

Search for lensing signatures in the gravitational-wave observations from the first half of LIGO-Virgo's third observing run

Iberian GWs Meeting, 9th June 2021

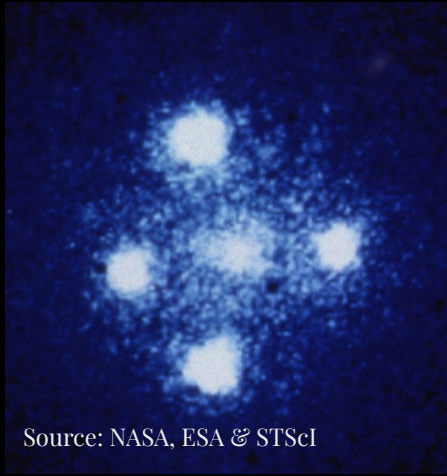
paper: [arXiv:2105.06384](https://arxiv.org/abs/2105.06384)

David Keitel (Universitat de les Illes Balears / IAC3)
*for the LIGO Scientific Collaboration
and the Virgo Collaboration*

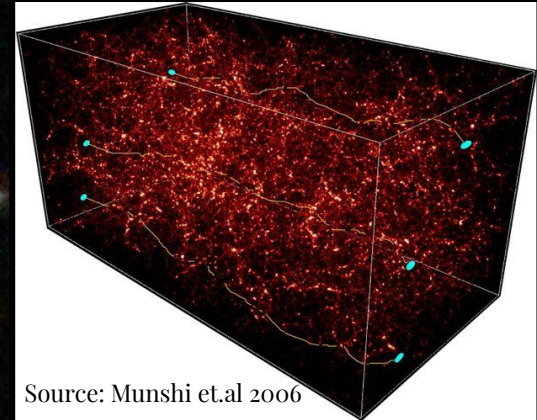
slides: <https://dcc.ligo.org/G2100963-v2>



Gravitational lensing of light



Source: NASA, ESA & STScI

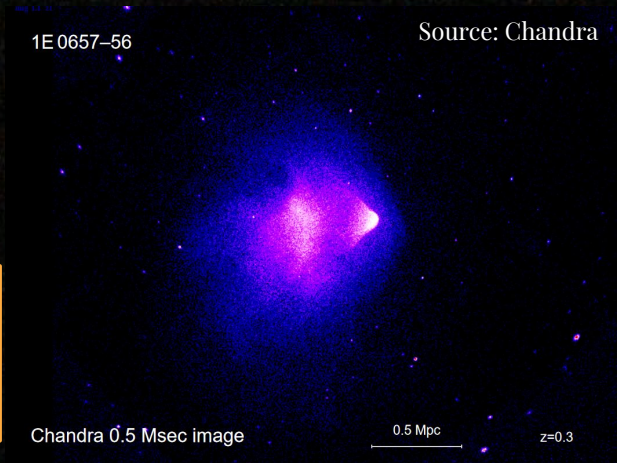


Source: Munshi et.al 2006



Source: NASA

an exceptionally productive tool
across many domains of
astrophysics and cosmology



1E 0657-56

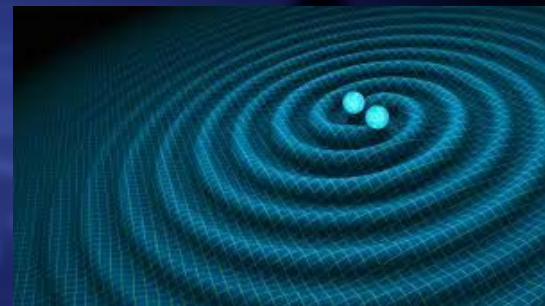
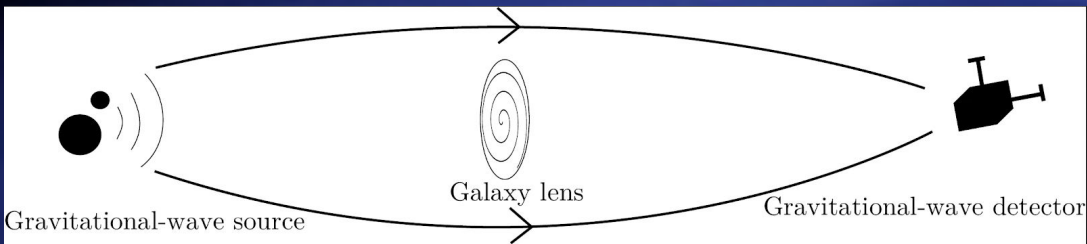
Source: Chandra

Chandra 0.5 Msec image

0.5 Mpc

$z=0.3$

Gravitational lensing of gravitational waves



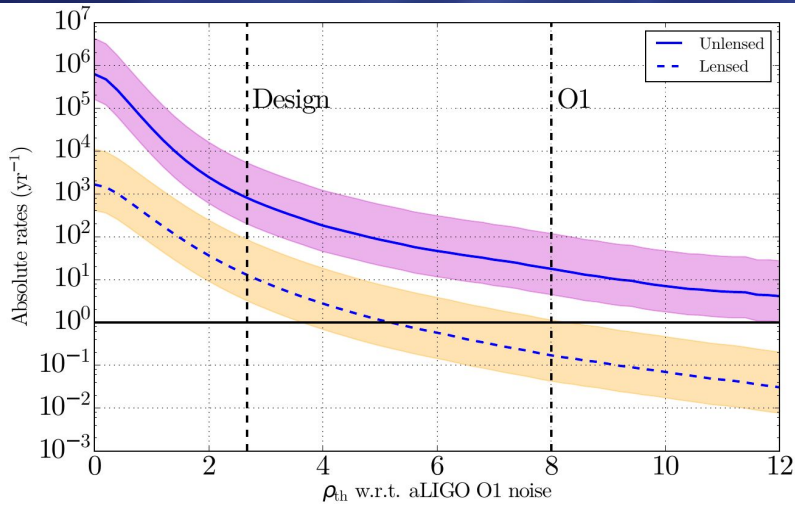
- GWs can be gravitationally lensed just like light
- detection methods and science cases very different than for EM lensing
- GWs experience
 - lensing magnification
 - multiple images
 - frequency-dependent deformations

example science cases in the literature:

- tests of fundamental physics (e.g. speed of light vs speed of GWs)
- localization of merging black holes
- precision cosmology studies from lensing time delays
- microlens population studies (e.g. primordial BHs?)

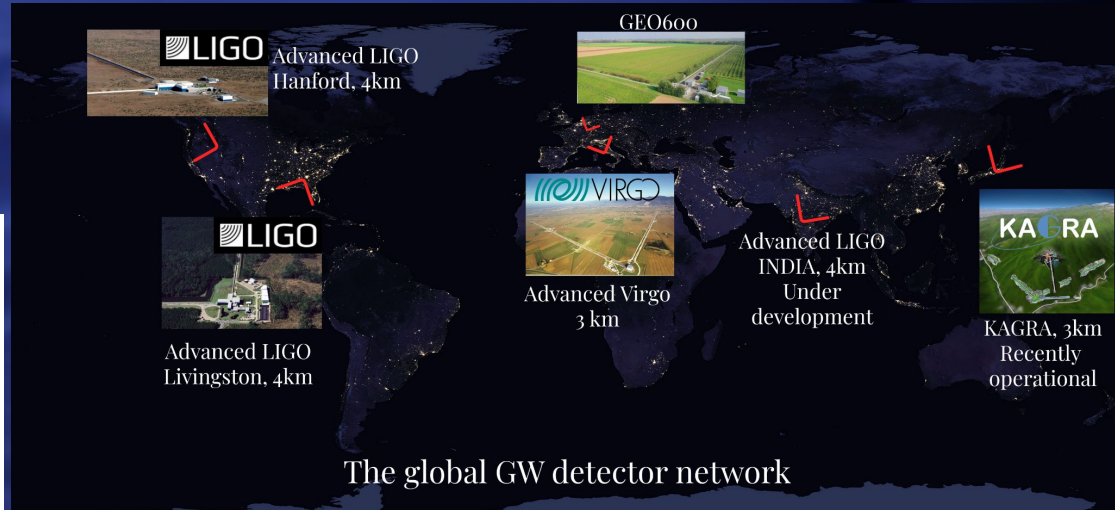
Why is GW lensing exciting (now)?

Sensitivity of global GW detector network rapidly increasing and more sites are getting added.



some recent forecasts in the literature predict strong lensing at a reasonable rate at design sensitivity

[plot: Ng et al. (2017); see also Li et al. (2018), Oguri (2018)]



interest in the community has grown rapidly
searches of O1-O2 data found some intriguing candidates, but no generally recognized evidence for any lensed GWs



[e.g. [Broadhurst+2018/2019/2020](#), [Hannuksela+2019](#), [Li+2019](#), [McIsaac+2019](#), [Dai+2020](#), [Liu+2020](#)]

recent paper: 1st LVC study on GW lensing

arXiv.org > gr-qc > arXiv:2105.06384

General Relativity and Quantum Cosmology

[Submitted on 13 May 2021]

Search for lensing signatures in the gravitational-wave observations from the first half of LIGO-Virgo's third observing run

The LIGO Scientific Collaboration, the Virgo Collaboration: R. Abbott, T. D. Abbott, S. Abraham, F. Acernese, K. Ackley, A. Adams, C. Adams, R. X. Adhikari, V. B. Adya, C. Affeldt, D. Agarwal, M. Agathos, K. Agatsuma, N. Aggarwal, O. D. Aguiar, L. Aiello, A. Ain, P. Ajith, K. M. Aleman, G. Allen, A. Allocca, P. A. Altin, A. Amato, S. Anand, A. Ananyeva, S. B. Anderson, W. G. Anderson, S. V. Angelova, S. Ansoldi, J. M. Antelis, S. Antier, S. Appert, K. Arai, M. C. Araya, J. S. Areeda, M. Arène, N. Arnaud, S. M. Aronson, K. G. Arun, Y. Asali, G. Ashton, S. M. Aston, P. Astone, F. Aubin, P. Aufmuth, K. AultONeal, C. Austin, S. Babak, F. Badaracco, M. K. M. Bader, S. Bae, A. M. Baer, S. Bagnasco, Y. Bai, J. Baird, M. Ball, G. Ballardin, S. W. Ballmer, M. Bals, A. Balsamo, G. Baltus, S. Banagiri, D. Bankar, R. S. Bankar, J. C. Barayoga, C. Barbieri, B. C. Barish, D. Barker, P. Barneo, F. Barone, B. Barr, L. Barsotti, M. Barsuglia, D. Barta, J. Bartlett, M. A. Barton, I. Bartos, R. Bassiri, A. Basti, M. Bawaj, J. C. Bayley, A. C. Baylor, M. Bazzan, B. Bécsy, V. M. Bedakihale, M. Bejger, I. Belahcene, V. Benedetto, D. Beniwal, M. G. Benjamin, T. F. Bennett, J. D. Bentley, M. BenYaala, F. Bergamin, B. K. Berger, S. Bernuzzi, C. P. L. Berry, D. Bersanetti et al. (1279 additional authors not shown)

We search for signatures of gravitational lensing in the gravitational-wave signals from compact binary coalescences detected by Advanced LIGO and Virgo during the first half of the third observing run. We compare the expected rate of lensing at current detector sensitivity and the implications of a non-observation of strong lensing or a stochastic gravitational-wave background. 1) the possibility of multiple images due to strong lensing by galaxies or galaxy clusters; 2) the possibility of multiple images due to strong lensing by galaxies or galaxy clusters; 3) the possibility of multiple images due to strong lensing by galaxies or galaxy clusters. Pairs of signals in the multiple-image analysis show similar parameters and, in this sense, are nominally consistent with the strong lensing hypothesis. However, when we search for lensing signatures in the observed gravitational-wave signals against lensing, these events do not provide sufficient evidence for lensing. Overall, we find no compelling evidence for lensing in the observed gravitational-wave signals.

<https://arxiv.org/abs/2105.06384>

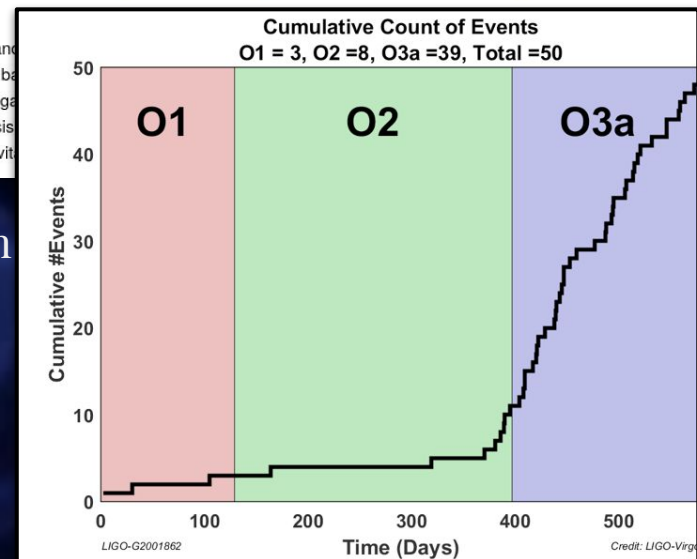
data release:

<https://dcc.ligo.org/P2100173/public>

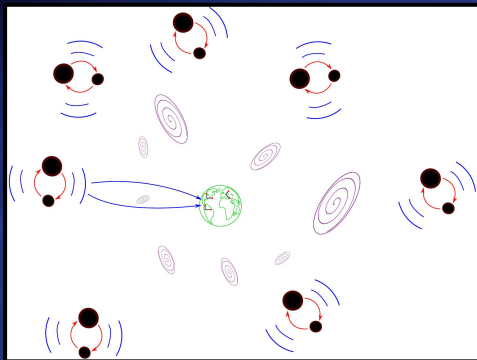
outreach summary:

<https://www.ligo.org/science/Publication-O3aLensing/>

paper focuses on
the *39 events*
from *O3a*
included in
GWTC-2

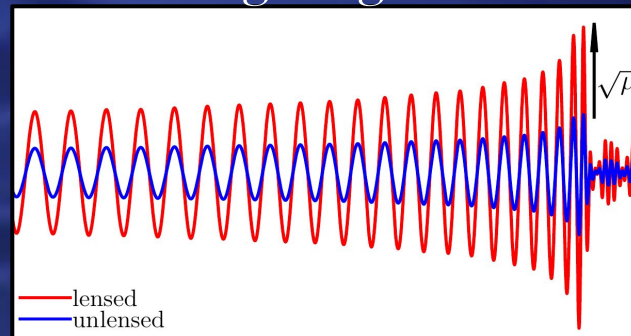


1st LVC study on GW lensing

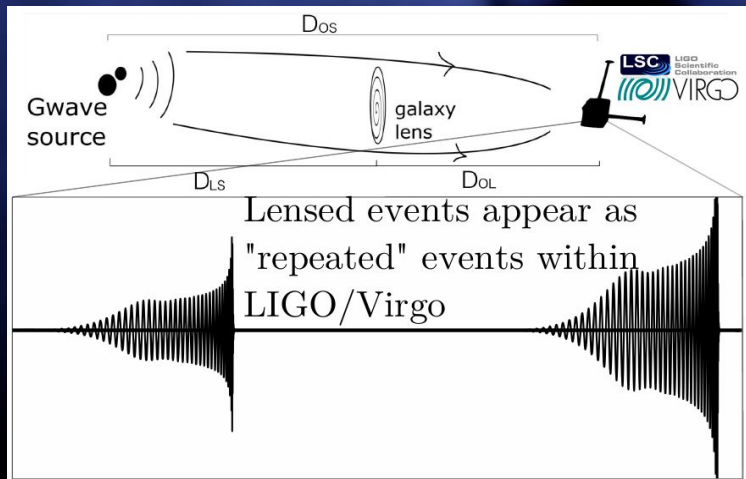


I. lensing statistics

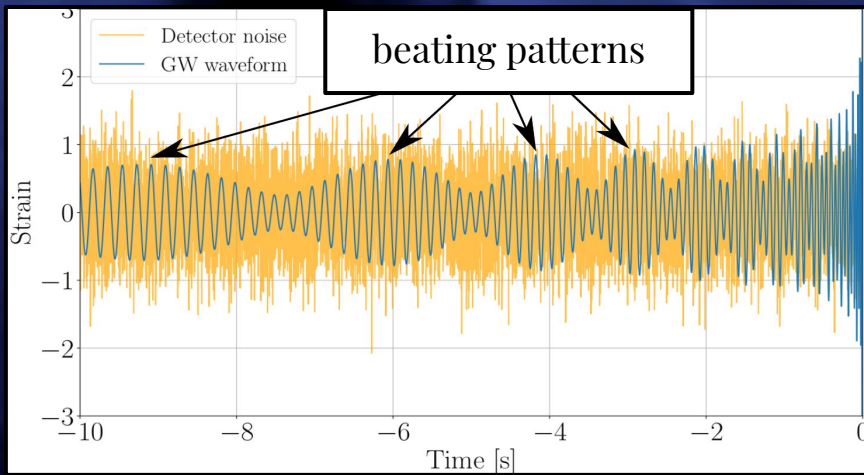
II. lensing magnification



III. strong lensing: multiple images



IV. microlensing distortions



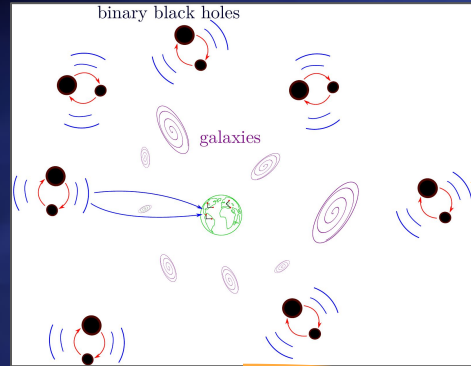
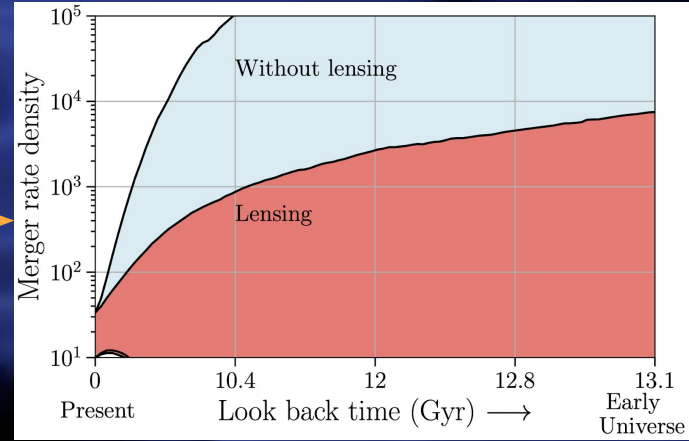
I. lensing statistics

given our understanding of

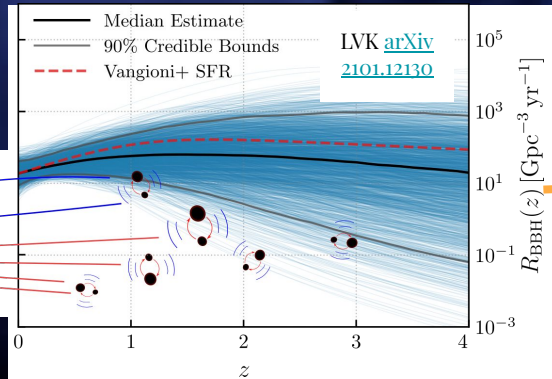
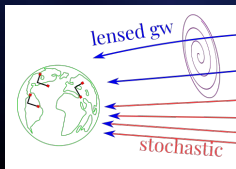
- BBH population
- lens populations

predicted rate of strong lensing:
 $1:10^{3-4}$ events

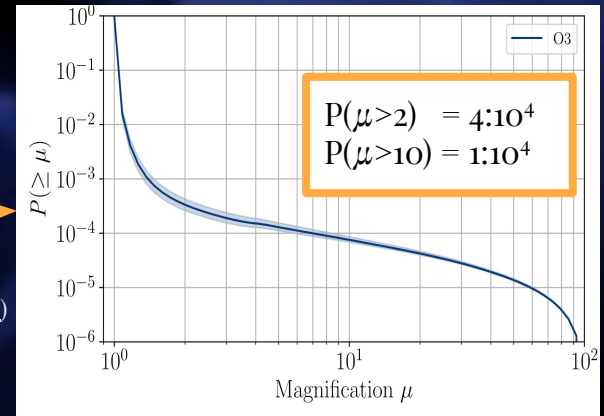
non-detection of lensed events can constrain
high-redshift merger rate density



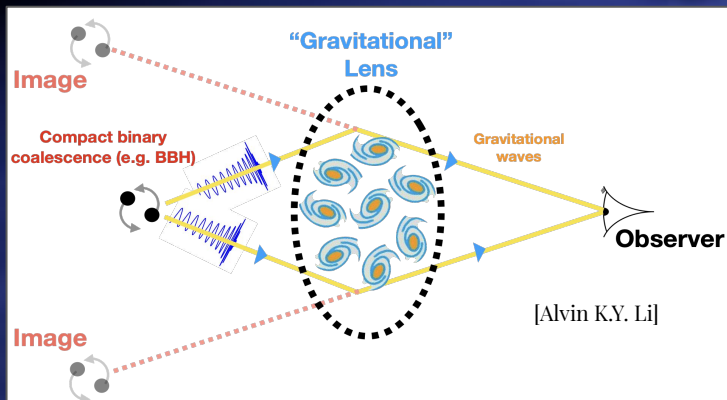
implications on unresolvable CBCs
from stochastic
background searches



- magnification model from [Dai+2017](#)
- parametric fit to weak ([Takahashi+2011](#)) and strong regime ([Hilbert+2008](#))
- based on method in ([Buscicchio+2020](#))



III. strong lensing: multiple images

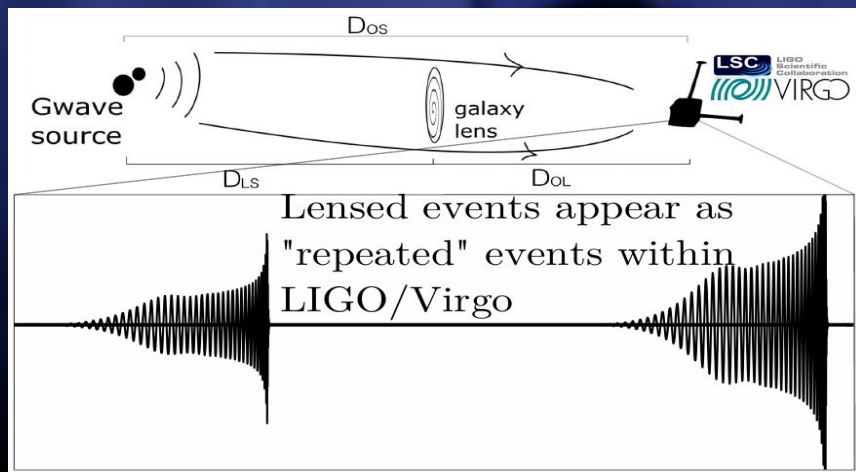


$$\tilde{h}_j^L(f; \theta, \mu_j, \Delta t_j, \Delta \phi_j) = \sqrt{|\mu_j|} h(f; \theta, \Delta t_j) e^{i\Delta \phi_j \text{sign}(f)}$$

magnification

time delay

Morse phase



- inferred luminosity distance and coalescence time different for lensed images of same event
- intrinsic parameters such as masses and spins are expected to be the same
- Morse phase depends on type I/II/III images

III.A posterior-overlap multi-image analysis

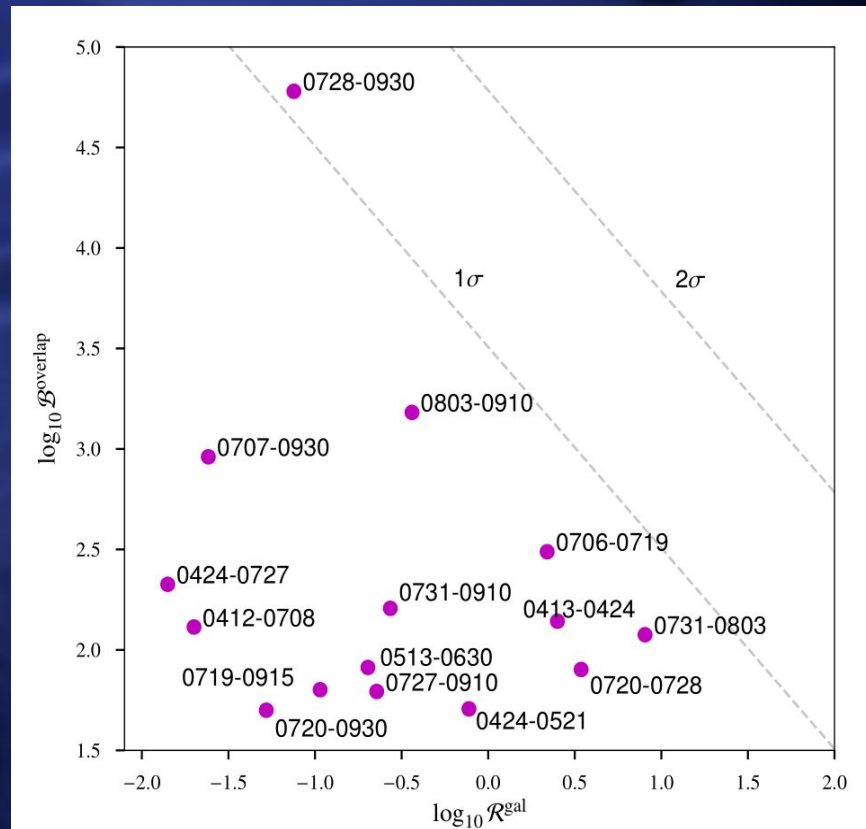
- identify most promising pair candidates (by parameter consistency) using **fast** posterior-overlap method
- **lensed vs. unlensed hypothesis** for a selected pair of signals:

$$\mathcal{B}^{\text{overlap}} = \int d\Theta \frac{p(\Theta|d_1) p(\Theta|d_2)}{p(\Theta)}$$

- also considering time delay for (galaxy-)lensed pairs vs. random coincidences:

$$\mathcal{R}^{\text{gal}} = \frac{p(\Delta t|\mathcal{H}_{\text{SL}})}{p(\Delta t|\mathcal{H}_{\text{U}})}$$

- no significant candidates by these two stats combined
- identified the **19 most promising** candidate pairs by $\mathcal{B}^{\text{overlap}}$ to follow up with more complete joint parameter estimation



III.B joint parameter estimation

- for 19 events with high posterior overlap
- joint Bayesian parameter estimation of the combined data sets with matching intrinsic parameters and sampling in the magnification, time delay and Morse phase
- compared against individual per-event runs with independent intrinsic parameters

$$\tilde{h}_j^L(f; \theta, \mu_j, \Delta t_j, \Delta \phi_j) = \sqrt{|\mu_j|} h(f; \theta, \Delta t_j) e^{i\Delta \phi_j \text{sign}(f)}$$

magnification

time delay

Morse phase

LALInference-based pipeline [Liu+2020]

- aligned-spin quadrupole waveform (IMRPhenomD, 11 parameters)
- Morse phase equivalent to shift in coalescence phase
- one run per possible phase shift (image type)
- also applied to O1+O2 events

bilby-based *hanabi* pipeline [Lo&Magaña2021]

- higher modes + precession included (IMRPhenomXPHM waveform, 15 parameters)
- Morse phase added in frequency domain
- sampling over image types
- includes source and lens population priors
- includes selection effects

III.B joint parameter estimation

Event 1	Event 2	$\log_{10} \mathcal{R}^{\text{gal}}$	$\log_{10}(C_{\text{U}}^{\text{L}})$ LALINFERENCE ($\Delta\phi$: 0, $\pi/2$, π , $3\pi/2$)	$\log_{10}(C_{\text{U}}^{\text{L}} _{\text{pop}})$ HANABI	$\log_{10}(\mathcal{B}_{\text{U}}^{\text{L}})$ HANABI
GW190412	GW190708_232457	-1.7	(+1.0, -9.7, -22.8, -4.4)	-5.6	-8.0
GW190421_213856	GW190910_112807	-	(+4.5, +2.5, -1.5, -0.0)	0.67	-1.8
GW190424_180648	GW190727_060333	-1.9	(+4.9, +0.0, +1.1, +4.0)	0.96	-1.5
GW190424_180648	GW190910_112807	-	(+2.5, +4.7, +4.3, +1.6)	0.62	-1.8
GW190513_205428	GW190630_185205	-0.7	(+0.8, +4.3, -1.9, -6.5)	-0.39	-2.8
GW190706_222641	GW190719_215514	0.34	(+2.4, +2.4, -0.0, -0.5)	0.81	-1.7
GW190707_093326	GW190930_133541	-1.6	(-4.6, -4.3, -3.5, -4.1)	-8.2	-11.
GW190719_215514	GW190915_235702	-1.	(+3.5, -2.1, -0.1, +4.1)	1.4	-1.1
GW190720_000836	GW190728_064510	0.54	(-1.4, -0.9, -4.5, -5.4)	-6.0	-8.5
GW190720_000836	GW190930_133541	-1.3	(-3.5, -2.8, -3.9, -3.9)	-8.2	-11.
GW190728_064510	GW190930_133541	-1.1	(-3.6, -2.5, -3.1, -2.9)	-7.	-9.8
GW190413_052954	GW190424_180648	0.4	(+0.6, -0.9, +0.4, -0.0)	0.35	-2.1
GW190421_213856	GW190731_140936	-2.1	(+3.1, -1.9, +2.5, +5.2)	1.7	-0.79
GW190424_180648	GW190521_074359	-0.1	(+1.3, +3.8, +3.7, +4.4)	-0.64	-3.1
GW190424_180648	GW190803_022701	-2.1	(+4.2, +1.9, +2.6, +3.1)	0.81	-1.7
GW190727_060333	GW190910_112807	-0.6	(+1.8, +3.3, +3.7, +3.4)	0.12	-2.3
GW190731_140936	GW190803_022701	0.9	(+4.1, +3.2, +2.2, +3.4)	1.1	-1.3
GW190731_140936	GW190910_112807	-0.6	(+0.1, +4.5, +0.8, -7.2)	0.92	-2.1
GW190803_022701	GW190910_112807	-0.4	(+4.0, +5.5, +4.7, +2.6)	1.5	-0.98

Coherence ratio C_{U}^{L} :
overlap information

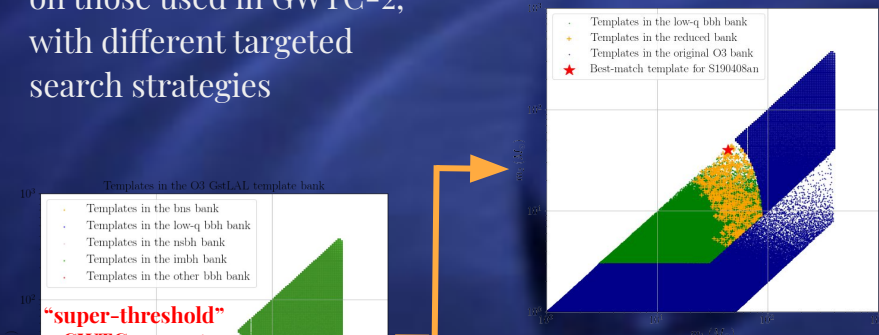
Population-weighted $C_{\text{U}}^{\text{L}}|_{\text{pop}}$: overlap
+ priors on BBH and lens populations

Bayes factor B_{U}^{L} :
overlap + pop. prior
+ selection effects

no evidence of strongly lensed super-threshold pairs in GWTC-2

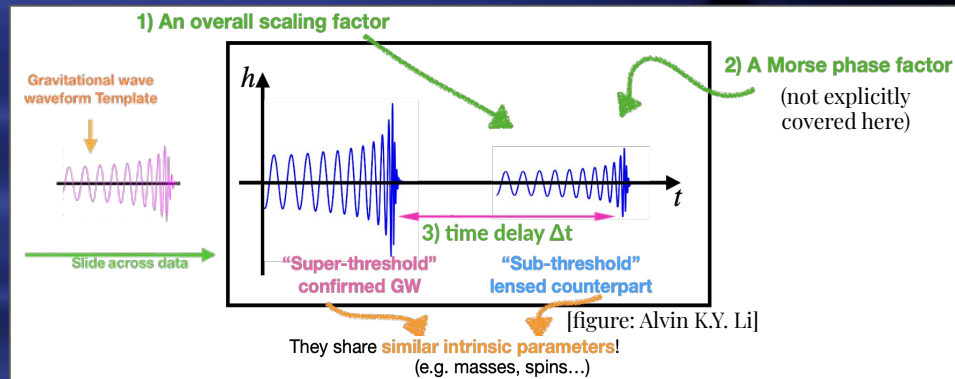
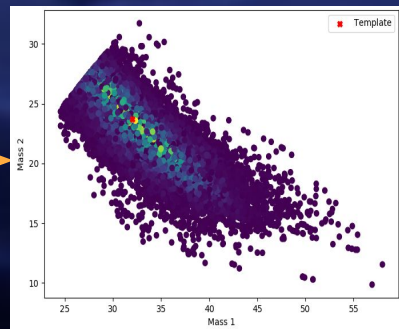
III.C search for sub-threshold lensed images

- strongly lensed events could have **fainter counterparts** not yet identified in wide parameter space searches
- targeted searches can **reduce the noise background**
- two matched-filter pipelines based on those used in GWTC-2, with different targeted search strategies



based on Table II of GWTC-2

figures for illustrative purposes only



1) GSTLAL-based lensing search [Li+2019]

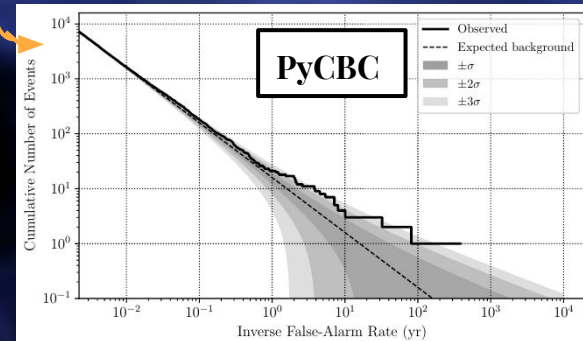
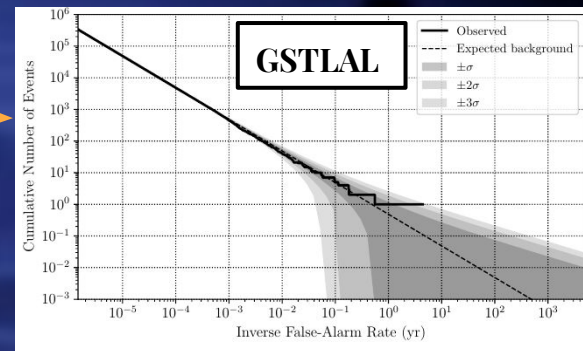
- targeted template banks based on recovery of injections with parameters drawn from GWTC-2 posterior samples

2) PyCBC-based lensing search [McIsaac+2019]

- a single template per target (max-posterior of GWTC-2 samples)

III.C search for sub-threshold lensed images

- false-alarm rates computed from estimated noise backgrounds
- combined results from the 39 searches of each pipeline: *slight observed excess* at high inverse FARs
- pure noise: ~ 2 events expected at $\text{FAR} < 1/16$ yr from $2 * 39$ searches
- *8 new triggers found with FAR < 1/16 yr* (6 of them unique)
- joint-PE follow-up *assuming the triggers are astrophysical* [*]
- some pairs *consistent* with shared parameters, but compared with results for GWTC-2 pairs: **no evidence** for lensing



[*] also found independently in 3-OGC [Nitz+, [arXiv:2105.09151](https://arxiv.org/abs/2105.09151)]

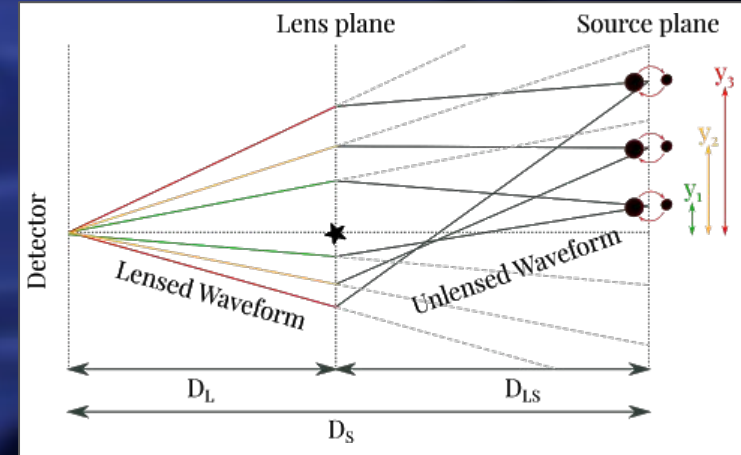
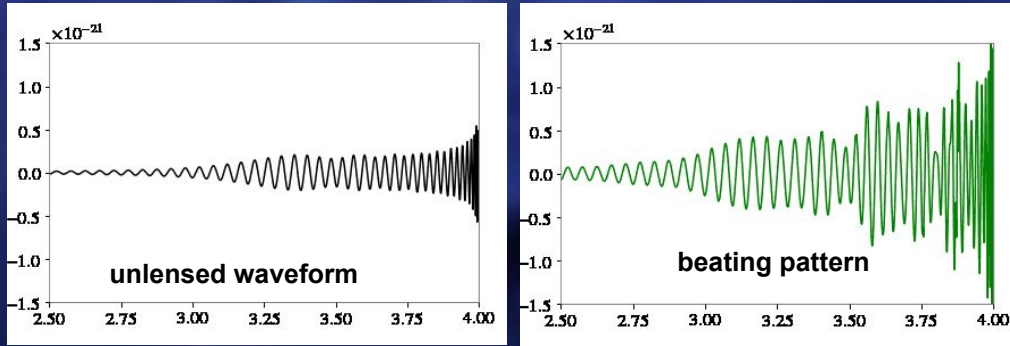
UTC time	GWTC-2 targeted event	$ \Delta t $ [d]	$(1+z)M$ [M_{\odot}]	FAR [yr^{-1}]		$O_{90\%CR}$ [%]	$\log_{10} C_U^L$ (LALINFERENCE) ($\Delta\phi$: 0, $\pi/2$, π , $3\pi/2$)
				PyCBC	GstLAL		
2019 Sep 25 23:28:45*	GW190828_065509	28.69	17.3	0.003	98.681	0.0%	–
2019 Apr 26 19:06:42	GW190424_180648	2.04	65.5	–	0.017	63.8%	(–5.8, –5.8, –5.9, –5.6)
2019 Jul 11 03:07:56	GW190421_213856	80.23	47.7	0.032	0.341	1.2%	(+2.3, +1.1, +1.1, +2.6)
2019 Jul 25 17:47:28 *	GW190728_064510	2.54	9.0	–	0.038	0.0%	–
2019 Jul 11 03:07:56	GW190731_140936	20.46	47.4	0.045	0.944	2.9%	(+2.6, –1.2, –1.6, +0.9)
2019 Aug 05 21:11:37	GW190424_180648	103.13	68.8	–	0.051	26.9%	(–1.1, +0.6, –0.3, –0.7)
2019 Jul 11 03:07:56	GW190909_114149	60.36	49.0	0.053	1.196	12.6%	(+3.5, +2.2, +3.4, +2.9)
2019 Sep 16 20:06:58*	GW190620_030421	88.71	53.3	0.055	1.389	49.5%	(+1.7, +3.6, +2.1, –3.2)

IV. microlensing search

- microlenses (size \sim GW wavelength) \rightarrow *frequency-dependent amplification*

$$h^{ML}(f; \theta_{ML}) = h^U(f; \theta) F(f; M_L^z, y)$$

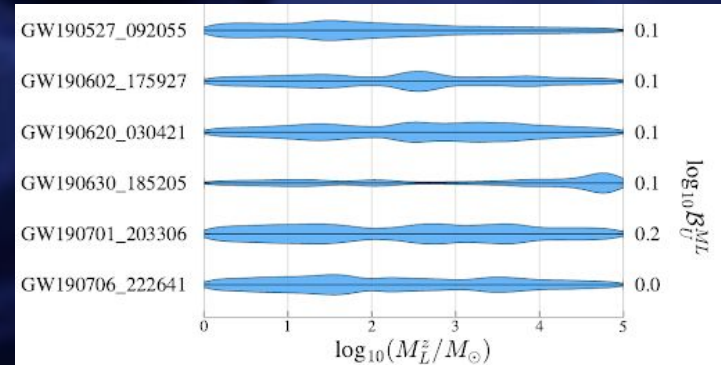
lensed images with time delays $<$ chirp time superpose \rightarrow *beating patterns*
 (more significant when GW passes closer to the lens / smaller y)



[Eungwang Seo, Apratim Ganguly]

- for 36 O3a events (clear BBHs): Bayes factors for lensed (by point-mass lens) vs. unlensed hypotheses, posteriors over lens mass M_L^z
- no well-recovered posteriors, all Bayes factors within the statistical fluctuations expected for unlensed events
- No microlensing effect observed.**

results snippet; see paper for full results on 36 events



conclusions

<https://arxiv.org/abs/2105.06384>

Four gravitational-wave analyses on O3a data:

- statistical forecasts, constraining the rate of lensing and mergers
- analysis of high-mass events under the hypothesis that they might be lensed
- three searches for multiple images from strong lensing
- search for microlensing-induced beating patterns

- First LVC analysis on a topic that is expected to be pursued further with new data (see the [LVK white paper](#)).

- As the current detector network expands and its sensitivity increases, our chances to detect lensing will improve!



Thank you!

I hope I left some time for questions... acknowledgments

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BACKUP SLIDES

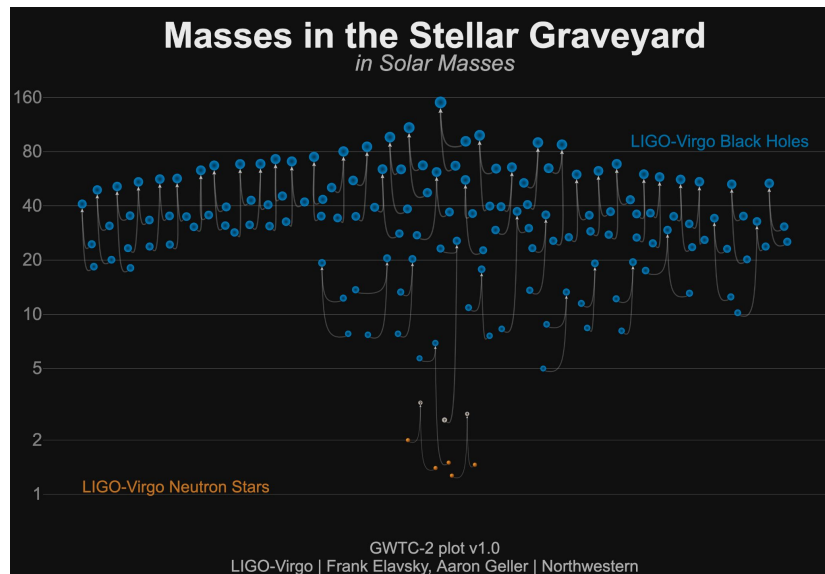
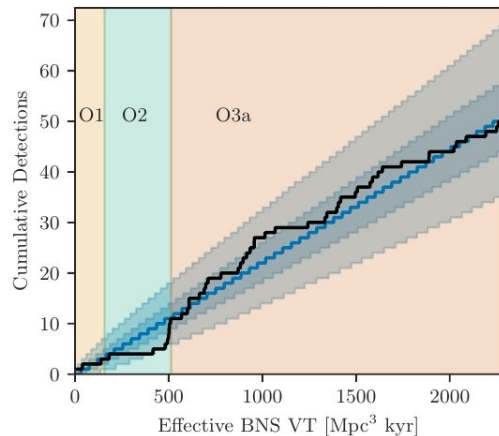
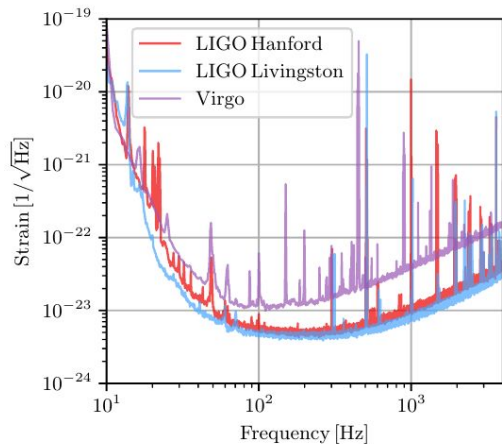
Previous searches for GW lensing

- interest in GW lensing for 2nd generation ground-based detectors grew with discoveries of heavy binary black hole (BBH) mergers [e.g. [Smith+2018](#), [Broadhurst+2018](#)]
- [Hannuksela+2019](#) studied magnification, multiple images, and microlensing in $O_1 \& O_2$ events (GWTC-1).
- lensing scenario also considered for various events/pairs in $O_1 \& O_2$, and low-latency candidates from O_3a [[Broadhurst+2019](#), [Dai+2020](#), [Liu+2020](#), [LVC2020](#), [Pang+2020](#), [Singer+2019](#), [Broadhurst+2020](#)]
- searches for faint sub-threshold counterpart images in $O_1 \& O_2$ [[Li+2019](#), [McIsaac+2019](#), [Dai+2020](#)]
- no generally recognized evidence for any lensed GW events so far, consistent with expected rates
- most promising pair candidate from $O_1 \& O_2$: GW170104–GW170814 [[Hannuksela+2019](#), [Dai+2020](#), [Liu+2020](#)], but would require unusual required lensing geometry; together with low expected lensing rate \gg unlikely to be lensed



LVC O3a results – where we are now

- second GW transient catalogue GWTC-2 [[arxiv:2010.14527](https://arxiv.org/abs/2010.14527)] includes 50 CBC events:
 - 3 from O1
 - 8 from O2
 - **39** from O3a(first half of 3rd observing run, 1 Apr 2019 – 1 Oct 2019)
- increased event rate enabled by unprecedented detector sensitivity thanks to instrumental upgrades and thorough commissioning at all 3 sites

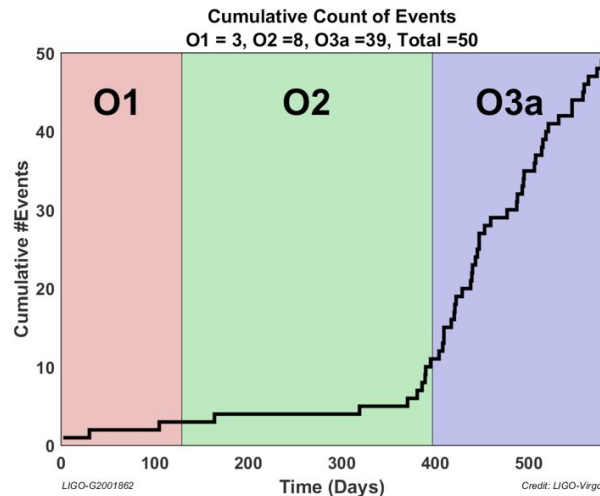
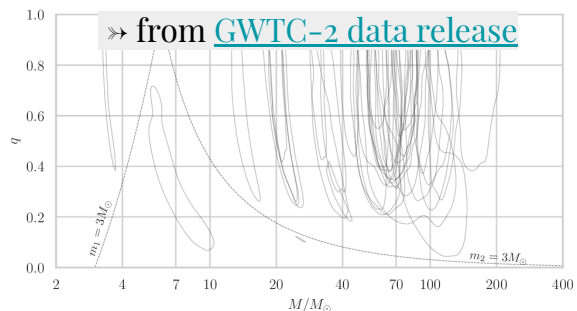


- Stay tuned for more results from O3b!
(Nov 2019 – Mar 2020)

Data and events considered

- paper focuses on the *39 events from O3a* included in GWTC-2

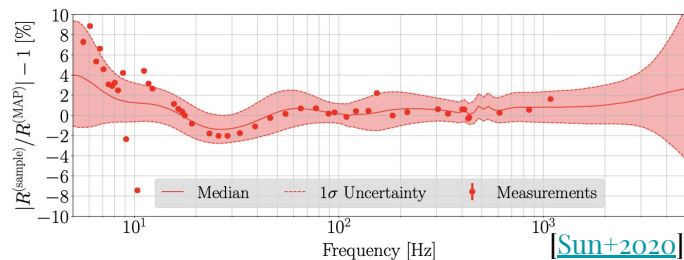
- some analyses use posterior samples from Bayesian inference of the source parameters



- interferometer strain data directly used in:

- **strong-lensing joint-PE analysis** and **microlensing analysis**: short snippets of data with same calibration version, PSDs, and data quality mitigation strategies as in GWTC-2

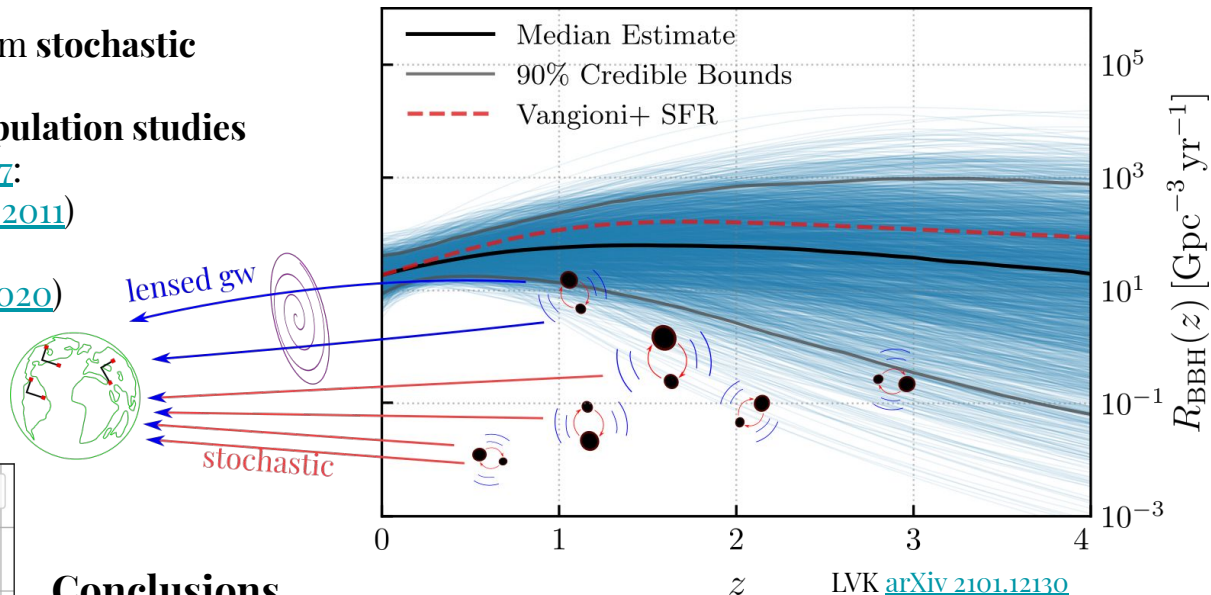
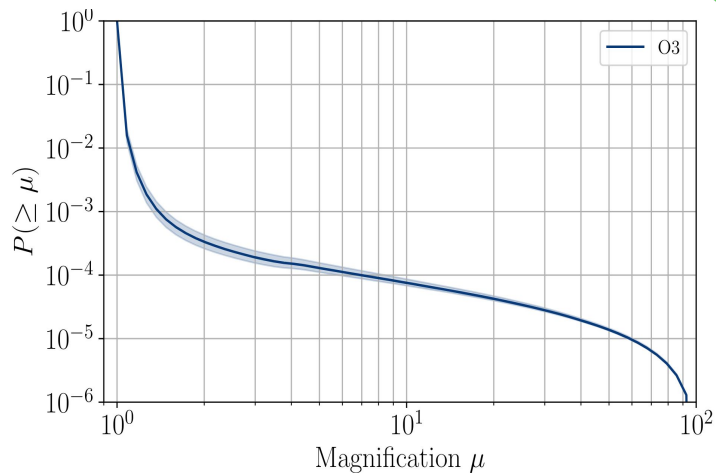
- **subthreshold counterpart searches**: full O3a strain data set, calibration as in PE analyses, and same data quality vetoes as in GWTC-2 searches



- data available from <https://www.gw-open-science.org/O3/O3a/>

I. Lensing statistics (stochastic)

- Implications on unresolvable CBCs from **stochastic background searches**
- Knowledge on detected CBCs from **population studies**
- Magnification model from [Dai et al 2017](#): parametric fit to weak ([Takahashi et al 2011](#)) and strong regime ([Hilbert et al 2008](#))
- Based on method in ([Buscicchio et al 2020](#))



Conclusions

- Expected rate
 - $P(\mu > 2) = 4:10^4$
 - $P(\mu > 10) = 1:10^4$
- Merger rate rate uncertainties in blue shading

III. Search for multiple images

$$\tilde{h}_j^L(f; \theta, \mu_j, \Delta t_j, \Delta \phi_j) = \sqrt{|\mu_j|} h(f; \theta, \Delta t_j) e^{i\Delta \phi_j \text{sign}(f)}$$

Magnification

Time delay

Morse phase

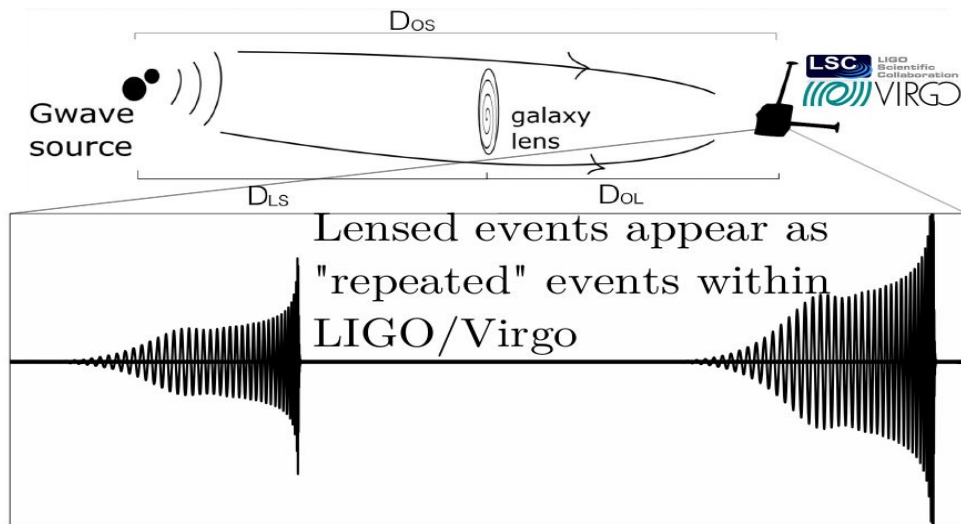


Image types:

Type I: minima $\Delta \phi_j = 0$

Type II: saddle point $\Delta \phi_j = \pi/2$

Type III: maxima $\Delta \phi_j = \pi$

- The inferred luminosity distance and coalescence time would be different for lensed images of an event.
- While intrinsic parameters such as masses and spins are expected to be the same.

III.C search for sub-threshold lensed images

- strongly lensed events could have **fainter counterparts** not yet identified in wide parameter space searches
- targeted searches can **reduce the noise background** thanks to a smaller trials factor when only looking for lensed waveforms that are identical up to the 3 points discussed before:

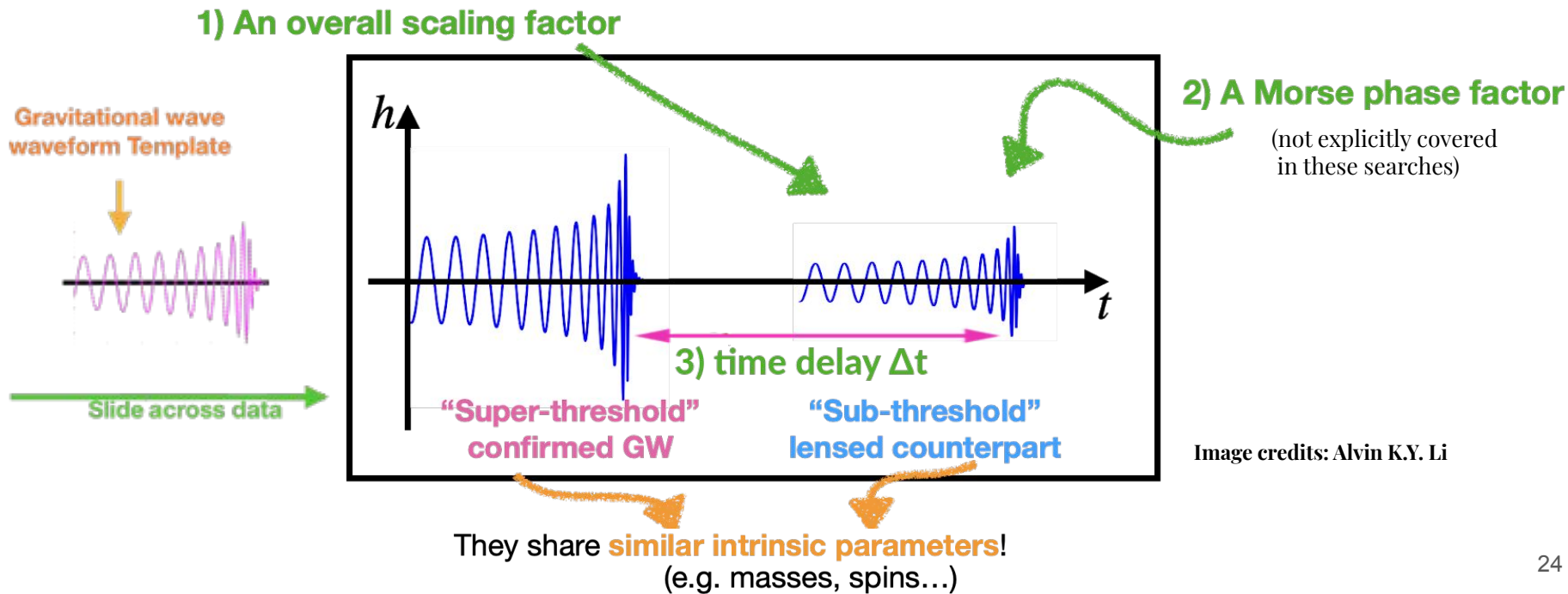
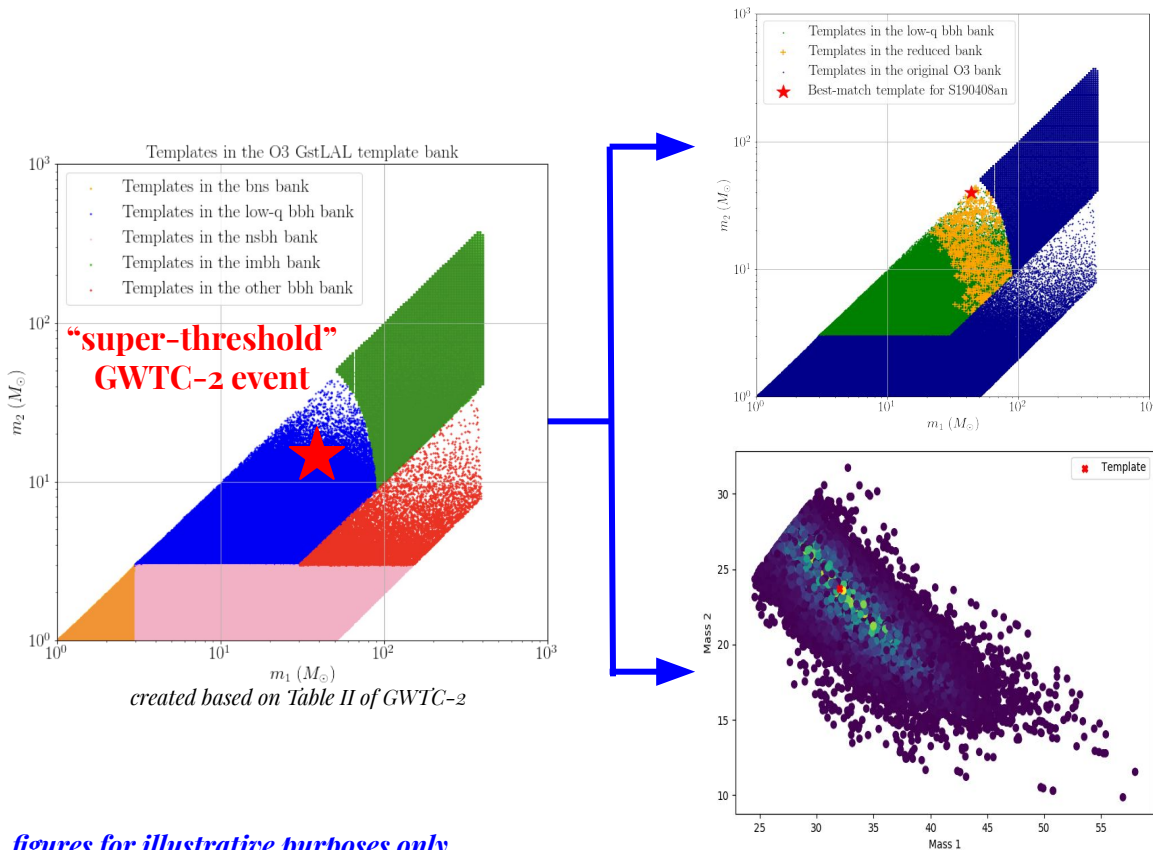


Image credits: Alvin K.Y. Li

III.C search for sub-threshold lensed images

two matched-filter pipelines based on those already used in GWTC-2, two different targeted search strategies



1) GSTLAL lensing search

- based on GSTLAL pipeline [Sachdev+2019; Hanna+2020; Messick+2017]
- lensing adaptation following [Li+2019](#)
- **targeted template banks** based on recovery of injections with parameters drawn from GWTC-2 posterior samples

2) PyCBC lensing search

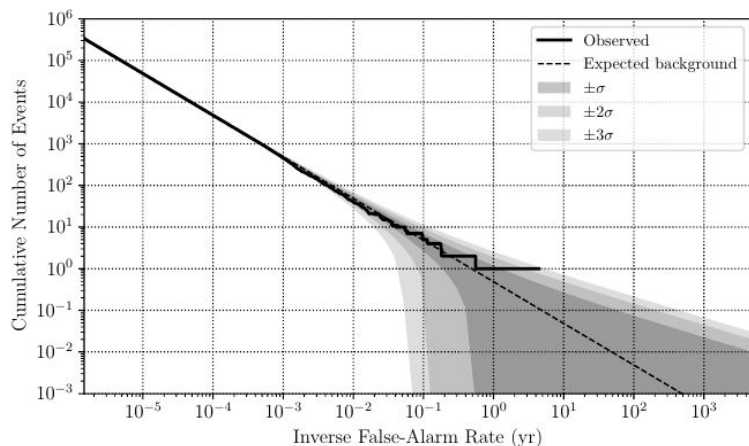
- based on PyCBC pipeline [Allen+2012; Allen+2005; DalCanton+2014; Usman+2016; Nitz+2017]
- lensing adaptation following [McIsaac+2019](#)
- **a single template** per target (max-posterior of GWTC-2 samples)

figures for illustrative purposes only

III.C Search for sub-threshold lensed images

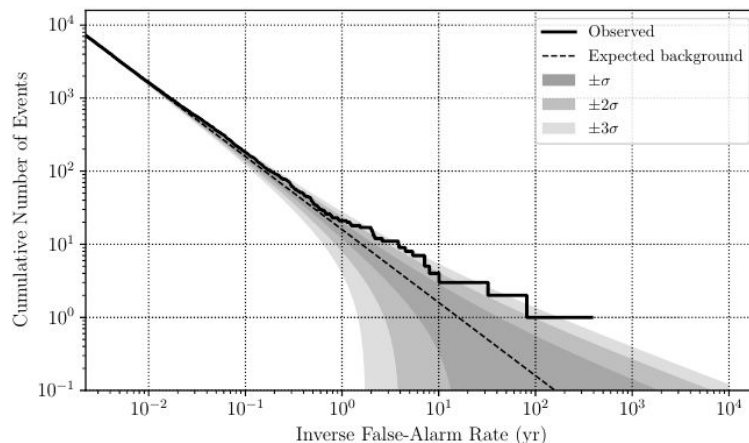
- each pipeline: 39 searches targeted at each of the GWTC-2 events
- combined search results for each pipeline:
 - excluding triggers corresponding to known GWTC-2 events
 - estimated background accounts for trials factor from analysing the data set multiple times

1) GSTLAL results



- FARs from search using combined version of the 39 template banks
- background from non-coincident noise triggers
- from both pipelines: *slight observed excess at low FARs* (high inverse FARs)

2) PyCBC results



- FARs from the individual searches
- for repeated triggers: inverse FARs summed
- background from time-shifted data

III.C Search for sub-threshold lensed images

- pure noise: ~ 2 events expected at $\text{FAR} < 1/16$ yr from combining 39 searches of 2 pipelines
- ***8 new triggers found with FAR < 1/16 yr*** (6 of them unique)
- 6 candidate pairs followed up with LALInference joint PE *assuming the triggers are astrophysical* (we did not calculate a p_{astro} here for whether they are!)
- some pairs *consistent* with shared parameters, but compared with results shown before for GWTC-2 pairs, the obtained C_U^L give ***no evidence*** for lensed pairs

UTC time	GWTC-2 targeted event	$ \Delta t $ [d]	$(1+z)M$ [M_\odot]	FAR [yr^{-1}]		$O_{90\%CR}$ [%]	$\log_{10} C_U^L$ (LALINFERENCE) ($\Delta\phi$: 0, $\pi/2$, π , $3\pi/2$)
				PyCBC	GstLAL		
2019 Sep 25 23:28:45	GW190828_065509	28.69	17.3	0.003	98.681	0.0%	–
2019 Apr 26 19:06:42	GW190424_180648	2.04	65.5	–	0.017	63.8%	(–5.8, –5.8, –5.9, –5.6)
2019 Jul 11 03:07:56	GW190421_213856	80.23	47.7	0.032	0.341	1.2%	(+2.3, +1.1, +1.1, +2.6)
2019 Jul 25 17:47:28	GW190728_064510	2.54	9.0	–	0.038	0.0%	–
2019 Jul 11 03:07:56	GW190731_140936	20.46	47.4	0.045	0.944	2.9%	(+2.6, –1.2, –1.6, +0.9)
2019 Aug 05 21:11:37	GW190424_180648	103.13	68.8	–	0.051	26.9%	(–1.1, +0.6, –0.3, –0.7)
2019 Jul 11 03:07:56	GW190909_114149	60.36	49.0	0.053	1.196	12.6%	(+3.5, +2.2, +3.4, +2.9)
2019 Sep 16 20:06:58	GW190620_030421	88.71	53.3	0.055	1.389	49.5%	(+1.7, +3.6, +2.1, –3.2)

- last pair (highest C_U^L) has $\log_{10}(\mathcal{B}_U^L) = -3.2$ from Hanabi

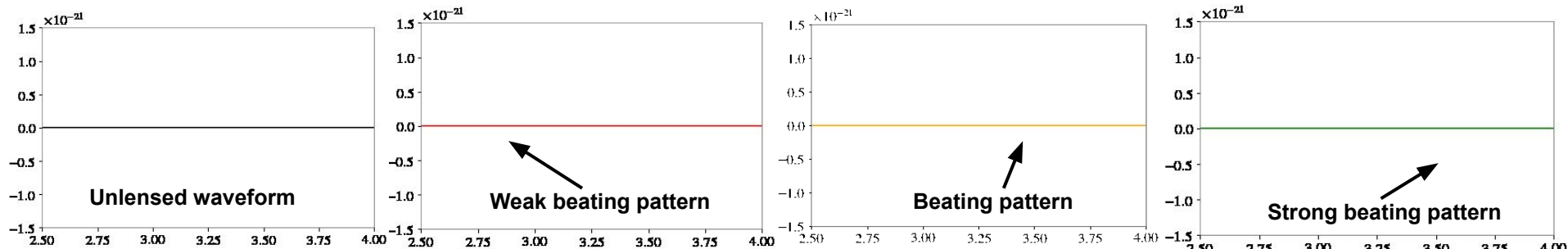
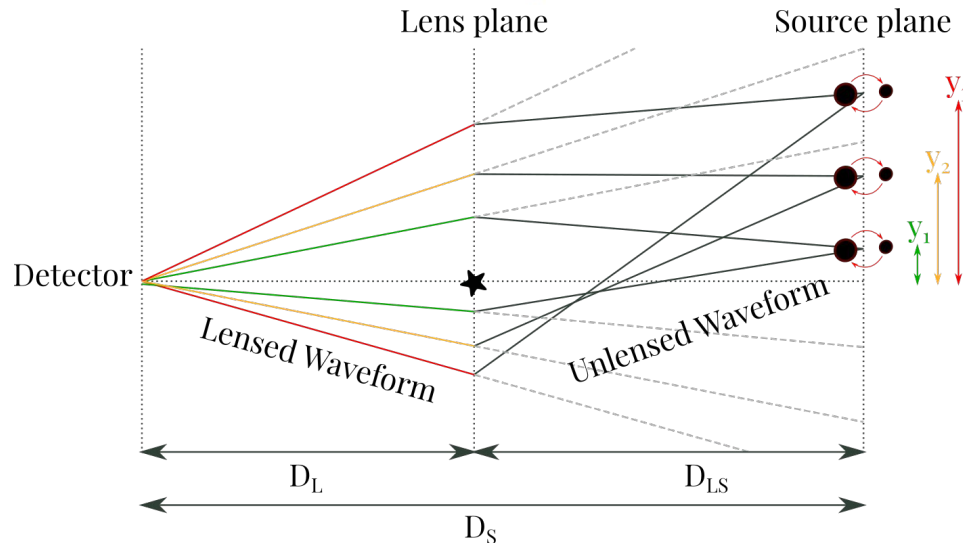
IV. Microlensing search

- Microlenses which are comparable to the GW wavelength can modulate the waveforms by **frequency-dependent amplification factors** which are a function of redshifted lens mass and source position:

$$h^{ML}(f; \theta_{ML}) = h^U(f; \theta) F(f; M_L^z, y)$$

- Lensed images with time delays shorter than the chirp time of the signal superpose to create **beating patterns** which are more significant when the GW passes closer to the lens (ie. smaller y).

3 microlens systems with same M_L^z and different source positions



IV. Microlensing search

- Investigate lensing signatures due to **isolated point mass lens** on O3a events by calculating Bayes' factors between the two (lensed & unlensed) hypotheses.

Results:

- For none of the events are the M_L^Z posteriors well recovered, no high Bayes' factors.
- Bayes factors for all events within the statistical fluctuations expected for unlensed events.
- No microlensing effect observed.**

