Anisotropic inflationary loop quantum cosmology: primordial gravitational waves and predictions for the CMB

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Overview

♦ Most of the models of the early universe are homogeneous and isotropic. Planck observations (2018) did not confirm with strong evidence any departure.

♦ However there is consensus that some anomalies at large scales (dipolar, quadrupolar, etc.) are present, indicating new (pre-)inflationary physics (L. Shamir (2020) reported that a distribution of galaxy spin directions show a quadrupolar-like alignment at more than 5σ).

♦ We will focus on the influence of anisotropies in the pre-inflationary universe (with special attention to tensor modes).

♦ Cosmological perturbation theory on inflationary Bianchi I spacetimes has been studied in great detail (Pereira, Pitrou, Uzan, 2007-2008).

♦ They discuss that anisotropies “break” scale invariance, isotropy (inducing high-order multipoles) and introduce scalar-tensor and tensor-tensor cross-correlations.
Overview

♦ But in classical GR, anisotropies can be large at the onset of inflation (and before). There is no well-posed initial value problem for perturbations.

♦ However, in bouncing inflationary cosmologies, this issue is alleviated (anisotropies are arbitrarily small in the far past).

♦ We complete a Fock quantization for perturbations (with anisotropies treated non perturbatively), and compute their power spectra at the end of inflation.

♦ We find upper bounds on the anisotropies (shear) via constraints on the quadrupolar anomaly reported by Planck Collaboration and discuss new observational effects (generation of $TB$ and $EB$ correlation functions).
Bianchi I spacetimes in LQC

♦ We consider LQC anisotropic bouncing models. Here, in the far past and future spacetime becomes isotropic.

♦ The effective dynamics is determined by (Ashtekar, Wilson-Ewing, Mena-Marugán, Martín-Benito, ...)

\[ G_{\mu\nu} = 8\pi G (T_{\mu\nu} + T_{\mu\nu}^{LQC}) , \]

(1)

♦ The energy density, mean Hubble parameter and shear are bounded above (Gupt, Singh, 2012-2013)

\[ \rho_{\text{max}} = 0.41 \rho_{\text{Pl}}, \quad H_{\text{max}} = \frac{8.34}{\ell_{\text{Pl}}}, \quad \sigma^2_{\text{max}} = \frac{11.57}{\ell_{\text{Pl}}^2}. \]

♦ The background is determined by initial conditions when the mean scale factor \( a(t) \) bounces at \( t = t_B \) (i.e. \( H(t_B) = 0 \)). There we fix \( \sigma^2(t_B), \Psi(t_B), \phi(t_B) \) and the choice for the scale factors \( a(t_B) = 1 \) and \( a_1(t_{\text{end}}) = a_2(t_{\text{end}}) = a_3(t_{\text{end}}) \).
Bianchi I spacetimes in LQC

We are interested in cosmologies with isotropic regimes connected by means of anisotropic ones: bouncing cosmologies.

We have focused in LQC bouncing models. The effective dynamics is determined by

\[ G_{LQC}^{\mu \nu} = 8\pi G T^{\mu \nu}, \]

The energy density, mean Hubble parameter and shear are bounded above (Gupt, Singh, 2012-2013)

\[ \rho_{max} = 0.41 \rho_{Pl}, \]
\[ H_{max} = 8.34 \ell_{Pl}, \]
\[ \sigma^2_{max} = 11.57 \ell_{Pl}^2. \]

The background is determined by initial condition at the bounce \( (H(t_B) = 0) \). There we fix \( \sigma^2(t_B) \), \( \Psi(t_B) \), \( \phi(t_B) \) and the choice for the scale factors \( a(t_B) = 1 \) and \( a_1(t_{end}) = a_2(t_{end}) = a_3(t_{end}) \).

\[ \log \begin{array}{c}
-10^4 & -10^3 & -10^2 & -10^1 & 0 & 10^1 & 10^2 & 10^3 & 10^4 & 10^5 & 10^6 & 10^7 \\
-10^{-23} & -10^{-21} & -10^{-19} & -10^{-17} & -10^{-15} & -10^{-13} & -10^{-11} & -10^{-9} & -10^{-7} & -10^{-5} & -10^{-3} & -10^{-1} & 10^{-1} \end{array} \]

\[ \frac{\dot{\phi}^2}{2}, V(\phi), \frac{\sigma^2}{2\kappa} \]

\[ (\text{log scale}) \quad (\text{linear scale}) \quad (\text{log scale}) \]
Bianchi I: gauge invariant perturbations

- The EOMs of each mode is now given by

$$\ddot{\Gamma}_\mu + 3H \dot{\Gamma}_\mu + \frac{k^2}{a^2} \Gamma_\mu + \frac{1}{a^2} \sum_{\mu' = 0}^{2} U_{\mu \mu'}(\hat{k}) \Gamma_{\mu'} = 0, \quad (2)$$

with $k^2/a^2 = (k_1^2/a_1^2 + k_2^2/a_2^2 + k_3^2/a_3^2)$. Besides, $\Gamma_0$ refers to the scalar mode, $\Gamma_1$ and $\Gamma_2$ to the two tensor (transverse and traceless) polarizations (+ and ×).

- It is more convenient to express the Fourier modes of tensor perturbations in the helicity basis (circular polarization)

$$\Gamma_{\pm 2}(\vec{k}) = \frac{1}{\sqrt{2}} \left( \Gamma_1(\vec{k}) \mp i \Gamma_2(\vec{k}) \right). \quad (3)$$

- Then, we express $\Gamma_s(\vec{k})$ as a linear combination of the elements of the (orthonormal) basis of complex solutions normalized to

$$\sum_{s = 0, \pm 2} v_s(\lambda)(\vec{k})\bar{v}_s(\lambda')(\vec{k}) - \bar{v}_s(\lambda)(\vec{k})v_s(\lambda')(\vec{k}) = -i \frac{4\kappa}{a^3 V_0} \delta^{\lambda\lambda'}. \quad (4)$$
Bianchi I: gauge invariant perturbations

♦ Quantum fields are given by
\[
\hat{\Gamma}_s(\vec{k}) = \sum_{\mu=0}^{2} v^{(\mu)}_s(\vec{k}) \hat{a}_\mu(\vec{k}) + \bar{v}^{(\mu)}_s(-\vec{k}) \hat{a}^\dagger_\mu(-\vec{k}),
\]

\[
[\hat{a}_\mu(\vec{k}), \hat{a}^\dagger_{\mu'}(\vec{k}')] = \delta_{\mu\mu'} \delta_{\vec{k},\vec{k}'}, \quad \hat{a}_\mu(\vec{k}) |0\rangle = 0. \tag{5}
\]

♦ For perturbations, we consider the 0th order adiabatic (also known as massless Minkowski) vacuum state for perturbations at 10^3 Planck secs. before the bounce

\[
v^{(1)}(\vec{k}) = \sqrt{\frac{4\kappa}{a^2 V_0}} \frac{1}{\sqrt{2k}} (1, 0, 0), \quad \dot{v}^{(1)}(\vec{k}) = \sqrt{\frac{4\kappa}{V_0 a^2}} \frac{1}{\sqrt{2k}} \frac{-ik}{\sqrt{2k}} (1, 0, 0),
\]

\[
v^{(2)}(\vec{k}) = \sqrt{\frac{4\kappa}{a^2 V_0}} \frac{1}{\sqrt{2k}} (0, 1, 0), \quad \dot{v}^{(2)}(\vec{k}) = \sqrt{\frac{4\kappa}{V_0 a^2}} \frac{1}{\sqrt{2k}} \frac{-ik}{\sqrt{2k}} (0, 1, 0),
\]

\[
v^{(3)}(\vec{k}) = \sqrt{\frac{4\kappa}{a^2 V_0}} \frac{1}{\sqrt{2k}} (0, 0, 1), \quad \dot{v}^{(3)}(\vec{k}) = \sqrt{\frac{4\kappa}{V_0 a^2}} \frac{1}{\sqrt{2k}} \frac{-ik}{\sqrt{2k}} (0, 0, 1). \tag{6}
\]
Bianchi I: Fock quantization of perturbations

♦ The relevant observables are the power spectra

\[ \langle 0 | \hat{\Gamma}_{ss'}(\vec{k},\vec{k}') | 0 \rangle = \mathcal{V}_0^{-1} \frac{2\pi^2}{k^3} \mathcal{P}_{ss'}(\vec{k}) \delta_{\vec{k}, -\vec{k}'}, \quad \mathcal{P}_{ss'}(\vec{k}) = \mathcal{V}_0 \frac{k^3}{2\pi^2} \sum_{\mu} \left[ v_s^{(\mu)}(\vec{k}) \bar{v}_{s'}^{(\mu)}(\vec{k}) \right] \]

♦ Power spectra satisfy: \( \mathcal{P}_{ss'}(\vec{k}) \) are real and positive if \( s = s' \), otherwise they are complex; \( \bar{\mathcal{P}}_{ss'}(\vec{k}) = \mathcal{P}_{ss'}(-\vec{k}) \) (reality conditions); \( \mathcal{P}_{ss'}(\vec{k}) = \mathcal{P}_{s's}(-\vec{k}) \) (commutation relations). A parity-invariant vacuum state implies \( \mathcal{P}_{ss'}(\vec{k}) = \mathcal{P}_{-s-s'}(-\vec{k}) \) (\( h_{ij} \) is parity invariant)

♦ We compute the power spectra \( \mathcal{P}_{ss'}(\vec{k}) \) at the end of inflation. For convenience

\[ \mathcal{P}_{ss'}(\vec{k}) = \sum_{L=|s-s'|}^{\infty} \sum_{M=-L}^{L} \mathcal{P}^{LM}_{ss'}(k) s-s' Y_{LM}(\hat{k}). \quad (7) \]

with \( s Y_{LM}(\hat{k}) \) the usual spin-weighted spherical harmonics. They are zero when \( L < |s| \) (Therefore, \( \mathcal{P}^{LM}_{ss'}(k) = 0 \) for \( L < |s-s'| \), i.e. only \( \mathcal{P}_{00} \) and \( \mathcal{P}_{22} = \mathcal{P}_{-2-2} \) will contribute when \( L = 0 \)).
Scalar power spectrum

\[ \mathcal{P}_R^{00}(k) \]

\[ A_s(k/k_*)^{n_s-1} \]

Observable region
Scalar power spectrum

\[
\frac{P_{00}(k)}{R(k)} \quad \frac{P_{20}(k)}{R(k)} \quad \Re\left[\frac{P_{21}(k)}{R(k)}\right] \quad \Re\left[\frac{P_{22}(k)}{R(k)}\right]
\]
Quadrupole of $P_{00}(\vec{k})$: constraints on the shear

- Planck collaboration provides constraints on $g_2$ associated to the quadrupolar moments $\mathcal{P}_R^{2M}(k)$, where $\mathcal{P}_R(\vec{k}) \propto \mathcal{P}_{00}(\vec{k})$.

- We can constraint the background parameter space, namely $\sigma^2(t_B)$, $\Psi(t_B)$, and $\phi(t_B)$.

- We find that the minimum allowed value of $\phi(t_B)$ (number of $e$-folds) grows with $\sigma^2(t_B)$ (amount of anisotropies), but it does not strongly depends on $\Psi(t_B)$ (distribution of anisotropies).

\[
\sigma^2(t_B) = 5.45, \quad \Psi(t_B) = 0.0, \quad \phi(t_B) = 1.1.
\]
Tensor power spectrum

\[ \frac{k}{k^*} \]

\[ 10^{-3} \quad 10^{-2} \quad 10^{-1} \]

\[ 10^{-9} \quad 10^{-10} \quad 10^{-11} \]

\[ 10^{-8} \]

\[ \mathcal{P}_{00}^{22}(k) \]
\[ \mathcal{P}_{20}^{22}(k) \]
\[ \mathcal{R}[\mathcal{P}_{21}^{22}(k)] \]
\[ \mathcal{R}[\mathcal{P}_{22}^{22}(k)] \]
Tensor-tensor cross-correlations

Observable region

$\mathbb{R}[P^4_{-22}(k)]$  $\mathbb{R}[P^6_{-22}(k)]$
$\mathbb{I}[P^5_{-22}(k)]$  $\mathbb{I}[P^7_{-22}(k)]$

$k/k^*$

10^{-3} 10^{-2} 10^{-1}
Scalar-tensor cross-correlations

$\frac{k}{k^*}$

Observable region

$\mathcal{P}^{20}_R(k)$
$\mathcal{P}^{30}_R(k)$
$\mathcal{P}^{40}_R(k)$
$\mathcal{P}^{50}_R(k)$
**BB angular correlation functions**

In the case of the correlation functions $TT$, $EE$, $BB$ and $TE$ one has $C_{\ell \ell', mm'}^{XX'} = 0$ if $\ell + \ell'$ is odd.
**TB-EB** angular correlation functions

In the case of the TB and EB correlation functions $C^{BY'}_{\ell\ell',mm'} = 0$ if $\ell + \ell'$ is even.
Summary

♦ We study quantum gauge-invariant cosmological perturbations for anisotropic inflationary spacetimes.

♦ We compute the power spectra within a concrete bouncing inflationary scenario. Here, tensor perturbations show a stronger coupling to anisotropies (enhanced particle production at large scales).

♦ We find upper bounds on anisotropies thanks to the constraints on the quadrupolar anomaly given by Planck Collaboration.

♦ Given the constraints above, we see that $BB$ correlation function shows higher power at low multipoles than the isotropic standard scenario (as a consequence of the enhancement of power of tensor modes at large scales).

♦ Moreover, anisotropies generate angular ($TB$ and $EB$) correlation functions, which would identically vanish in the isotropic limit.