New twists in compact binary waveform modelling: a fast time domain model for precession

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Image credit: Rafel Jaume
Compact Binary GW Signals

Compact binary coalescing signals:

- High dimensional parameter space: individual masses, spins, orientation parameters, sky location, matter parameters, eccentricity …
- Knowledge about signals from different approaches during different stages of the evolution:
  - Early inspiral: Post-Newtonian theory, Self-force
  - Numerical relativity: late inspiral, plunge, merger and ringdown
  - Ringdown: BH perturbation theory

Waveform models are crucial for extracting the best info from the detectors data:

- Parameter estimation of source properties
- Tests of general relativity
- Searches/event rates, …

Accurate, general and efficient waveform models needed for the challenges of next observing runs and future observatories (LISA, ET)
**Phenomenological waveform modelling program**

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<tr>
<th>Phenom(eno)logical waveform modelling: accurate and fast representations of GW signals.</th>
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<td>• Extreme compression of available information (PN theory, BH perturbation theory, Numerical Relativity) in terms of fast closed-form expressions for the waveforms.</td>
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In Fourier domain (best suited for most data analysis applications).

Continuous development towards modelling generic CBC signals:

- Non-spinning (PhenomA/B)
- Spin aligned (dominant mode): PhenomC/D/X
- Precessing: PhenomP/Pv2/Pv3/XP
- Higher harmonics: PhenomHM/XHM/Pv3HM/XPHM
- Eccentricity: PhenomXE
- Matter: PhenomNRTidal/NSBH

Motivation for a time domain Phenom family:

- Dispense with the Stationary Phase Approximation (SPA) for modelling the precession transfer functions.
- Closer relation to system dynamics (aims to help in the modelling of generic systems).
- Easier to approximate precessing ringdown.
- Cleaner inspiral-merger-ringdown separation for testing GR.
- While maintaining Phenom philosophy:
  - Efficient and accurate representation of the waveforms.
Phenom modelling in the time domain: non-precessing

GW polarisations decomposed in (spin-weighted) spherical harmonic basis:

\[ h_+(t) - i h_\times(t) = \sum_l \sum_{-l \leq m \leq l} h_{lm}(t) \, 2Y_{lm}(\iota, \phi) \]

Model separately each mode \((2, \pm 2), (2, \pm 1), (3, \pm 3), (4, \pm 4), (5, \pm 5)\):

- Piecewise \(C^1\) expressions for amplitude and phase (derivative) of each mode.

- Inspiral: PN analytical expressions (3.5PN spinning TaylorT3 for orbital frequency, 3PN amplitudes from Blanchet+, 2PN corrections from Buonanno+, 1.5PN corrections from Arun+ + 3.5PN for \((2,2)\) amplitude from Faye+)

- Intermediate/plunge: phenomenological expressions based on hyperbolic functions.

- Ringdown: adaptation of analytical expressions based on QNM decomposition from Damour+


**IMRPhenomT/P:** Estellés et al 2020

**IMRPhenomTHM:** Estellés et al 2020

**IMRPhenomTPHM:** Estellés et al 2021
Phenom modelling in the time domain: precessing

Precessing extension based on “twisting-up” technique:

\[ h_x(t) - i h_y(t) = \sum_{l} \sum_{-l \leq m \leq l} h_{lm}(t) 2Y_{lm}(\phi) \]

- Inertial frames modes obtained from rotation of non-inertial (co-precessing) modes with simpler morphology:

\[ h^I_{lm}(t) = \mathcal{D}^I_{\alpha\beta\gamma}(\alpha(t), \beta(t), \gamma(t)) h^{coprec}_{lm}(t) \]

- Euler angles encode precessing motion of the orbital plane:

\[ \alpha = \arctan(\hat{\ell}_y, \hat{\ell}_x) \]
\[ \beta = \hat{J} \cdot \hat{\ell} = \hat{\ell}_z \]
\[ \gamma = -\dot{\alpha} \cos \beta \]

- Co-precessing modes approximated from non-precessing model (with modified precessing final state):

\[ h^{coprec}_{lm}(t; \chi_1, \chi_2) \approx h^{nonprec}_{lm}(t; \chi_1, \chi_2) \]

\( \hat{\ell} \): Unit vector perpendicular to the orbital plane (Newtonian orbital angular momentum).

\( J \): Total angular momentum of the system.

Credit: Maria de Lluc Planas
Euler angles: analytical approaches

Main analytical approaches to precessing Euler angles:

- Next-to-next-to-leading order (NNLO) (Bohe+) effective single spin.
- Multiscale analysis (MSA) (Chatziioannou+) double spin.

Aimed for more direct comparisons with other Phenom models.

Evaluated with non-precessing analytical orbital frequency:

\[ v(t) = \Omega^{1/3}_{\text{orb}}, \quad \Omega_{\text{orb}} = \frac{1}{2} \omega^{T}_{22} \]

Improvements over previous implementations:

- Numerical evaluation of minimal rotation condition (recovering of nonprecessing limit in MSA).
- Smooth plunge behaviour with linear continuation.
Euler angles: numerical evolution

Numerical evolution approach:

- Solve evolution equations for $\hat{\Omega}$ (implies evolving individual spin vectors):
  \[
  \frac{d\hat{\Omega}}{dt} = \Omega(v(t), q, S_1, S_2) \times \hat{\Omega}
  \]
  \[
  \dot{\Omega} = -\dot{S}_1 - \dot{S}_2
  \]

- Orbit averaged $N^4LO$ PN expressions included.

- Tracking all degrees of freedom: improvement over previous analytical expressions.

- Efficient evaluation in terms of analytical non-precessing orbital frequency: fast implementation.

- Simple analytical approximation attached at ringdown:
  \[
  \alpha^{RD}(t) \simeq (\omega^{RD}_{122} - \omega^{RD}_{121})t + \alpha^{RD}_0
  \]
Comparison with other state-of-the-art waveform models

Unfaithfulness comparison with other state-of-the-art precessing multimode models: IMRPhenomXPHM, SEOBNRv4PHM and NRSur7dq4.

Great agreement with TD models (median $\sim 0.2\%$): more consistent treatment of merger-RD.

Better agreement of numerical approach with SEOB: more accurate inspiral than analytical approaches.

Disagreement for large mass asymmetry and high spins norm: possibly caveat of non-precessing orbital frequency.
Comparison with Numerical Relativity

Unfaithfulness comparison with Numerical Relativity precessing simulations:

- Bulk of cases below 1% mismatch.
- 1 outlier (SXS:0165) with challenging parameters. Need to include further physics (mode asymmetry).

Parameter estimation of NR injected signals:

- Correct recovery at $M_T = 100M_\odot$.
- Individual masses in 90% contour levels for higher masses.
- Need of more detailed systematic studies towards identifying recovery bias.
Parameter estimation: GW190412 re-analysis

GW190412: first reported GW event with confident mass asymmetry: interesting to compare different multimode models.

Non-precessing IMRPhenomTHM employed in published re-analysis (Colleoni+): great consistency with other NR-calibrated models.

Precessing re-analysis:

- Recovered medians and CI consistent with previous results.
- Slightly better agreement with SEOB results (in terms of medians and CIs). (SEOB results obtained with RIFT PE code)
- Higher SNR, likelihood and BF that for Fourier domain models.
More on parameter estimation

Re-analysis of GWTC-1 with a new generation of phenomenological waveform models (Maite Mateu-Lucena, presented today at 12:40):

- Re-analysis with nonprecessing model IMRPhenomTHM for all events and precessing IMRPhenomTPHM for some of them.
- Consistent results with IMRPhenomXPHM, better inference for GW170729.

A detailed analysis of GW190521 with phenomenological waveform models (Marta Colleoni, Friday at 15:10):

- Discussion of recovery of high mass ratio support, higher mode content effects, probability of PISN mass gap, association with AGN flare ZTF19abanrhr ...
Conclusions & future work

New precessing multimode model in the time domain for BBH signals:

- Phenom philosophy: fast and accurate representation of the waveforms.
- Improved inspiral Euler angle description: numerical evolution of spin evolution equations.
- Simple analytical approximation for the ringdown.
- Fast implementation.
- Candidate model for BBH coalescing signals.
- Reviewed by the LVC, publicly available with LALSuite.

Caveats and future improvements:

- Improve efficiency:
  - Inefficient evaluation time for low mass signals.
  - Bottleneck in ringdown evaluation for highly redshifted massive systems.
- Improve physics:
  - Consistent evolution of orbital frequency in terms of the evolving spin magnitudes.
  - Include mode asymmetry effects.
  - Better understanding of precessing ringdown.