Departamento de Matemáticas
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An overview on the Prus-Szczepanik condition

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Workshop on Fixed Point Theory and its Applications
On the occasion of Enrique Llorens' 70th birthday
(Valencia, December 2016)

- *Prus & Szczepanik (2005)*

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Fixed Point Theorem

X has FPP if

Prus-Szczepanik condition

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Fixed Point Theorem

X has FPP if

there exists $\varepsilon \in (0,1)$ such that

$\forall x \in S_X \quad b_1(1,x) < 1 - \varepsilon \quad$ or $\quad d(1,x) > \varepsilon$

Prus-Szczepanik condition

- *Hernández, Llorens, Mazcuñán & Muñiz (2014)*

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X has the Prus-Szczepanik condition, $\mathbf{PSz}$, if

there exists $\varepsilon \in (0,1)$ such that

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$b_1(1,x) < 1-\varepsilon$: X has $\mathbf{PSzA}$

$d(1,x) > \varepsilon$: X has $\mathbf{PSzB}$

❖ *Hernández, Llorens, Mazcuñán & Muñiz (2014)*

Characterization of PSzA

X has $PSzA \Leftrightarrow M(X) > 1$

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Characterization of PSzA

X has $PSzA \Leftrightarrow M(X) > 1$

Characterization of PSzB

X has $PSzB \Leftrightarrow r_X(1) > 0$

❖ *Hernández, Llorens, Mazcuñán & Muñiz (2014)*

Summary

X has PSz can be seen as the local union of

- $M(X) > 1$
- something related to Opial modulus

❖ *Prus & Szczepanik (2005)*

Stability Theorem

Y has FPP if $d(X,Y) < M_1(X)$

where

- X has $PSz \quad \approx \quad M_1(X) > 1$

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Stability Theorem

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- X has $PSz \quad \approx \quad M_1(X) > 1$
- $M_1(X) \geq M(X)$

❖ *Prus & Szczepanik (2005)*

Summary

Prus & Szczepanik stability condition

$$d(X,Y) < M_1(X)$$

improves Dominguez's condition

$$d(X,Y) < M(X)$$

because $M_1(X) \geq M(X)$ as it adds to $M(X)$, locally, something related to Opial modulus.

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¿Sounds familiar?

❖ *Llorens & Jiménez (2000)*

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Stability Theorem

Y has FPP if

$$d(X,Y) < \sup\left\{ \frac{1+a}{R\Big(a/d(X,Y)C_X\big(d(X,Y)\big),X\Big)} : a \geq 0 \right\}$$

where

$$C_X(B) = \sup\{c \geq 0 : r_X(c) \leq B - 1\}$$

❖ *Prus & Szczepanik (2005)*

P & Sz win

Llorens & Jiménez stability condition $\Rightarrow d(X,Y) < M_1(X)$

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Win but almost tie