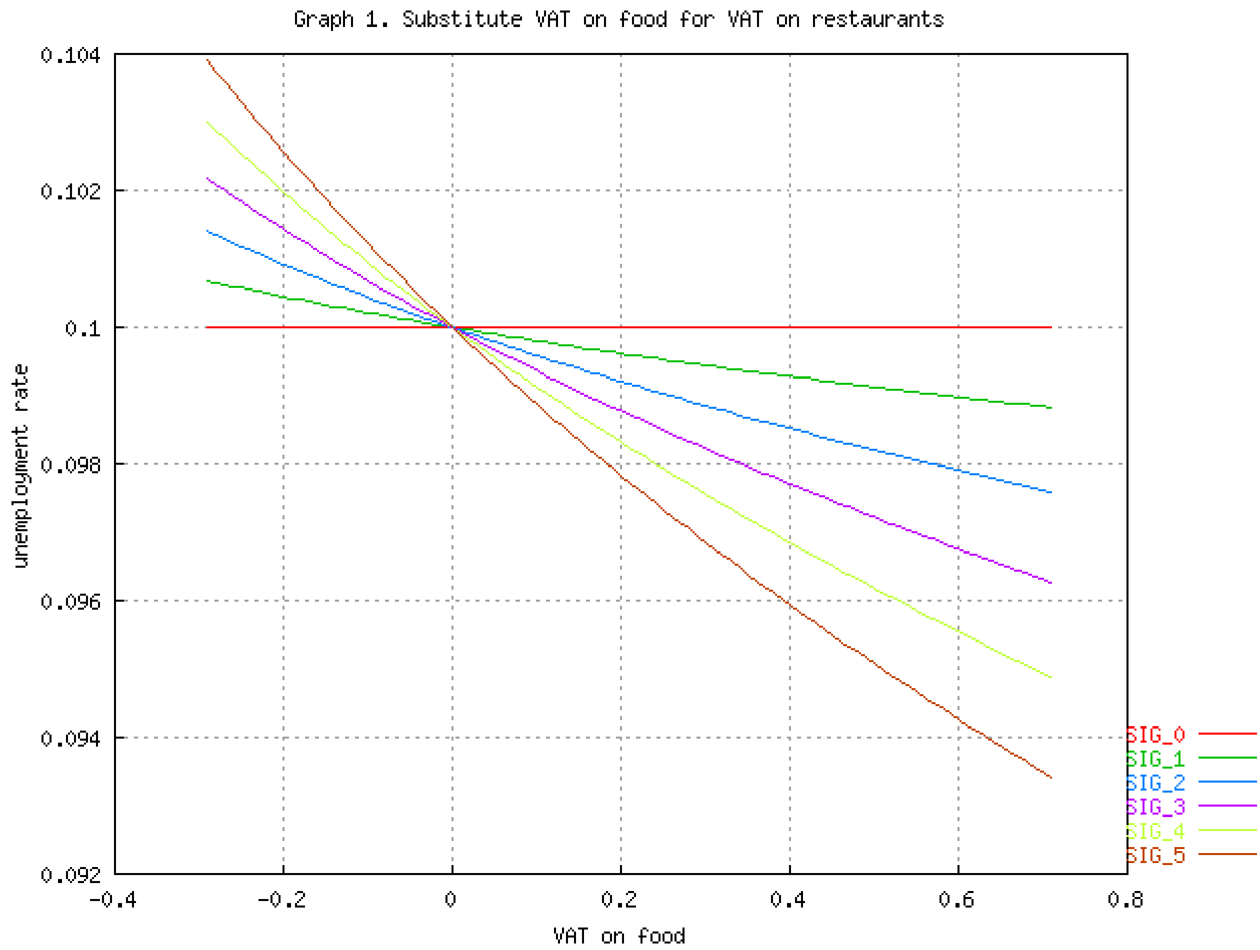


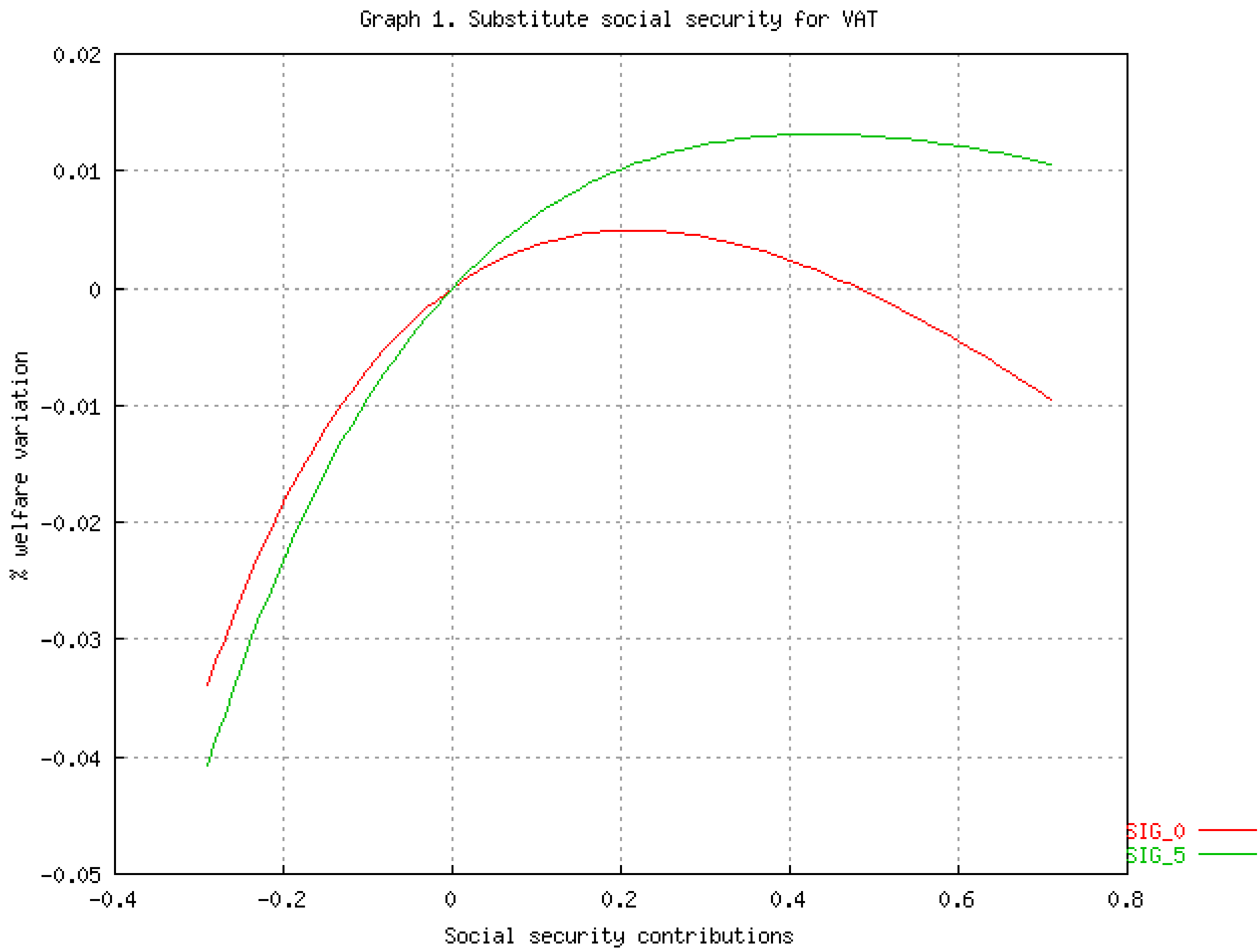
Graph 1. Substitute social security for VAT
0.1002
0.1001
0.1
0.0999
0.0998
0.0997
0.0996
0.0995
0.0994
unemployment rate
-0.4
-0.2
0
0.2
0.4
0.6
0.8
Social security contributions
SIG_0
SIG_1
SIG_2
SIG_3
SIG_4
SIG_5

Figure 1:



Graph 1. Substitute VAT on food for VAT on restaurants
unemployment rate
0.104
0.102
0.1
0.098
0.096
0.094
0.092
-0.4
-0.2
0
0.2
0.4
0.6
0.8
VAT on food
SIG_0
SIG_1
SIG_2
SIG_3
SIG_4
SIG_5

Figure 2:



Graph 1. Substitute social security for VAT
0.02
0.01
0
-0.01
-0.02
-0.03
-0.04
-0.05
% welfare variation
-0.4
-0.2
0
0.2
0.4
0.6
0.8
Social security contributions
SIG_0
SIG_5

Figure 3:

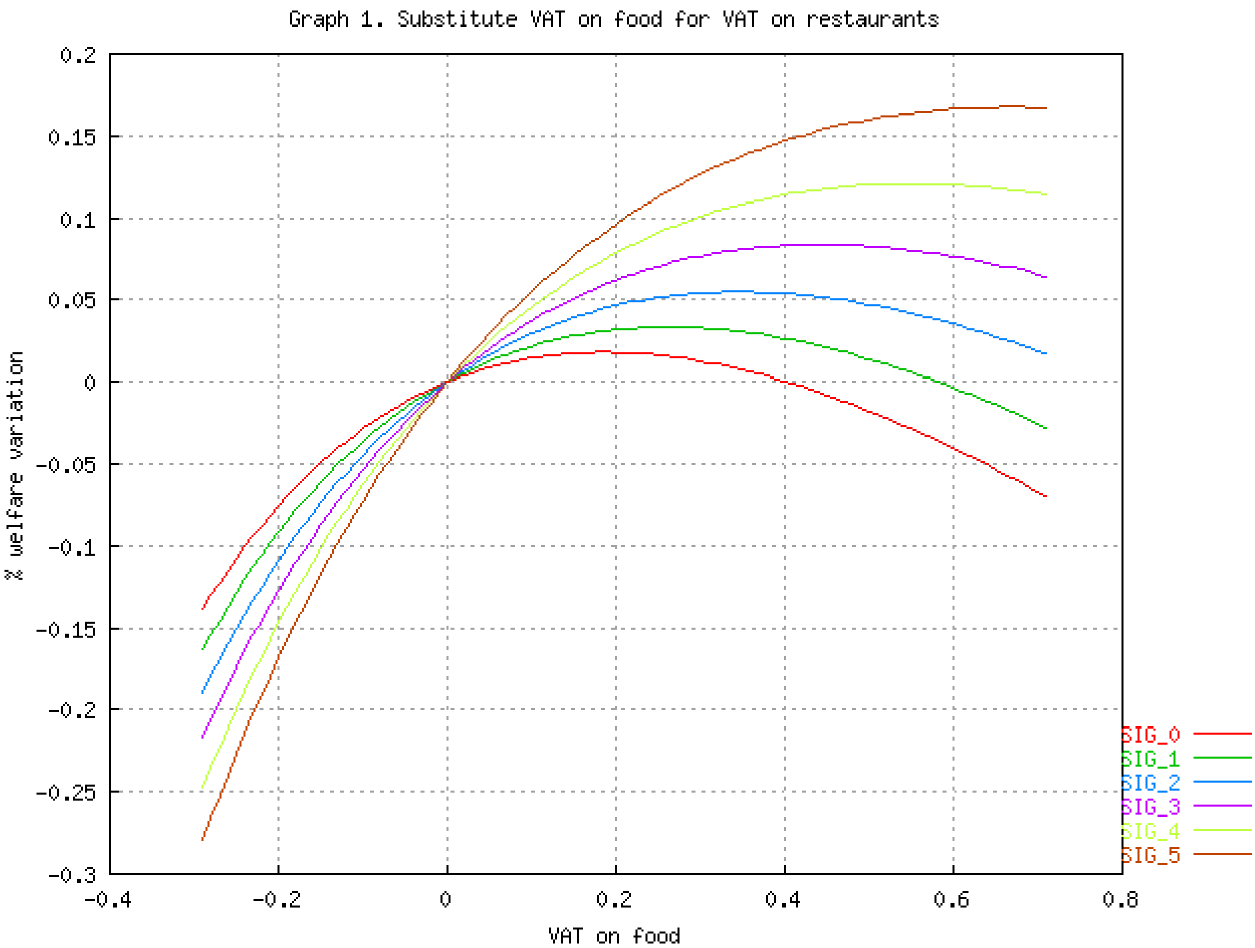

Graph 1. Substitute VAT on food for VAT on restaurants
% welfare variation
VAT on food
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
-0.25
-0.3
-0.4
-0.2
0
0.2
0.4
0.6
0.8
SIG_0
SIG_1
SIG_2
SIG_3
SIG_4
SIG_5

Figure 4: