SINGULAR ISSUES ABOUT STRICTLY SINGULAR OPERATORS

JESÚS M. F. CASTILLO

1. Singular operators where you thought was none

Theorem 1. *There is a separable isomorphically polyhedral $\mathcal{L}_\infty$ space that is not isomorphically Lindenstrauss. Moreover, it is a subspace of an isomorphically polyhedral Lindenstrauss space.*

JMF Castillo, P.L. Papini, *On isomorphically polyhedral $\mathcal{L}_\infty$-spaces* J. Funct. Anal. 270 (2016) 2336 –2342

2. Singular operators save the day

Theorem 2.

(1) *For every non-trivial L_∞-centralizer Ω on L_2 there is a complex structure on L_2 that can not be extended to an operator on $d_\Omega L_2$.*
(2) *For every non-trivial ℓ_∞-centralizer Ω on ℓ_2 there is a complex structure on ℓ_2 that can not be extended to an operator on $d_\Omega \ell_2$.*

JMF Castillo, V. Ferenczi, Y. Moreno, W. Cuellar, *Complex structures on twisted Hilbert spaces*, preprint

3. Where are the singular operators when you need one?

The Henson-Moore classification problem of $\mathcal{L}_\infty$-spaces by isomorphic ultrapowers is

Problem. How many ultratypes of $\mathcal{L}_\infty$-spaces are there?

Only two different ultra-types of $\mathcal{L}_\infty$-spaces are known: that of $C(K)$-spaces and that of spaces of almost universal disposition. I personally believe that:

There is a continuum of different ultratypes of $\mathcal{L}_\infty$-spaces

A. Avilés, F. Cabello Sánchez, JMF Castillo, M. González, Y. Moreno, *Separable injective Banach spaces* , Lecture Notes in Mathematics 2132 (2016) Springer-Verlag.