MAXIMAL AVERAGING OPERATORS: FROM GEOMETRY TO BOUNDEDNESS THROUGH DUALITY

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The course will present the general principle that boundedness of maximal averages characterizes pointwise convergence in a variety of contexts. The main focus of the course will be on understanding maximal operators through geometry and duality. The general principle that we will try to explore is that there is an equivalence between geometric covering properties of families of sets, and boundedness of corresponding maximal operators. This equivalence is established through duality and thus reduces the problem of boundedness of maximal operators to that of understanding geometric covering lemmas. The opposite implication is also very useful, namely the fact that analytic properties of operators imply geometric and combinatorial properties of families of sets.

We will begin by exploring the simplest instances of the phenomenon described above: the case of maximal averages with respect to Euclidean balls, and cubes. Here the main tools are relatively simple covering lemmas. A next step will be to understand the family of rectangles in 2 dimensions, where the geometry and combinatorics are slightly more involved; this is reflected in the fact that the corresponding maximal operator, namely the "strong maximal function", has weaker boundedness properties. Depending on the dynamic and response of the work in the group during the school we might (or might not) touch upon another example which has a higher level of complexity, the directional maximal operator: here the family of sets is a set of parallelograms pointing in a given set of directions in the plane.

All the examples above will illustrate the general principle and exhibit the interplay between geometry, functional analysis, and analytic properties of maximal averages.