Single-output color pattern recognition using a fractional correlator

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Abstract. A novel method for performing color image pattern recognition using a fractional correlator (FC) is proposed. The input plane is illuminated with three different coherent sources of wavelengths corresponding to RGB (red, green, and blue) colors. The output plane provides a single output peak, which is a result of an incoherent addition between the three correlations obtained per each color. By using the fractional correlator, which is a partially space variant correlator, we achieve space-variance-controlled color pattern recognition. The use of the three-color illumination can drastically increase the discrimination ability of the suggested correlator. © 1997 Society of Photo-Optical Instrumentation Engineers. [S0091-3286(97)02808-0]

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1 Introduction

Pattern recognition is one of the main uses of optical systems due to the ability to perform correlation in real time. Most of the pattern recognition systems are based on the 4-f VanderLugt (VL) correlator1 or the joint transform correlator (JTC). Many applications deal with monochromatic images, however, color pattern recognition has become important due to the increased discrimination ability obtained. Different solutions for color pattern recognition have been proposed based on the VL correlator and the JTC.

Electronic devices capture a polychromatic image by grabbing its transmittance for the red, green, and blue (RGB) color illumination in three separate color channels. All channels are processed in parallel, yielding in the output plane a set of separated correlation outputs each of a different wavelength. Such systems were developed both for the VL correlator2-7 and the JTC (Refs. 5 to 7). This leads to a cumbersome process of analyzing each channel and composing all channels to a detection decision. Several systems with a single output have been introduced as well. Such VL-based systems tried to overcome this obstacle by time multiplexing of the different color channels in a monochromatic correlator and then integrating the output to obtain a single output.8,9 Another method tries to encode the color image as a phase-only distribution, thus having a single output. This method was implemented both on the VL (Ref. 10) correlator and the JTC (Ref. 11). The major drawback of this approach is that it suffers from the need to preprocess the signal in order to encode it. A JTC-based multichannel single-output scheme12 was also proposed. However, since it is based on the JTC configuration, its space variance is not controlled.

It has been shown that in certain applications the full shift invariance property is not recommended and can even cause fault detection. Such applications include the detection of an image appearing within another image in a specified area and a rejection of the same image (or detection of other image) appearing in a different area.13 For instance, the detection of a stamp of a postcard. The stamp should appear only at the upper right corner while only small degree of spatial shift variance should be allowed within that region.

In this paper we propose the use of the fractional correlator (FC) based on the fractional Fourier transform (FRT). The fractional parameter of this transform is the fractional order $P$. The order controls the amount of spatial shift variance of the transform. If $P = 1$, the FRT becomes the conventional Fourier transform, which is fully spatially shift invariant. If $P = 0$, the FRT domain becomes the input domain, thus the transform is fully space shift variant. Using any other order of $P$ between 0 to 1 results with a transform that is partially spatially shift variant. The input of the proposed setup consists of a colored picture. This input is illuminated with three different coherent wavelengths according to RGB colors, coming from three different lasers. Note that the choice of the three RGB colors and not any other color combination arises from convenience of being compatible to other existing electronic equipment and from the fact that in most cases, a great deal of the information exists around these colors. The filter placed in the fractional domain contains the complex conjugate of the summations of the $P$ order FRT of the RGB transmittance of the reference signals. Each fractional Fourier transformed transmittance is scaled properly according to its corresponding wavelength. The FC performs an FRT with a fractional order of $P$ over the input signal, multiplies it by the filter, and performs an inverse Fourier transform. If the input object is the reference object, then in the output plane an incoherent summation is obtained for the correlation peaks of each of the color channels (the summation is incoherent since the different wavelengths do not interfere).
Note also that by using more channels the detection is enhanced. In this case, by using three channels the discrimination of the detection peak can be enhanced in comparison to the detection ability of an ordinary monochromatic correlator. Each detection channel contributes to the peak if it was matched, and the incoherent addition of all channels yields a larger peak. If the polychromatic input is mainly represented at the red wavelength, for instance, then a detection will still be made due to the red matched component of the filter (as obtained from a monochromatic correlator).

Section 2 presents background of the FRT and the FC. Some computation considerations are discussed in Sec. 3. The structure of the suggested modified filter is described in Sec. 4. Section 5 illustrates several computer simulations.

2 FRT and the FC

The FRT can be accomplished with optics by two means with different interpretations. One interpretation is based on the Hermite-Gaussian functions and is accomplished by propagating through a graded-index (GRIN) medium. The other interpretation is based on the Wigner transform and is accomplished by free-space propagation and lenses. Both interpretations are fully equivalent. Based on the Wigner transform, the mathematical definition for the FRT can be formulated as

\[
\mathcal{F}_{p,\lambda}[u_0(x_0)](x) = C_1 \int_{-\infty}^{\infty} u_0(x_0) \exp \left( \frac{i \pi (x_0^2 + x^2)}{\lambda f_1 \tan \phi} \right) \times \exp \left( -2i \pi \frac{L x x_0}{\lambda f_1 \sin \phi} \right) \, dx_0,
\]

where \( P \) is the fractional order, \( f_1 \) is a scaling constant of the transformed function, \( \lambda \) is the wavelength, and \( C_1 \) is a normalizing constant that equals

\[
C_1 = \frac{\exp(-i\{\pi \, \text{sgn} \left( \sin \phi \right)/4 - \phi/2\})}{\left| \sin \phi \right|^{1/2}}.
\]

Optically, the second interpretation is implemented by a cascaded operation of free-space propagation of distance \( Z = R f_1 \), a lens with focal length \( f = f_1 / Q \), and another free-space propagation of distance \( Z = R f_1 \), as depicted in Fig. 1, where:

\[
R = \tan \frac{\phi}{2} \quad Q = \sin \phi.
\]

Performing the FC on two distributions is accomplished by multiplying their fractional transforms and then again performing another fractional transform over the result. Analytically, the operation of FC on an input function \( f(x) \) with a reference pattern \( g(x) \) is defined as follows:

\[
C_{P_1, P_2, P_3}(x') = *_{P_1} \{ \mathcal{F}_{P_1}[f(x)] \mathcal{F}_{P_2}^*[g(x)] \},
\]

where \( P_1, P_2, \) and \( P_3 \) are the orders of the FRTs to be performed. Since we use the modified matched RGB filter, hence for a real picture, one can easily see that instead of using a \( P \)-order FRT and then computing its complex conjugate, it is equivalent to perform a \( -P \) fractional order. Therefore, according to what was already mentioned and with respect to the definition of the asymmetric definition of the FC (Refs. 19 and 20), the chosen fractional orders are

\[
P_1 = P \quad P_2 = -P \quad P_3 = 1.
\]

In this case, if the input coincides with the reference object, a perfect phase matching between the FRT of the input and the reference object occurs in the fractional domain. Thus the Fourier transform \((P_3 = 1)\) will obtain the desired peak of detection.

3 Computation Considerations

To obtain the modified matched RGB filter we derive the relation between two different \( P \) ordered FRTs with different wavelengths. Our goal is to write \( \mathcal{F}_{P_1, \lambda_1}[u_0(x_0)](x) \) as a function of \( \mathcal{F}_{P_1, \lambda_1}[u_0(x_0)](x') \), where \( x' \) is a scaled \( x \) coordinate where \( p_i \neq p_j \) and \( \lambda_i \neq \lambda_j \). All of the mathematical formulas are developed here for 1-D FRTs only for reasons of compactness but are applicable for 2-D FRTs also.

\[
\mathcal{F}_{P_1, \lambda_1}[u_0(x_0)](x) = C_1 \int_{-\infty}^{\infty} u_0(x_0) \exp \left( \frac{i \pi (x_0^2 + x^2)}{\lambda f_1 \tan \phi_i} \right) \times \exp \left( -2i \pi \frac{L x x_0}{\lambda_i f_1 \sin \phi_i} \right) \, dx_0,
\]
Therefore one can rewrite Eq. \( \tilde{F}_{p,\lambda}[u_0(x_0)](x) = C_1 \int_{-\infty}^{\infty} u_0(x_0) \exp \left[ \frac{i \pi x^2}{\lambda f_1 \tan \phi_j} \right] \times \exp \left[ \frac{-2 \pi i x_0}{\lambda f_1 \sin \phi_j} \right] \, dx_0, \) (7)

\[ \phi_j = P_i \frac{\pi}{2}. \] (8)

After defining the following relations:

\[ K = \frac{\lambda_i}{\lambda_j}, \] (9)

\[ L = (\cos^2 \phi_i + K^2 \sin^2 \phi_i)^{1/2}, \] (10)

the following expressions can be formulated:

\[ \tan \phi_j = K \tan \phi_i, \] (11)

\[ \lambda_j \sin \phi_i = \lambda_i \sin \phi_j L. \] (12)

Therefore one can rewrite Eq. (7):

\[ \tilde{F}_{p,\lambda}[u_0(x_0)](x) = C_1 \exp \left[ \frac{i \pi (x_0^2 + x^2)}{\lambda f_1 \tan \phi_j} \right] \times \int_{-\infty}^{\infty} u_0(x_0) \exp \left[ \frac{i \pi (x_0^2 + x/L)^2}{\lambda f_1 \tan \phi_j} \right] \times \exp \left[ \frac{-2 \pi i x_0}{\lambda f_1 \sin \phi_j} \right] \, dx_0. \] (13)

The final result obtained between two FRTs with different orders and different wavelengths is

\[ \tilde{F}_{p,\lambda}[u_0(x_0)](x) = c(x) \tilde{F}_{p,\lambda}[u_0(x_0)] \left( \frac{x}{L} \right), \] (14)

where

\[ c(x) = \exp \left[ \frac{i \pi (x/L^2)}{\lambda f_1 \tan \phi_j} \right], \] (15)

\[ P_j = \frac{2}{\pi} \phi_j. \] (16)

Using the relation of Eq. (14) one can find the desired modified matched RGB filter. According to Eq. (1), to compute three FRTs with fractional orders of \( P \) for three different wavelengths, we need to use the following algorithm: Scale the input according to the ratio between the different wavelengths, calculate three FRTs with the same wavelength and same fractional order \( P \) and then finally scale the output by using the same scaling ratios for the different wavelengths. Instead, we use a different algorithm based on the relation of Eq. (14) for the computation process. Here, the computation includes performing three FRTs with the same wavelength but different fractional orders, scaling the obtained result by \( L \), and multiplying it by the function \( c(x) \). Using such mathematical analysis, better accuracy is gained since the scaling (loss of information) is done only at the end of the computation. Note that mathematically both interpretations for calculating the filter are equal, but computationally the preceding method is more accurate.

### 4 Structure of the Modified Matched RGB Filter

The filter placed in the FRT plane includes a summation of a complex conjugate of three terms: the FRT of the red transmittance of the reference object, the FRT of the green transmittance, and the FRT of the blue transmittance. Denoting the red, green, and blue transmittances of the reference object by \( u_R(x_0, y_0) \), \( u_G(x_0, y_0) \), and \( u_B(x_0, y_0) \), respectively, yields

\[ u_p(u, v) = \tilde{F}_{p,\lambda}^R[u_R(x_0, y_0)] + \tilde{F}_{p,\lambda}^G[u_G(x_0, y_0)] + \tilde{F}_{p,\lambda}^B[u_B(x_0, y_0)] \] (17)

where \( u_p(u, v) \) is the filter. The calculation of each of the three terms in the summation is done according to Eq. (14) [calculation of three FRTs with three different fractional orders but identical wavelength, scaling the obtained result, and multiplying it by \( c(x) \)]. When the reference object is placed in the input plane, the correlation expression obtained in the output plane can be formulated as

\[ C(\xi, \eta) = C_{RR}^{ij} \tilde{F}_{p,\lambda_1}^R(\xi, \eta) + C_{GG}^{ij} \tilde{F}_{p,\lambda_2}^G(\xi, \eta) + C_{BB}^{ij} \tilde{F}_{p,\lambda_3}^B(\xi, \eta) \]

\[ + \sum_{i\neq j} C_{ij}^{ij} \tilde{F}_{p,\lambda_1,\lambda_2}^R(\xi, \eta), \] (18)

where \( C_{RR}^{ij} \tilde{F}_{p,\lambda_1}^R(\xi, \eta) \) is the fractional correlation between the red transmittance of the input and the reference objects with the fractional order of \( P \) and the wavelength of \( \lambda_1 \) (red). Here \( C_{ij}^{ij} \tilde{F}_{p,\lambda_1,\lambda_2}^R(\xi, \eta) \) is the overall expression for the cross-terms obtained in the correlation expression. It includes terms of the fractional correlation between the green transmittance of the input and the red transmittance of the reference object or the blue transmittance of the input and the green transmittance of the reference, etc. The expression \( C_{ij}^{ij} \tilde{F}_{p,\lambda_1,\lambda_2}^R(\xi, \eta) \) means

1. performing an FRT with the fractional order \( P \) and wavelength \( \lambda_i \), for the \( i \) transmittance of the input (\( i \) is red, green, or blue)
2. multiplying it by the complex conjugate of an FRT with the fractional order of \( P \) and wavelength of \( \lambda_j \), for the \( j \) transmittance of the reference object (\( j \) is red, green, or blue and is not the same as \( i \))
3. performing an inverse Fourier transform.

The cross-term contribution to the correlation peak is negligible, as shown in the computer simulations.
Note that from Eq. (18), it is seen that the discrimination ability of the suggested FC is improved. The output correlation contains a summation of three autocorrelation terms, each of which contributes a correlation peak that for itself may indicate a detection. The amount of the space variance is controlled by the fractional order \( P \) of the FC.

We suggest two ways to implement the modified matched RGB filter. Each way has its advantages and disadvantages. The first realization is based on the volume reflection hologram (VRH). The VRH is recorded with three tilted plane waves, for the three wavelengths, as illustrated in Fig. 2, so that when using the VRH as a filter its output will be reflected with an angle toward the Z axis. The input undergoes FRT and passes through the filter, which is coded by the VRH scheme. The filter’s output is then reflected backward with the angle as it was coded with. Finally, through the Fourier transform (FT) we obtain the three peaks at the output plane located in the same place. Therefore the final optical setup based on the VRH filter is illustrated in Fig. 3. The advantage of using the VRH implementation is the ability to get all three correlation peaks at the same place easily. A disadvantage of this scheme is that the filter preparation is complicated.

The second realization is based on a computer-generated hologram (CGH). Due to the fact that in this case the desired filter is reconstructed around the first order of diffraction, one can regard this effect as a multiplication of the original filter by a grating with a frequency of \( f_o \). Therefore, the correlation peaks obtained after the FT are shifted, each according to its wavelength, by \( f_o\lambda_j f \), where \( \lambda_j \) are the different wavelengths and \( f \) is the focal length of the lens. Neglecting the cross-terms, the expression obtained is

\[
C(x,y) = C_{\lambda_1}^{RR}(x-f_0\lambda_1 f, y) + C_{\lambda_2}^{GG}(x-f_0\lambda_2 f, y) + C_{\lambda_3}^{BB}(x-f_0\lambda_3 f, y).
\]  

An additional setup must be added to obtain all three correlation peaks at the same position. The setup is illustrated in Fig. 4. To reduce the irrelevant data, a mask that blocks all regions except the one in which the peaks are located is placed right after the output of the fractional correlator. At that point a \( 4f \) system is added with a filter at its Fourier plane, which is conjugate to the grating that multiplies the original filter due to the CGH implementation. The output of the system, after the Fourier transform is

\[
u(u_1, v_1) = \exp(j 2 \pi f_0 u_1) \mathcal{F}\left[ \sum_{j=1}^{3} C_{\lambda_j}^{j}(x,y) \right].
\]

After multiplication with the conjugate grating \( \exp(-j 2 \pi f_0 u_1) \) one obtains

\[
u(u_1, v_1) = \mathcal{F}\left[ \sum_{j=1}^{3} C_{\lambda_j}^{j}(x,y) \right].
\]

Finally, the last FT yields

\[
C(x_1, y_1) = C_{\lambda_1}^{RR}(x_1, y_1) + C_{\lambda_2}^{GG}(x_1, y_1) + C_{\lambda_3}^{BB}(x_1, y_1).
\]

Thus, the shifting factor of the peaks is eliminated. At the output, the peaks are obtained all in the same place at the first order of diffraction and at higher orders. The advantages of using the CGH implementation is the flexibility in the filter preparation.

5 Computer Simulations

To examine the correlator’s ability to detect color input with shift variance control, several computer simulations were performed. The wavelengths, chosen to correspond with RGB colors, are \( \lambda_1 = 0.633 \, \text{\mu m} \) (red), \( \lambda_2 = 0.514 \, \text{\mu m} \) (green), and \( \lambda_3 = 0.450 \, \text{\mu m} \) (blue). The \( P \) order is chosen between 0 to 1 according to the amount of shift variance that is desired in each simulation.

The first simulation is designed to determine the correlator’s output for a given order of \( P = 0.5 \) with the RGB modified matched filter for an input that contains only one of the RGB color transmittances at a time. That way, one can examine the contribution of each channel. This empha-

Fig. 2 Recording the VRH with illumination of three tilted plane waves for the three wavelengths.

Fig. 3 Setup for performing single-output color pattern recognition using an FC based on VRH realization.

Fig. 4 Setup for performing single-output color pattern recognition using an FC based on CGH realization.
sizes the importance of gathering the correlation information from all the channels. Even if one of the channels does not contain a correlation peak contribution that is sufficient for the detection, it can be compensated by the other two channels. Note again that if we have more channels, the correlator’s detection reliability improves. Another aspect of this experiment is our ability to see that the cross-terms of Eq. (18) are negligible. Indeed, one can see that for a single input color, the decomposition of the matched filter’s output is mainly introduced from the corresponding color of the matched filter while the other cross-terms do not contribute to the output peak. The reference scene (and thus the input scene) used is presented in Fig. 5. The results are depicted in Figs. 6, 7, 8, and 9.

The second simulation is designed to examine the fractional correlator working as a simple color correlator with $P = 1$. This means that it is an ordinary case when the correlator is based on the conventional Fourier transform so that the system is fully space shift invariant. The input is a colored picture that has three different objects (the three different butterflies), as shown in Fig. 10. The filter is matched only to one of the objects within the picture (the lower left butterfly). Since this system is fully space shift invariant, we obtain a correlation peak even though the filter and the objects are not placed at the same place. Results are depicted in Figs. 11, 12, 13, and 14. Note that the red channel alone results in a false detection since one of the other butterflies in the picture contains a large amount of energy around the red color. A simple monochromatic correlator based only on the red channel will obtain a false detection in this case. On the other hand, a summation of the three channels yields a detection. This elucidates the importance of gathering information from all the channels.

The third simulation is designed to determine the order $P$ for which the proposed system is partially shift variant. The input scene is the same as in the second simulation. The partial shift variance is defined such that for an input object that is placed within the input scene at a distance of 10 pixels from the reference object used to calculate the filter, the peak’s height will be reduced to half of its height when the input object is placed exactly at the same position as the reference object used to calculate the filter. The order was found to be $P = 0.777$. Figure 15 presents the obtained output when the input object within the picture is placed at the same position as the reference object when $P = 0.777$. Figure 16 presents the obtained output when the input object within the picture is placed at a distance of 10 pixels from the reference object when $P = 0.777$. 

Fig. 5 Reference object used for all the computer simulations.

Fig. 6 (a) Performance with order $P = 0.5$; the contribution of the red channel and the negligence of its cross-terms. (b) Cross section of this result.
Fig. 7 Same as Fig. 6 for the green channel.

Fig. 8 Same as Fig. 6 for the blue channel.

Fig. 9 Same as Fig. 6, where the obtained output is a summation of all channels.
Fig. 10 Input scene used for the second and third computer simulations.

Fig. 11 (a) Performance with order $P=1$ as a simple full space shift invariant color correlator; The contribution of red channel. (b) Cross section of this result.

Fig. 12 Same as Fig. 11 for the green channel.
Fig. 13 Same as Fig. 11 for the blue channel.

Fig. 14 Same as Fig. 11, where the obtained output is a summation of all channels.

Fig. 15 (a) Performance with order $P = 0.777$. The output is obtained when the reference object is placed at the same position as the input object. (b) Cross section of this result.
6 Conclusions
A novel method for color image pattern recognition using an FC was proposed. This method benefits from the fact that it has multichannel input and a single channel output without the need of pre- or postprocessing. The use of multiple channels enhances the discrimination of the detection peak. Since this method is based on the FC its shift variance can be controlled. Computer simulation demonstrated the ability of the proposed approach.

References

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