Gravitational Waves in Non-Minimal Matter-Curvature Coupling Theories

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Outline

- Tachyonic Crash Course on Gravitational Waves (GWs) and gravity theories beyond General Relativity (GR)
- The non-minimal coupling between matter and curvature (NMC)
- GWs in NMC theories

General Relativity

Einstein-Hilbert action:

$$S = \int \left[\kappa R + \mathcal{L}\right] \sqrt{-g} d^4 x \quad , \tag{1}$$

where $\kappa = M_P^2/2$, R is the Ricci scalar curvature, and $\mathcal{L}$ is the matter Lagrangian density. Variation relatively to the metric $g_{\mu\nu}$ yields the field equations:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu} \ , \tag{2}$$

where $R_{\mu\nu}$ is the Ricci tensor, and $T_{\mu\nu}$ is the energy-momentum tensor built from $\mathcal{L}$.

Spacetime tells matter how to move
Matter tells spacetime how to curve

What is a Gravitational Wave?

Solution of linearised Einstein's equations (although they exist at full nonlinear theory):

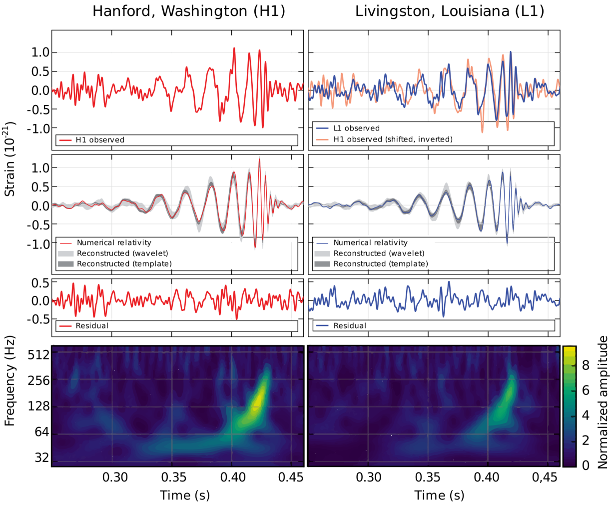
$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \rightarrow \Box\left(h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h\right) = -\frac{8\pi G}{c^4}T^{(0)}_{\mu\nu} \ , \qquad (3)$$

with $h = h^{\mu}_{\mu}$. In vacuum $T_{\mu\nu} = 0$.

Sources:

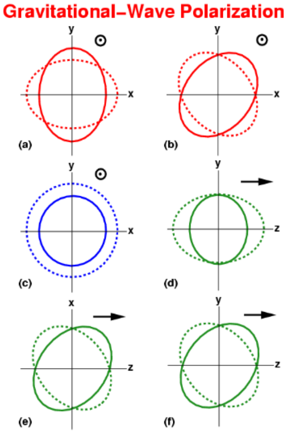
- Black Holes, Neutron Stars, and White Dwarfs binaries;
- Some inflationary models;
- ...

Observation of GWs



Polarisation modes of GWs

A metric theory may present up to six polarisation states [Eardly et al.,1973]



Two tensor modes: $+$ and $\times$ polarisations;

Two scalar modes: breathing and longitudinal modes;

Two vector modes.

GWs in the presence of matter fields

In GR (and theories where only the gravitational sector is modified) we can extend the analysis by resorting to Green functions' method.
Other approaches:

- the Campbell-Morgan formalism of GR (2 polarisation modes) [Ingraham,1997];
- semiclassical theory of electromagnetic response analogue (modified dispersion relation); [Cetoli,Pethick,2011];
- Cyclotron damping and Faraday rotation in collisionless magnetised plasmas [Gali et al 1983, Servin et al 2001];
- presence of a cosmological constant (field equations lose their residual gauge freedom)[Bernabeu et al 2011, Ashtekar et al 2015].

Why not GR?

Successes:

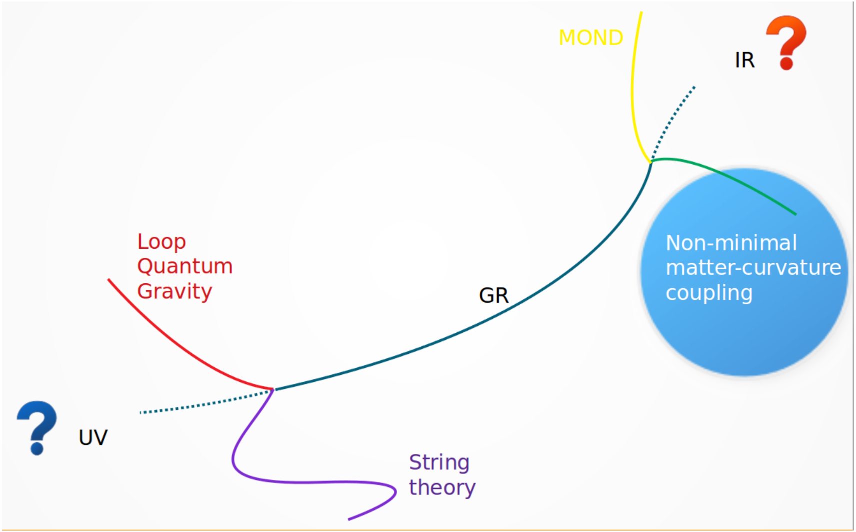
- Solar System constraints;
- GPS ...

But there were still some conundrums:

- Large scale data requires DM and DE;
- It lacks a consistent high energy version.

Alternative theories of gravity:

- f(R)
- Horndeski gravity;
- Jordan-Brans-Dicke;
- NMC [Bertolami, Böhmer, Harko, Lobo 2007]...



[Gomes, **PhD thesis**]
[Bertolami, **What if ... General Relativity is not the theory?**, 2011]

The non-minimal coupling between matter and curvature (NMC) [Bertolami, Böhmer, Harko, Lobo 2007]

$$S = \int \left[\kappa f_1(R) + f_2(R)\mathcal{L}\right]\sqrt{-g}\,d^4x \quad , \tag{4}$$

where $\kappa = M_P^2/2$.
Varying the action relatively to the metric $g_{\mu\nu}$:

$$\begin{aligned} 2\left(\kappa F_1 - F_2\rho\right)\left(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R\right) =& f_2 T_{\mu\nu} + \kappa\left(f_1 - F_1 R\right)g_{\mu\nu} + \\ & + F_2\rho R g_{\mu\nu} + 2\Delta_{\mu\nu}\left(\kappa F_1 - F_2\rho\right) \end{aligned} \tag{5}$$

where $F_i \equiv df_i/dR$, and $\Delta_{\mu\nu} \equiv \nabla_\mu\nabla_\nu - g_{\mu\nu}\Box$.
One recovers GR by setting $f_1(R) = R$ and $f_2(R) = 1$.

Using the Bianchi identities, one finds the covariant non-conservation of the energy-momentum tensor:

$$\nabla_\mu T^{\mu\nu} = \frac{F_2}{f_2} \left(g^{\mu\nu} \mathcal{L} - T^{\mu\nu} \right) \nabla_\mu R \qquad (6)$$

For a perfect fluid, the extra force due to the NMC can be expressed as:

$$f^\mu = \frac{1}{\rho + p} \left[\frac{F_2}{f_2} (\mathcal{L} - p) \nabla_\nu \mathrm{R} + \nabla_\nu p \right] h^{\mu\nu}, \qquad (7)$$

with $h^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu$ being the projection operator, and u^μ is the 4-velocity of the fluid.

Degeneracy-lifting of the Lagrangian choice [O. Bertolami, F. S. N. Lobo, J. Páramos, 2008]

Mimicking Dark Matter (galaxies, clusters) [O. Bertolami, J. Páramos, 2010; O. Bertolami, P. Frazão, J. Páramos, 2013]

Cosmological Perturbations [O. Bertolami, P. Frazão, J. Páramos, 2013]

Modified Layzer-Irvine equation and virial theorem [O. Bertolami, C. Gomes, 2014]

Inflationary dynamics [C. Gomes, O. Bertolami, J.G. Rosa, 2017]

Boltzmann equation [O. Bertolami, C. Gomes, 2020]

Jeans instability [C. Gomes, 2020]

...

Gravitational waves in NMC theories

[Bertolami, Gomes, Lobo, Eur.Phys.J.C 78 (2018) 4, 303]

Linearised field equations around a Minskowskian background for $\mathcal{L} \approx const.$:

$$\begin{aligned}
&(F_1 + 2F_2\mathcal{L}_m)\,\delta R_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}F_1\delta R - \frac{1}{2}h_{\mu\nu}f_1 \\
&- \left[\partial_\mu\partial_\nu - \eta_{\mu\nu}\Box\right]\left(\delta f' + \delta h'\right) = f_2\delta T_{\mu\nu} + F_2 T_{\mu\nu}\delta R \ . \qquad (8)
\end{aligned}$$

and from the trace equation:

$$3\Box\left(\delta f' + \delta h'\right) = \delta f + \delta h \ , \qquad (9)$$

where the fluctuations:

$$\begin{aligned}
\delta f &\equiv (F_1 - 2F_2\mathcal{L}_m + F_2 T)\,\delta R \ , \qquad & \delta h &\equiv f_2\delta T \ , \\
\delta f' &\equiv (F_1' + 2F_2'\mathcal{L}_m)\,\delta R \ , \qquad & \delta h' &\equiv 2F_2\delta\mathcal{L}_m \ . \qquad (10)
\end{aligned}$$

Cosmological constant case, $\mathcal{L} = -\Lambda$:

$$\Box(h_{\mu\nu} - \frac{1}{4} h\eta_{\mu\nu}) = \frac{f_1 - 2f_2\Lambda}{F_1 - 2F_2\Lambda} h_{\mu\nu} \ , \tag{11}$$

where the scalar mode was absorbed into the factor $1/2 \to 1/4$ in the "Λ" gauge:

$$\partial^\mu \left[h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu} h - \eta_{\mu\nu} \left(\frac{\delta f' + \delta h'}{F_1 - 2F_2\Lambda} \right) \right] = 0 \ . \tag{12}$$

This yields a solution of the form:

$$h_{\mu\nu} = A^+ e^{ik_\alpha x^\alpha} e^+_{\mu\nu} + A^\times e^{ik_\alpha x^\alpha} e^\times_{\mu\nu} \ , \tag{13}$$

where A^+ and $A^\times$ are the amplitudes of the “plus” and “cross” polarisations, and $e^+_{\mu\nu}$, $e^\times_{\mu\nu}$ are the usual polarisation tensors. The dispersion relation reads:

$$k_\alpha k^\alpha \equiv \omega^2 - k^2 = \frac{f_1 - 2f_2\Lambda}{F_1 - 2F_2\Lambda} \tag{14}$$

And a propagating scalar mode:

$$\Box\Omega = m_\Omega^2 \Omega \ , \tag{15}$$

with

$$\Omega \equiv \frac{\delta f'}{F_1 - 2F_2\Lambda} = \frac{F_1' - 2F_2'\Lambda}{F_1 - 2F_2\Lambda}\delta R \ , \tag{16}$$

Need for speed:

- The "speed" of the gravitational wave (from the parametrisation $\omega^2 = m_g^2 + c_{gw}^2 k^2 + a\frac{k^4}{\Delta}$) is constrained to $c_{gw} \in [0.55, 1.42]$ [Yunes et al. 2016, Cornish et al 2017]. For these theories $c_{gw} = 1$.
- the group velocity $v_g \equiv \frac{\partial\omega}{\partial k} \approx 1 - \frac{m_{gw}^2}{2k^2}$ is constrained to $v_g \in [1 - 3\times 10^{-15}, 1 + 7\times 10^{-16}]$ [Abbott et al, 2017]. For these theories $v_g \to 1^-$.

Dark-energy-fluid case, $\mathcal{L} = -\rho$:

$$\Box\left(h_{\mu\nu} - \frac{1}{4} h\eta_{\mu\nu}\right) = \frac{f_1 - 2f_2\rho}{F_1 - 2F_2\rho} h_{\mu\nu} \ , \tag{17}$$

where the scalar mode was absorbed into the factor $1/2 \rightarrow 1/4$ in the "Λ" gauge:

$$\partial^\mu \left[h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu} h - \eta_{\mu\nu} \left(\frac{\delta f' + \delta h'}{F_1 - 2F_2\Lambda} \right) \right] = 0 \ . \tag{18}$$

This yields a solution of the form:

$$h_{\mu\nu} = A^+ e^{ik_\alpha x^\alpha} e^+_{\mu\nu} + A^\times e^{ik_\alpha x^\alpha} e^\times_{\mu\nu} \ , \tag{19}$$

with:

$$k_\alpha k^\alpha \equiv \omega^2 - k^2 = \frac{f_1 - 2f_2\rho}{F_1 - 2F_2\rho} \tag{20}$$

Two scalar modes which can be decoupled into:

$$\Box\omega_f = m^2_{\omega_f}\omega_f \ , \tag{21}$$

$$\Box\omega_h = m^2_{\omega_h}\omega_h \ , \tag{22}$$

with

$$\omega_f \equiv \frac{\delta f'}{F_1 - 2F_2\rho} = \frac{F_1' - 2F_2'\rho}{F_1 - 2F_2\rho}\delta R \ , \tag{23}$$

and

$$\omega_h \equiv \frac{\delta h'}{F_1 - 2F_2\rho} = \frac{-2F_2}{F_1 - 2F_2\rho}\delta\rho \ . \tag{24}$$

Newman-Penrose formalism

Complex null tetrad:

$$\mathrm{k} = \frac{1}{\sqrt{2}}\left(\mathrm{e}_t + \mathrm{e}_z\right) \ , \qquad \mathrm{l} = \frac{1}{\sqrt{2}}\left(\mathrm{e}_t - \mathrm{e}_z\right) \ , \tag{25}$$

$$\mathrm{m} = \frac{1}{\sqrt{2}}\left(\mathrm{e}_x + i\mathrm{e}_y\right) \ , \qquad \bar{\mathrm{m}} = \frac{1}{\sqrt{2}}\left(\mathrm{e}_x - i\mathrm{e}_y\right) \ , \tag{26}$$

which obey $-\mathrm{k}\cdot\mathrm{l} = \mathrm{m}\cdot\bar{\mathrm{m}} = 1$ and $\mathrm{k}\cdot\mathrm{m} = \mathrm{k}\cdot\bar{\mathrm{m}} = \mathrm{l}\cdot\mathrm{m} = \mathrm{l}\cdot\bar{\mathrm{m}} = 0$, respectively.
Note that $T_{abc\ldots} = T_{\mu\nu\lambda\ldots}a^{\mu}b^{\nu}c^{\lambda\cdots}$, where $a, b, c, \ldots$ are vectors of the null-complex tetrad basis $(\mathrm{k}, \mathrm{l}, \mathrm{m}, \bar{\mathrm{m}})$, whilst $\mu, \nu, \ldots$ run over the spacetime indices.

The Newman-Penrose quantities in the tetrad basis read [Newman,Penrose,1962]:

$$\Psi_0 \equiv C_{kmkm} = R_{kmkm}$$
$$\Psi_1 \equiv C_{klkm} = R_{klkm} - \frac{R_{km}}{2}$$
$$\Psi_2 \equiv C_{km\bar{m}l} = R_{km\bar{m}l} + \frac{R}{12}$$
$$\Psi_3 \equiv C_{kl\bar{m}l} = R_{kl\bar{m}l} + \frac{R_{l\bar{m}}}{2}$$
$$\Psi_4 \equiv C_{l\bar{m}l\bar{m}} = R_{l\bar{m}l\bar{m}}$$
$$\Phi_{00} \equiv \frac{R_{kk}}{2}$$
$$\Phi_{11} \equiv \frac{R_{kl} + R_{m\bar{m}}}{4}$$
$$\Phi_{22} \equiv \frac{R_{ll}}{2}$$
$$\Phi_{01} \equiv \frac{R_{km}}{2} = \Phi_{10}^* \equiv \left(\frac{R_{k\bar{m}}}{2}\right)^*$$
$$\Phi_{02} \equiv \frac{R_{mm}}{2} = \Phi_{20}^* \equiv \left(\frac{R_{\bar{m}\bar{m}}}{2}\right)^*$$
$$\Phi_{12} \equiv \frac{R_{lm}}{2} = \Phi_{21}^* \equiv \left(\frac{R_{l\bar{m}}}{2}\right)^*$$
$$\tilde{\Lambda} \equiv \frac{R}{24},$$

NP quantities built from the decomposition of the Riemann Tensor in terms of irreducible parts: Weyl tensor, Ricci tensor and scalar curvature.

In **GR**, only Ψ_4 is nonzero $\rightarrow$ polarisations $+$ and $\times$

In **NMC with c.c.** other scalar, vector and tensor modes are also possible ($\Phi_{00}, \Phi_{11}, \Phi_{22}, \tilde{\Lambda} \neq 0$), but full characterisation only when the full solution is known (needed for the Ψ_i).

Conclusions:

- In the far-field (no matter): NMC become pure $f(R)$;
- Other regions: matter $\rightarrow$ so NMC plays a role - Λ, DE-like fluid;
- Extra longitudinal modes (highly non-trivial!)

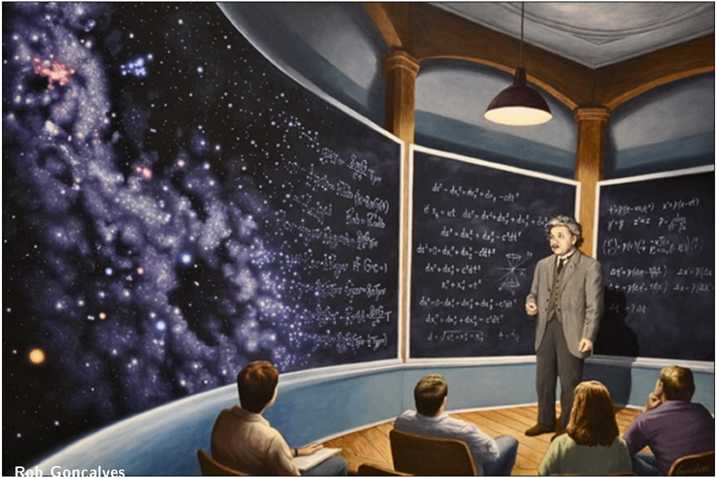
$$\omega = \omega\left(\delta R, \delta \mathcal{L}\right) \tag{27}$$

  which can decouple into two independent modes, under certain conditions.
- Beyond linear level one has to implement the Newman-Penrose formalism (decomposition of the Riemann tensor into its irreducible parts): extra polarisation modes appear.

The Story Untold

- In GR, both metric and "Palatini" approaches lead to the same field equations, and polarisation modes. However, for alternative theories of gravity, this is not the case.
  - metric f(R) theories may present up to six polarisation modes.
  - "Palatini" f(R) only exhibits two tensor modes.
- When matter is included: do matter fields feel the connection built from metric field or the independent connection (e.g. fermions)? Three approaches: metric, Palatini and metric-affine.
- Take home message: matter matters!

Thank you for your attention!



Rob Goncalves