

Equilibrium and Dynamics of Remnants of Binary Neutron Star Mergers

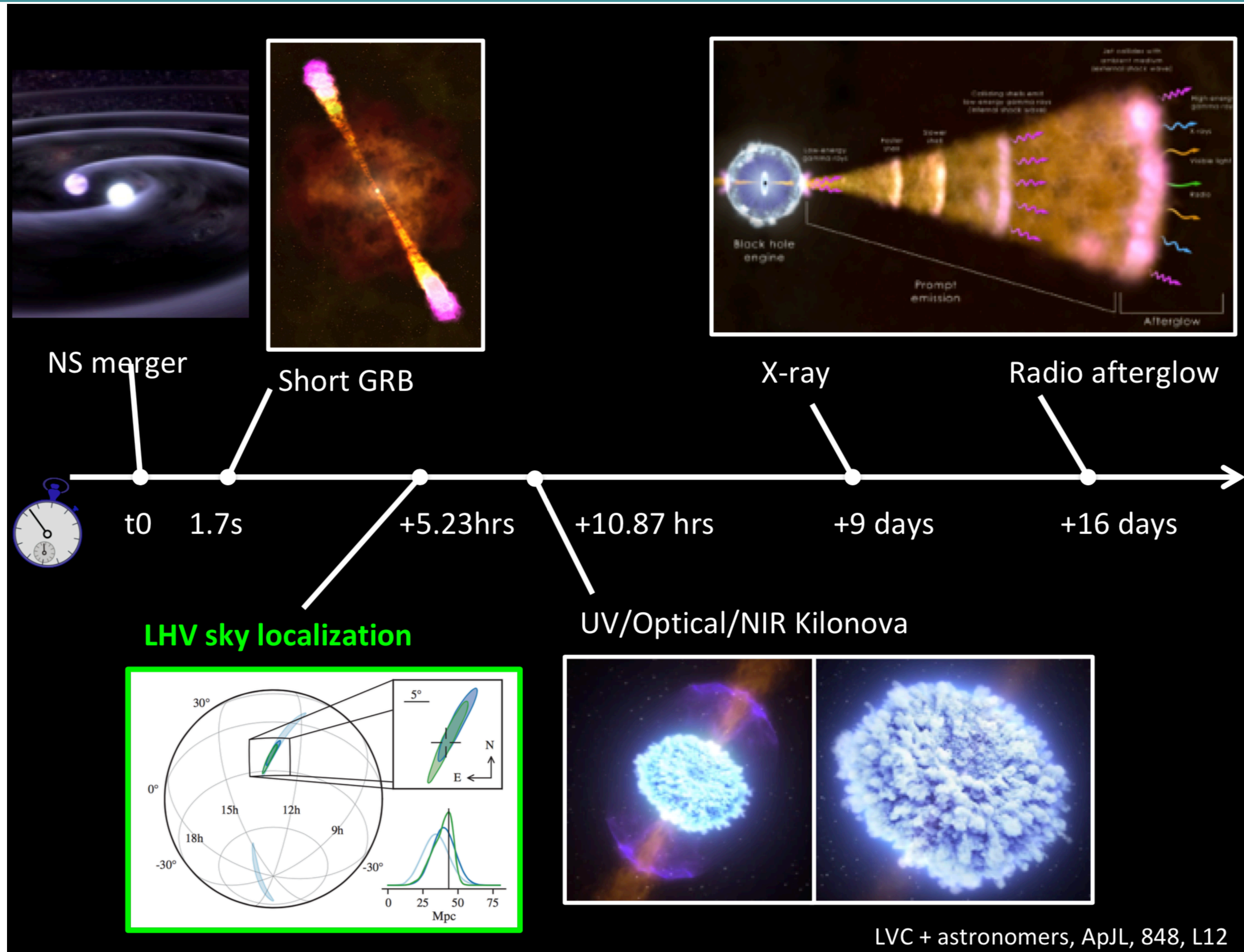
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GW170817 BINARY NEUTRON STAR MERGER



LVC + astronomers, ApJL, 848, L12

M. Branchesi

PLANNED OBSERVATIONS FOR THE NEXT FEW YEARS

Next observing run: O4
(starting date
~ summer 2022)

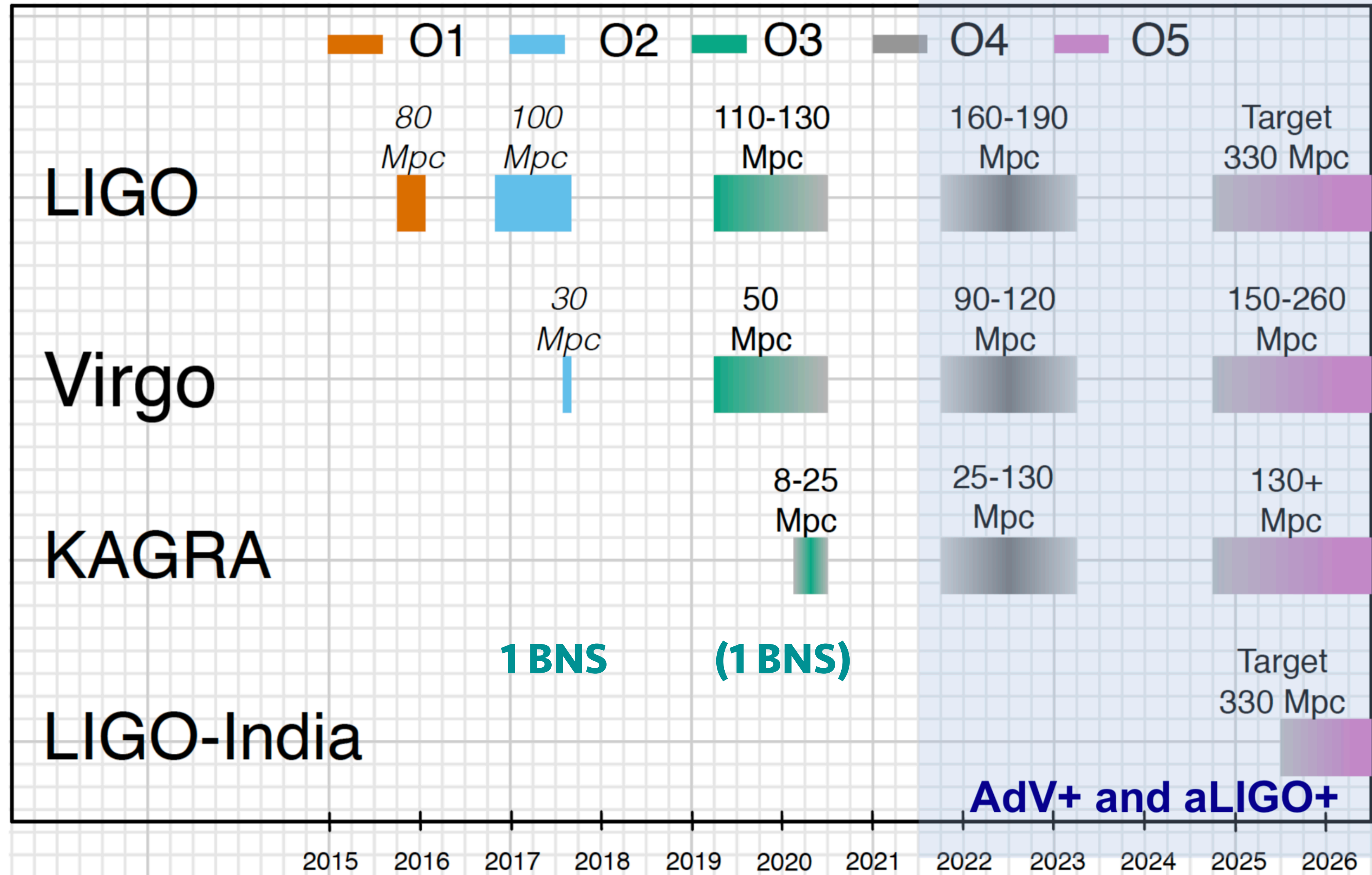
We may expect

$$10^{+52}_{-10}$$

BNS detections with the
HLVK network.

O5 may reach ~2 x further,
or ~8 x volume

[Abbott et al. LRR \(2020\)](#)



DETECTION EFFICIENCY OF BNS MERGERS WITH 3G DETECTORS

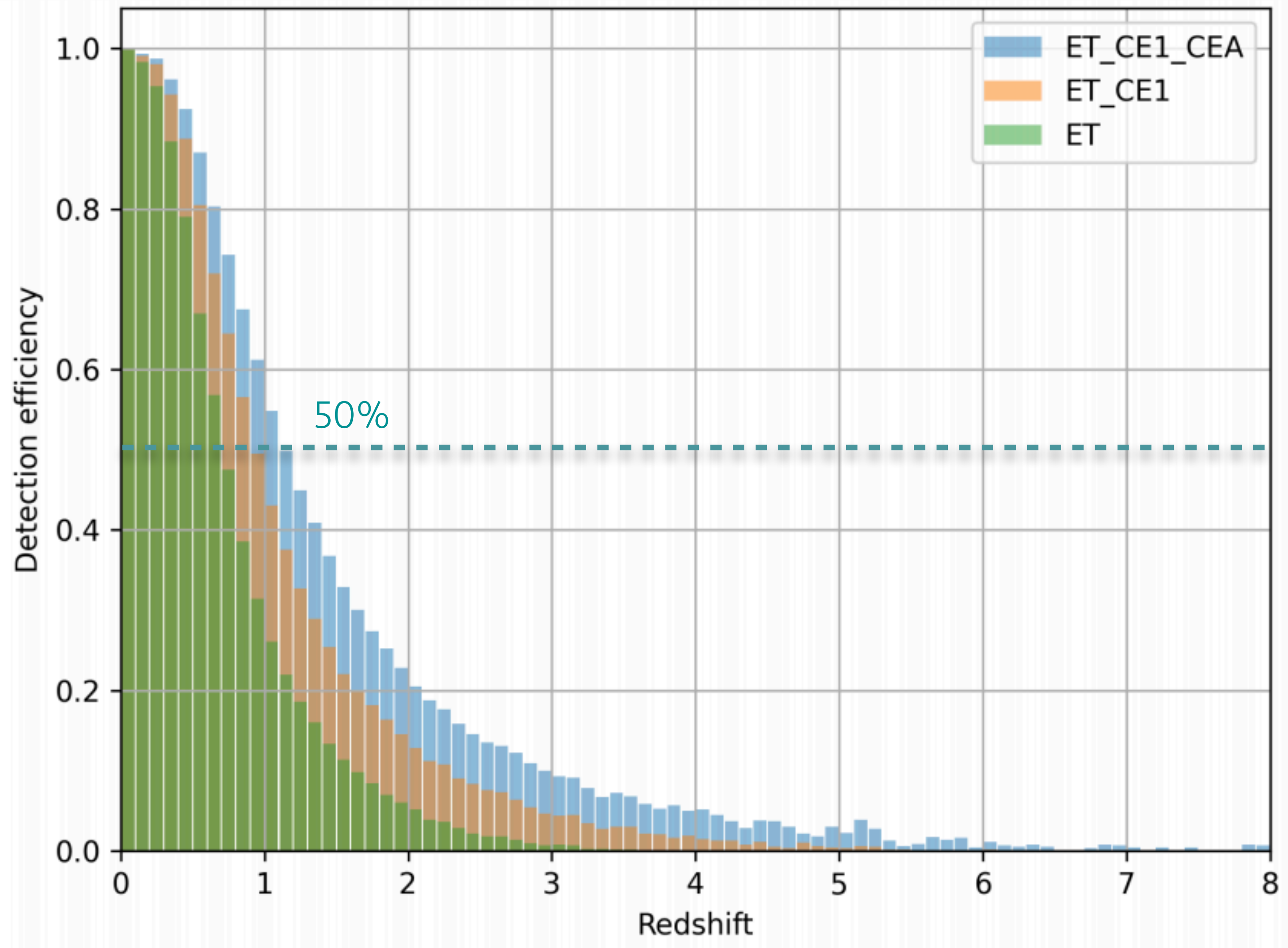
Detection efficiency of BNS mergers with various networks of 3G detectors.

(Assuming a population randomly oriented and located in space and detection SNR of 8.)

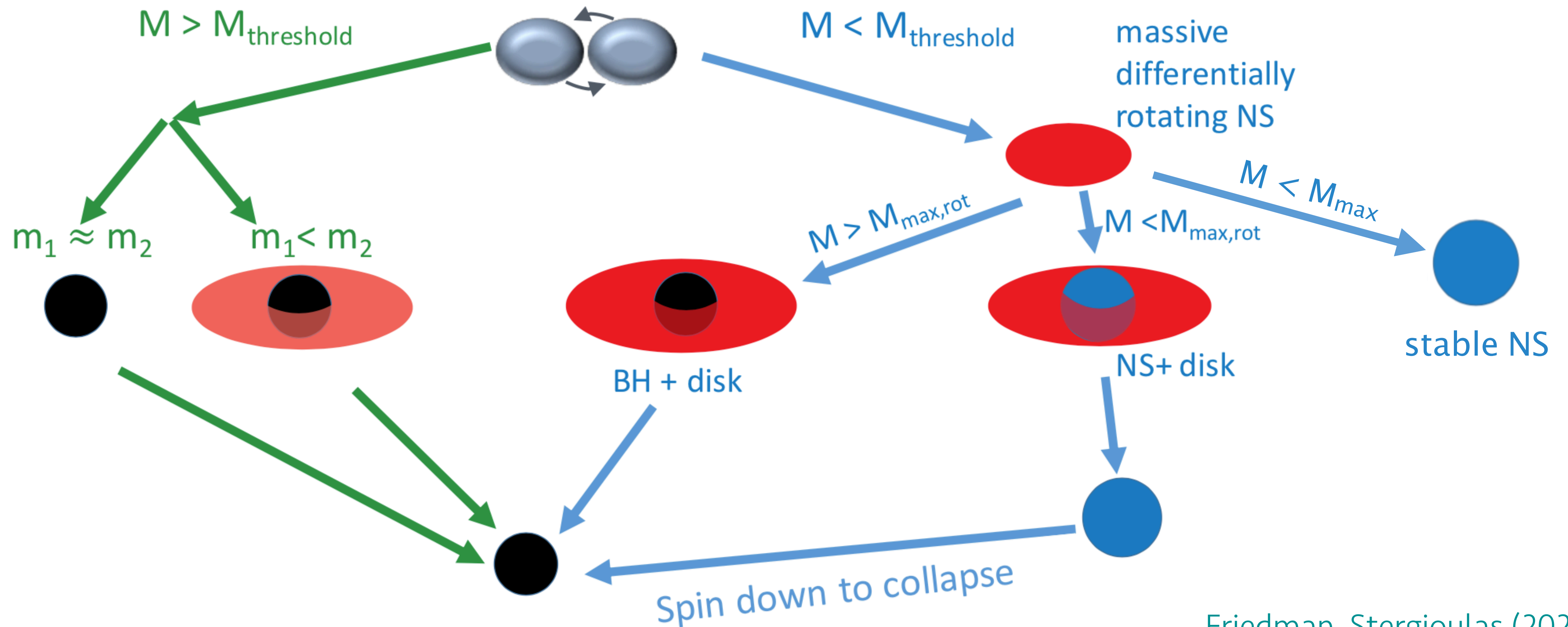
Rosati et al. [arxiv2104.09535](https://arxiv.org/abs/2104.09535) (2021)

Joint GW + sGRB detections with THESEUS: up to *a few tens per year* (incl. aligned and misaligned jet w.r.t. the observer)

Ciolfi et al. [arxiv2104.09534](https://arxiv.org/abs/2104.09534) (2021)



POSSIBLE OUTCOMES OF BNS MERGERS



Friedman, Stergioulas (2020)

Significant differences in post-merger E/M emission, depending on the outcome of BNS mergers
The accurate determination of M_{thres} is important for GW and multi-messenger astronomy.

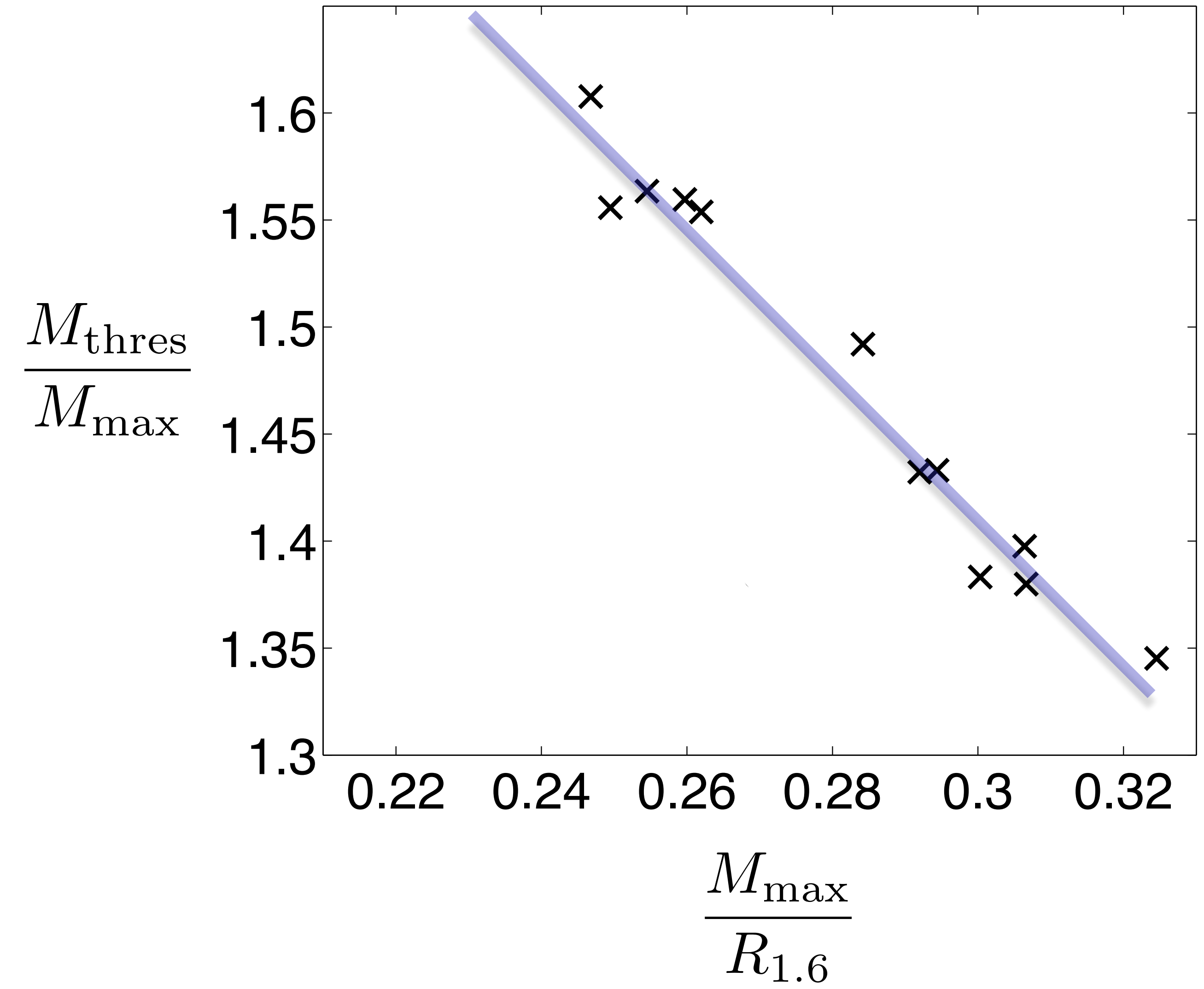
THRESHOLD MASS TO PROMPT COLLAPSE

Simulations with SPH+CFC code in GR:

- Equal mass mergers
- 12 fully temperature-dependent EOS
- 340k SPH particles

Empirical relation for the threshold mass to prompt collapse, in terms of the maximum TOV mass and the radius of $1.6 M_{\odot}$ stars:

$$M_{\text{thres}} = \left(-3.606 \frac{GM_{\text{max}}}{c^2 R_{1.6}} + 2.38 \right) \cdot M_{\text{max}}$$



Bauswein, Baumgarte, Janka (2013)

EXTENSION OF EMPIRICAL RELATION FOR THRESHOLD MASS

Extended set of simulations with SPH+CFC code in GR:

- 33 hadronic EOS (fully temperature dependent or with an added thermal part)
- Mass ratio $0.7 \leq q \leq 1$
- Threshold mass determined within $0.025M_{\odot}$

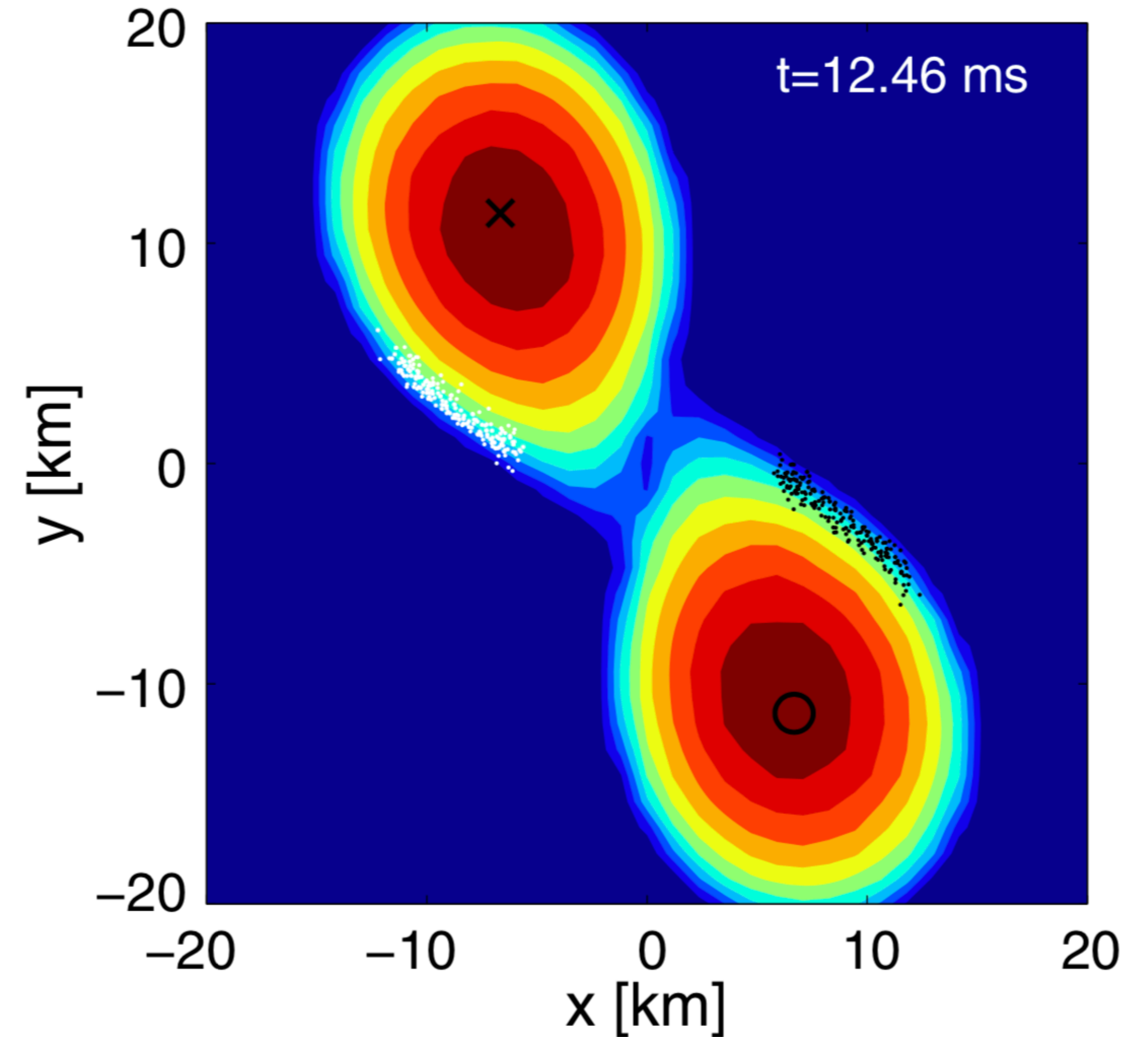
New, bilinear empirical relations of the form

$$M_{\text{thres}} (M_{\odot}) = aM_{\text{max}} + bR_{1.6} - c$$

or

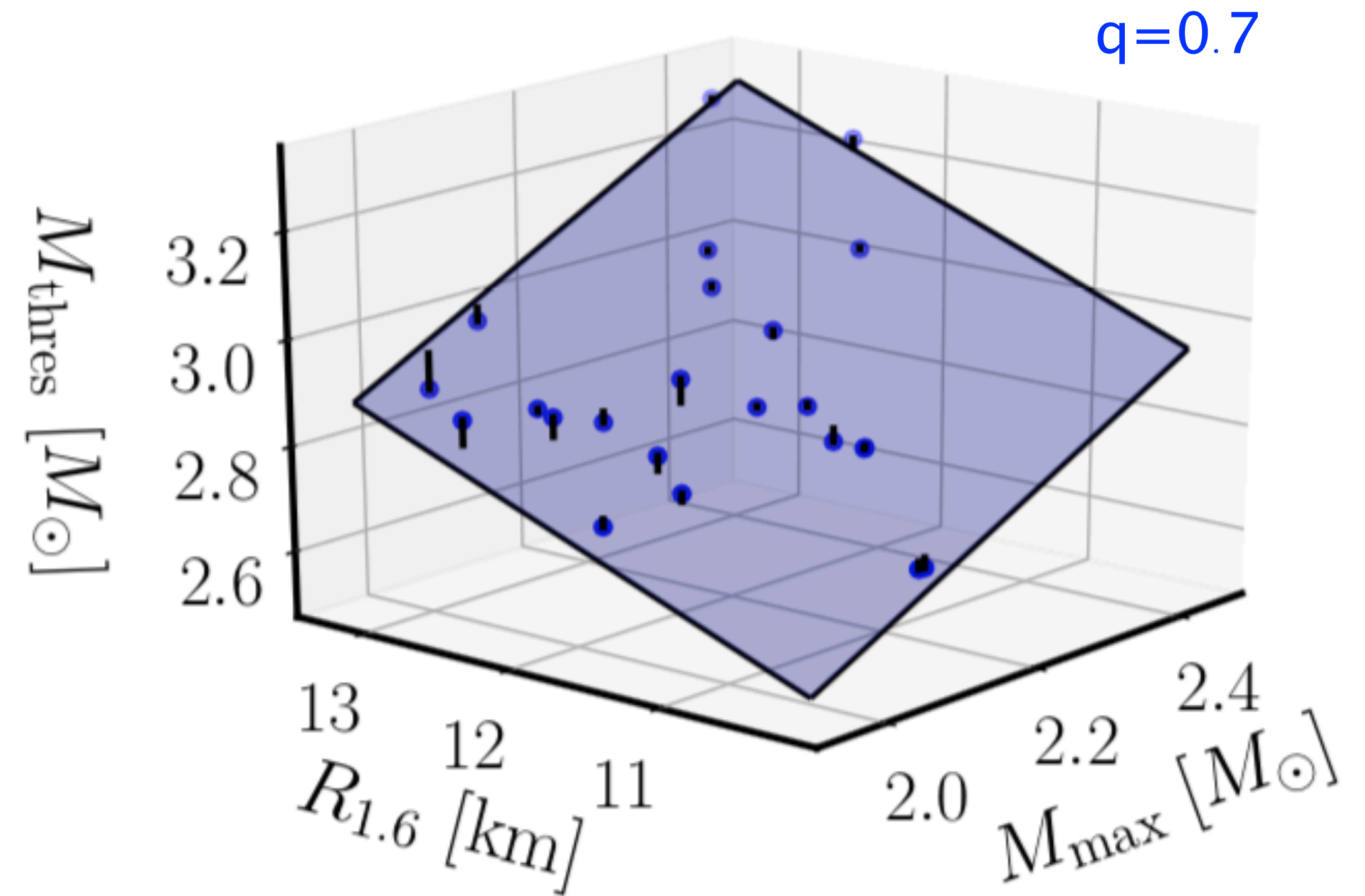
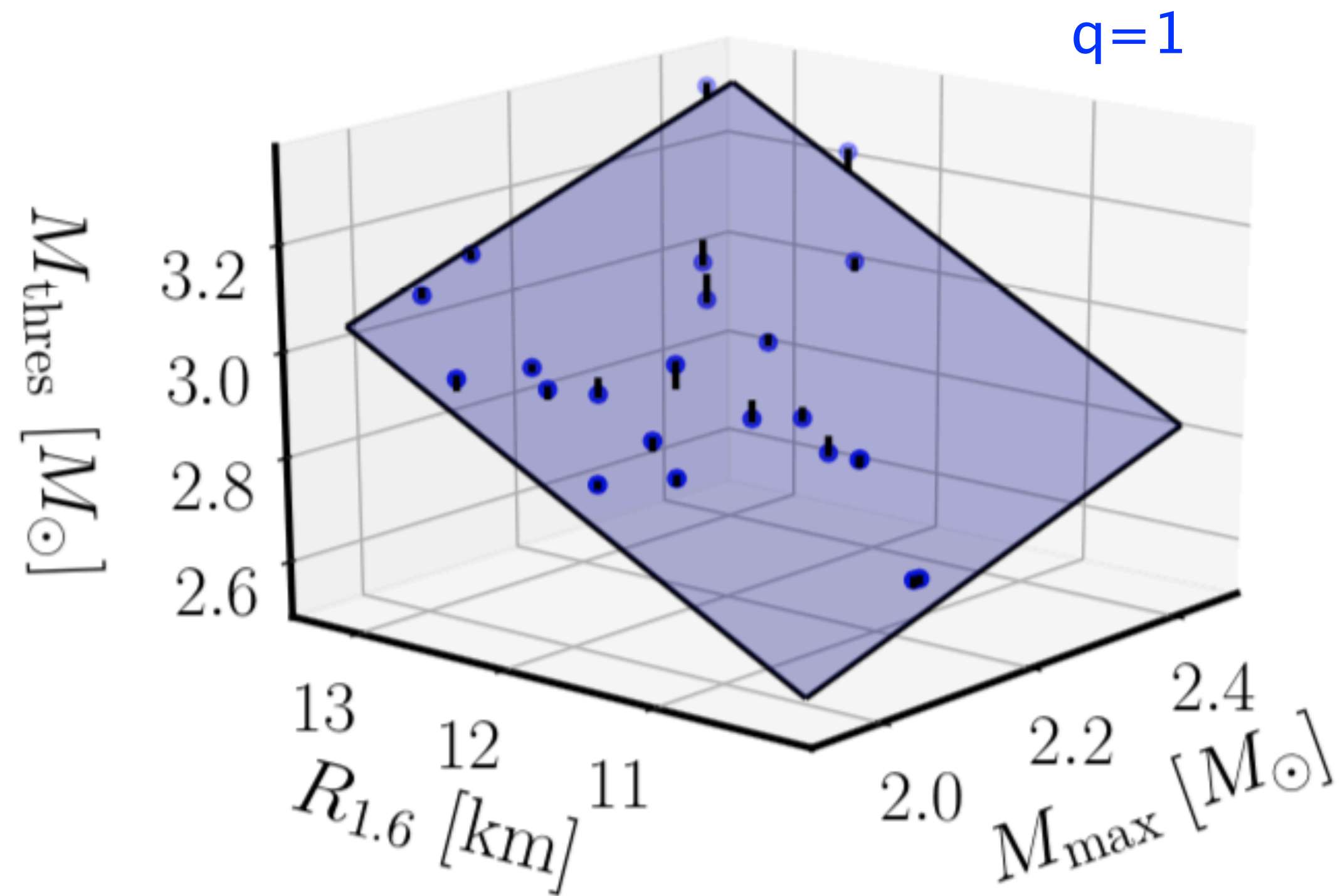
$$M_{\text{thres}} (M_{\odot}) = aM_{\text{max}} + b\Lambda_{1.4} - c$$

Bauswein et al. (2021)



THRESHOLD MASS TO PROMPT COLLAPSE FOR DIFFERENT MASS RATIOS

New bilinear empirical relations of the form $M_{\text{thres}} = aM_{\text{max}} + bR_{1.6} + c$



$$M_{\text{thres}}(M_{\odot}) = 0.547M_{\text{max}} + 0.165R_{1.6} - 0.198 \quad (\pm 0.042)$$

(max. dev.)

$$M_{\text{thres}}(M_{\odot}) = 0.832M_{\text{max}} + 0.116R_{1.6} - 0.276 \quad (\pm 0.067)$$

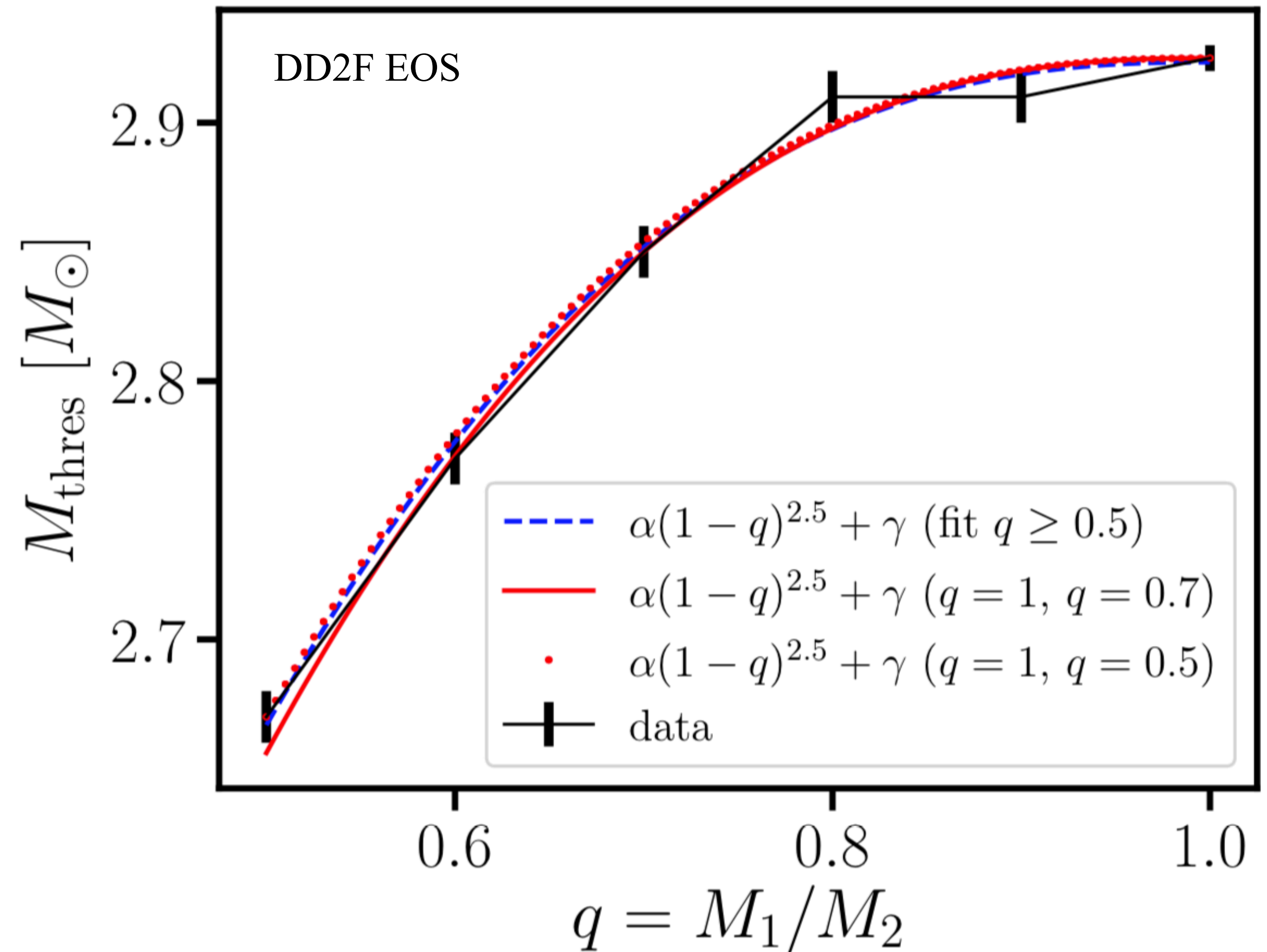
(max. dev.)

q-DEPENDENT EMPIRICAL RELATION FOR SPECIFIC EOS

For a few EOS we construct a fit in the range $0.5 \leq q \leq 1$ of the form

$$M_{\text{thres}}(q) = \alpha(1 - q)^n + \gamma$$

and find that $2.5 \leq n \leq 3.5$ within this sample.



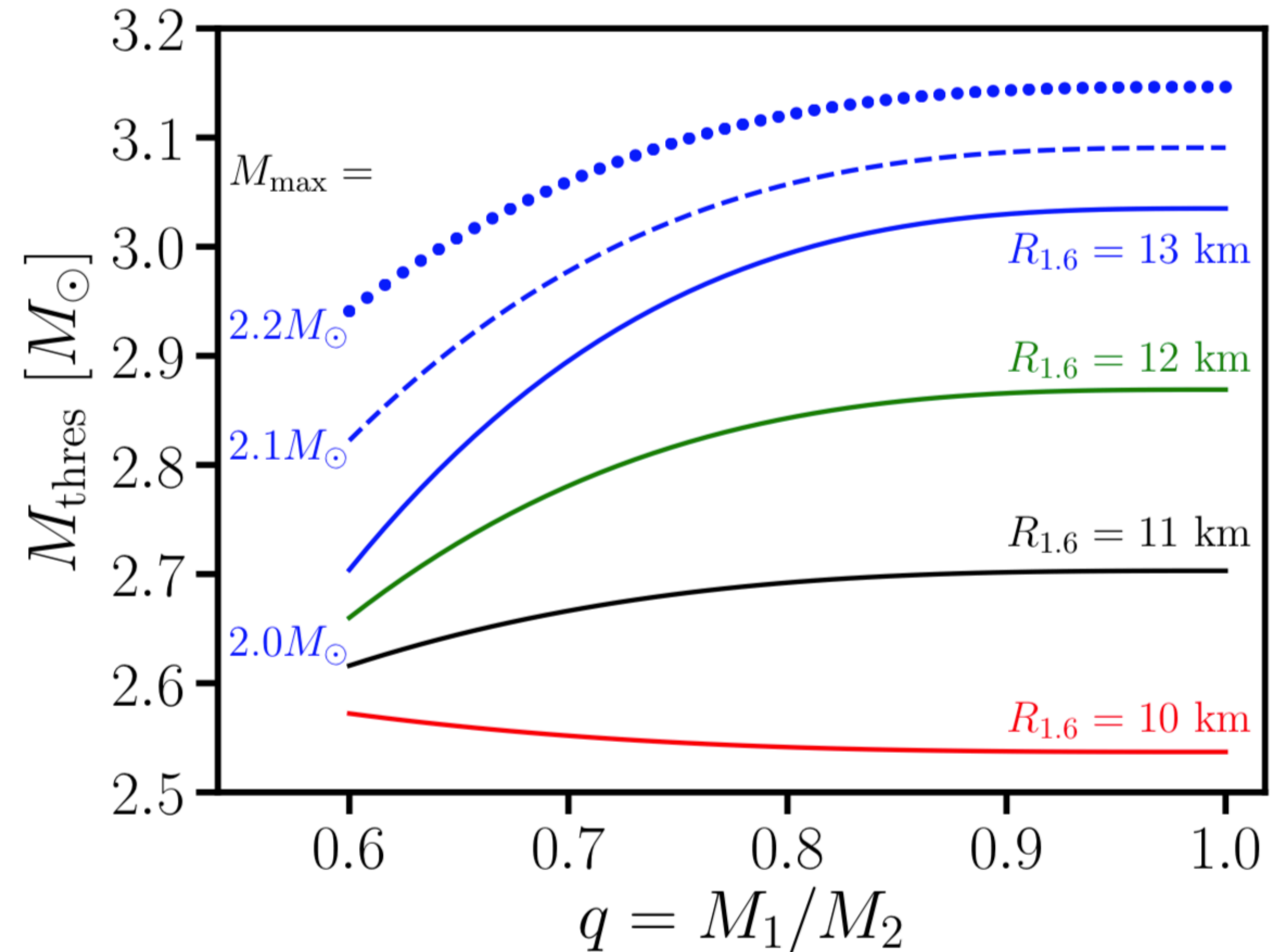
GENERALIZED EMPIRICAL RELATION FOR THRESHOLD MASS

We take $n=3$ as an average value and arrive at the **generalized empirical** relation that also depends on the mass ratio:

$$M_{\text{thres}}(q, M_{\text{max}}, R_{1.6}) = c_1 M_{\text{max}} + c_2 R_{1.6} + c_3 + c_4 \delta q^3 M_{\text{max}} + c_5 \delta q^3 R_{1.6} + c_6 \delta q^3$$

$$\delta q \equiv 1 - q$$

For the base sample $q=1$ and $q=0.7$ with 23 hadronic EOS, the maximum residual is only $0.067M_{\odot}$.



ROTATION PROFILE OF BNS REMNANTS

Key properties:

- Maximum angular velocity about twice the central angular velocity

$$\Omega_{\max} \sim 2\Omega_c$$

- Angular velocity at equator comparable to central angular velocity

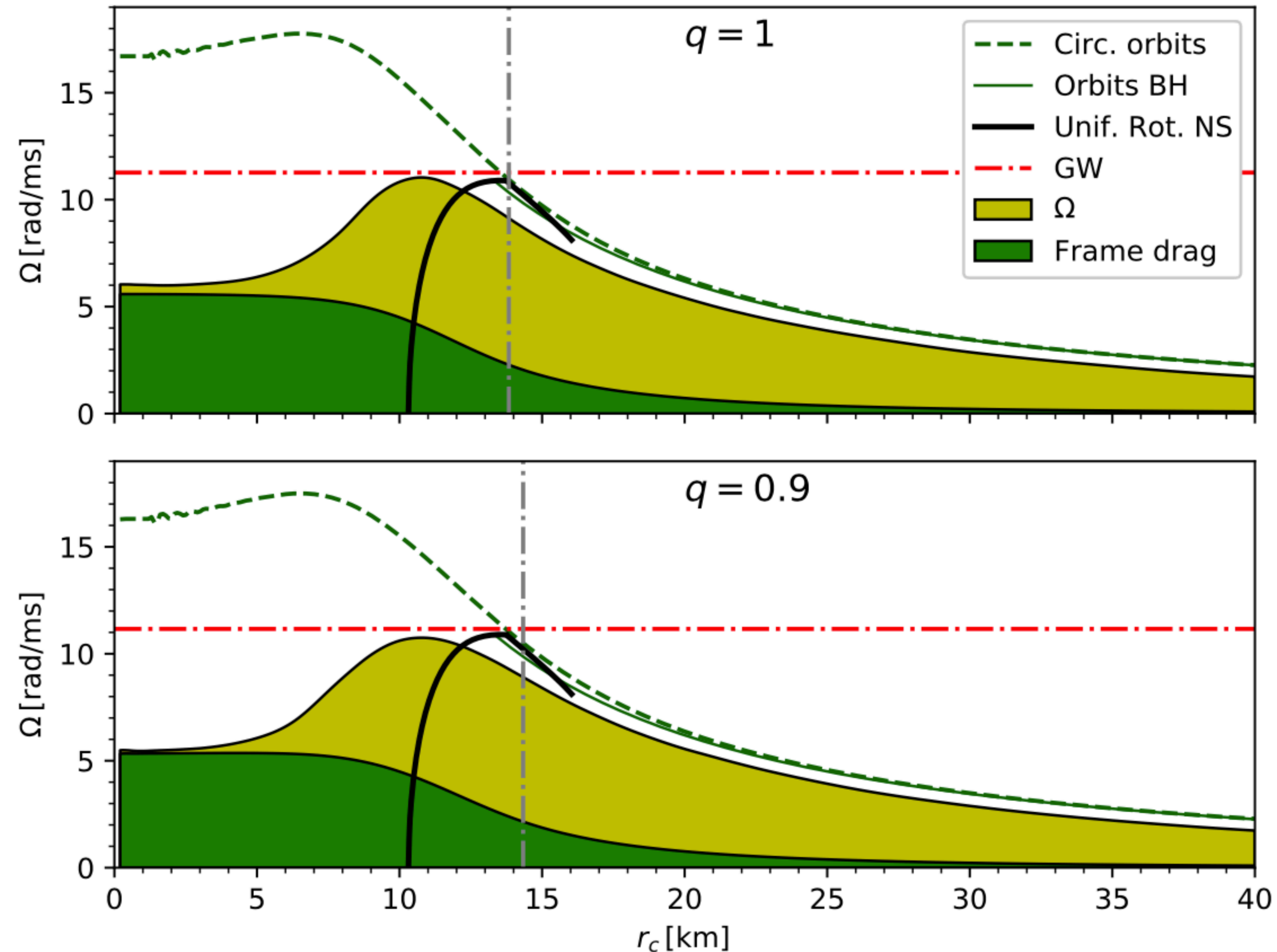
$$\Omega_e \sim \Omega_c$$

- Central angular velocity mostly due to frame dragging

$$\Omega \sim \omega$$

- Frequency of main post-merger GW peak about half of the maximum angular velocity

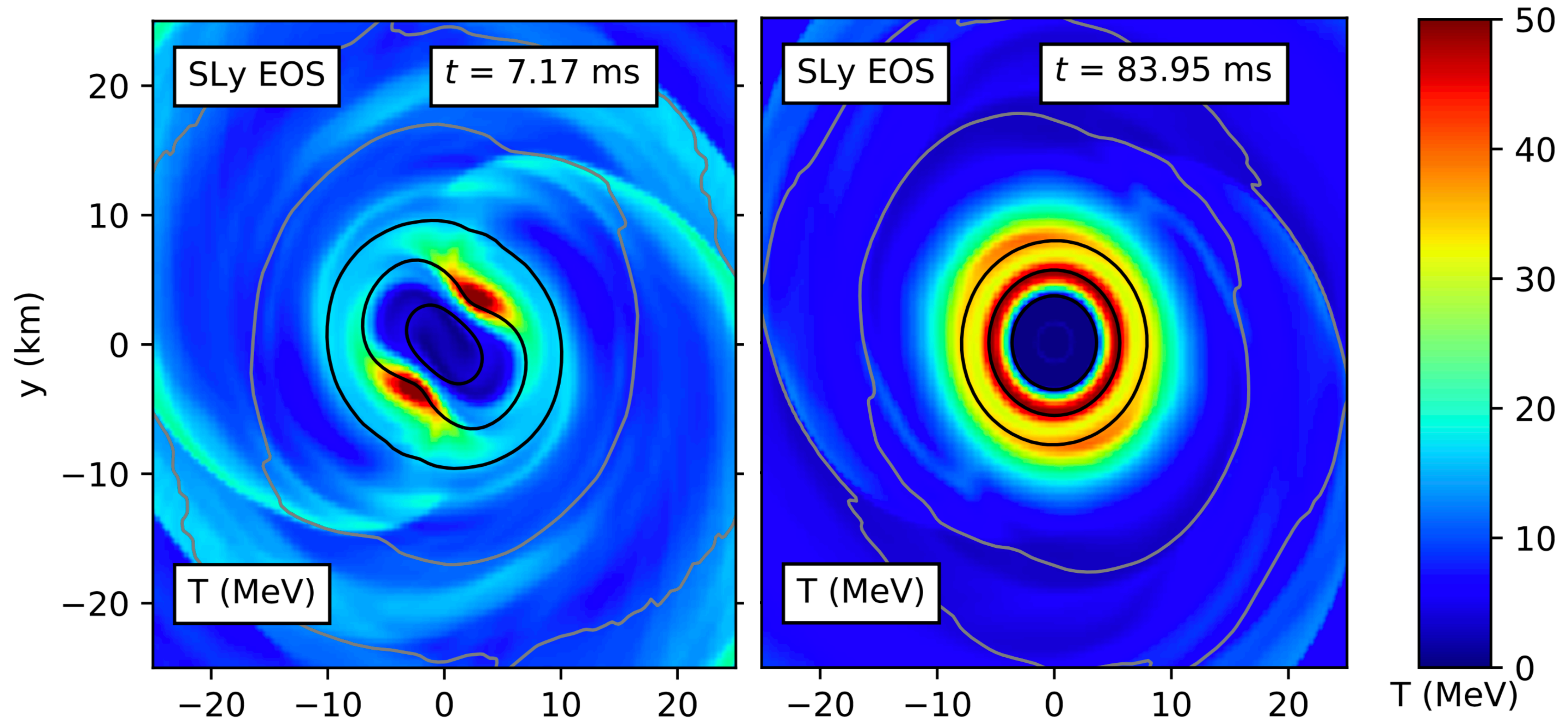
$$\Omega_{\max} \sim \Omega_{\text{GW}}/2$$



Kastaun, Galeazzi (2015); Kastaun, Ohme (2021)

THERMAL PROFILE OF POST-MERGER REMNANTS

Cold core + hot envelope of several tens MeV.



De Pietri et al. (2019)

ANGULAR MOMENTUM OF POST-MERGER REMNANTS

The angular momentum of post-merger remnants at the moment of collapse is given by an EOS-insensitive empirical relation of the form (Bauswein, Stergioulas, 2017)

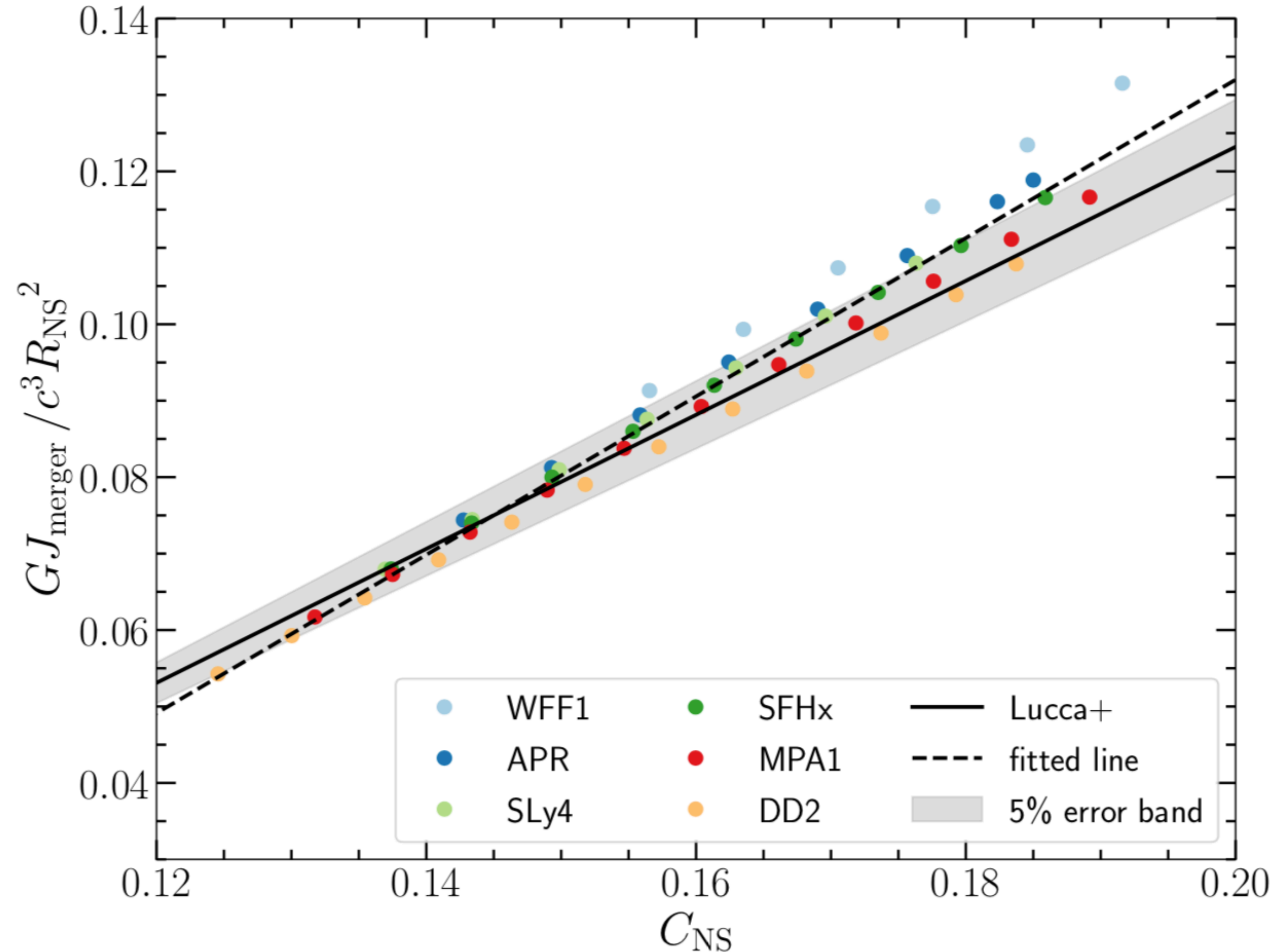
$$\frac{cJ_{\text{merger}}}{GM_{\odot}^2} \simeq a \frac{M_{\text{tot}}}{M_{\odot}} - \left(b + \frac{R_{1.5} - R_{1.5}^{\text{DD2}}}{10 \text{ km}} \right)$$

with $a=4.041$ and $b=4.658$.

An alternative form is (Lucca et al. 2021)

$$\frac{GJ_{\text{merger}}}{c^3 R_{\text{NS}}^2} = 0.875 C_{\text{NS}} - 5.209$$

Where $C_{\text{NS}} = GM_{\text{NS}}/c^2 R_{\text{NS}}$ is the compactness of a TOV model with $M_{\text{NS}} = M_{\text{tot}}/2$



Iosif, Stergioulas (2021)

EQUILIBRIUM MODELS OF POST-MERGER REMNANTS

We construct merger sequences of equilibrium models of post-merger remnants, with the following characteristics:

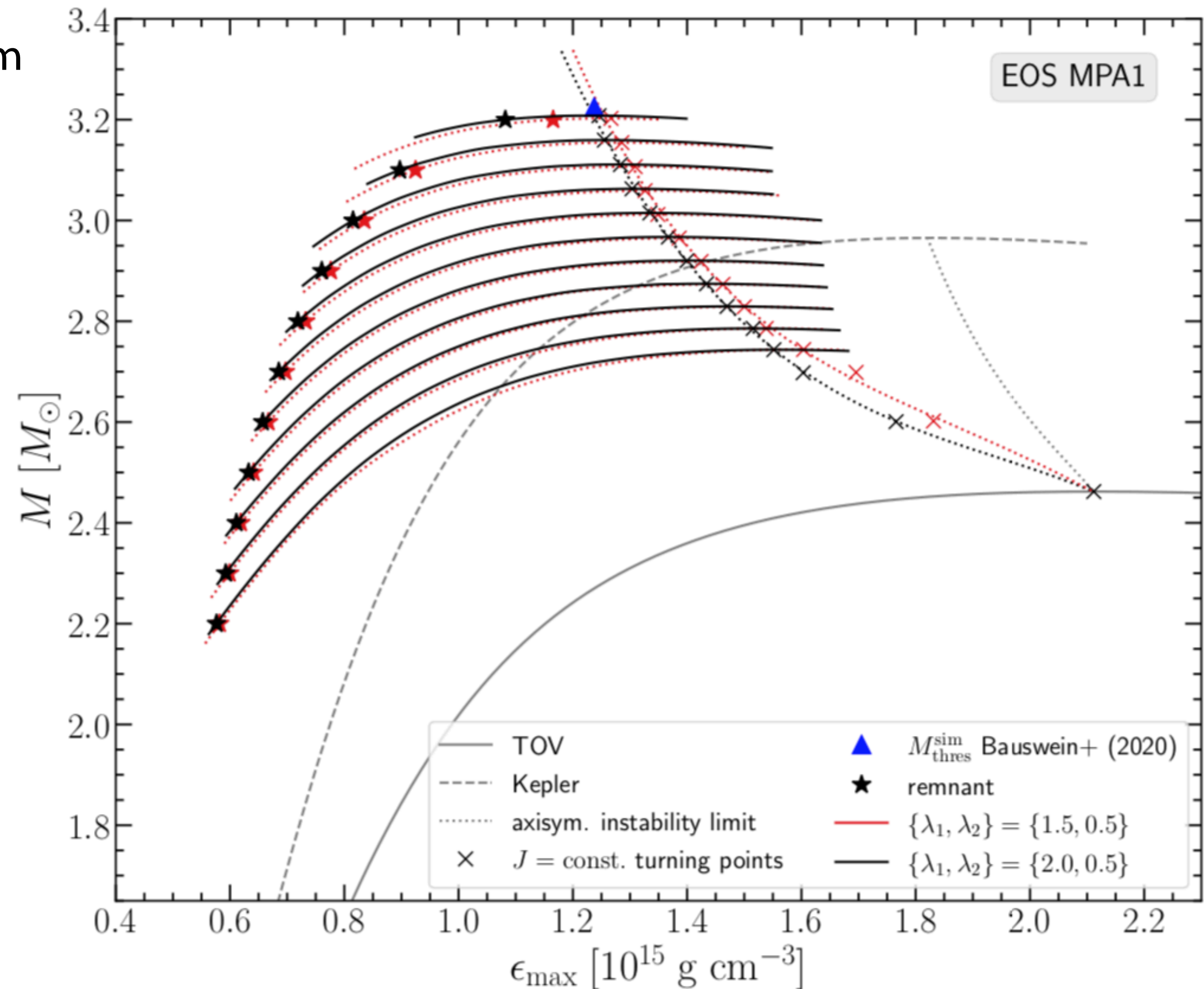
- 4-parameter rotation law by Uryu et al. (2017), with $p=1$, $q=3$.

$$\Omega = \Omega_c \frac{1 + \left(\frac{F}{B^2 \Omega_c}\right)^p}{1 + \left(\frac{F}{A^2 \Omega_c}\right)^{q+p}} \quad F \equiv u^t u_\phi$$

- The remaining two parameters A, B are redefined as

$$\lambda_1 \equiv \frac{\Omega_{\max}}{\Omega_c} \quad \lambda_2 \equiv \frac{\Omega_e}{\Omega_c}$$

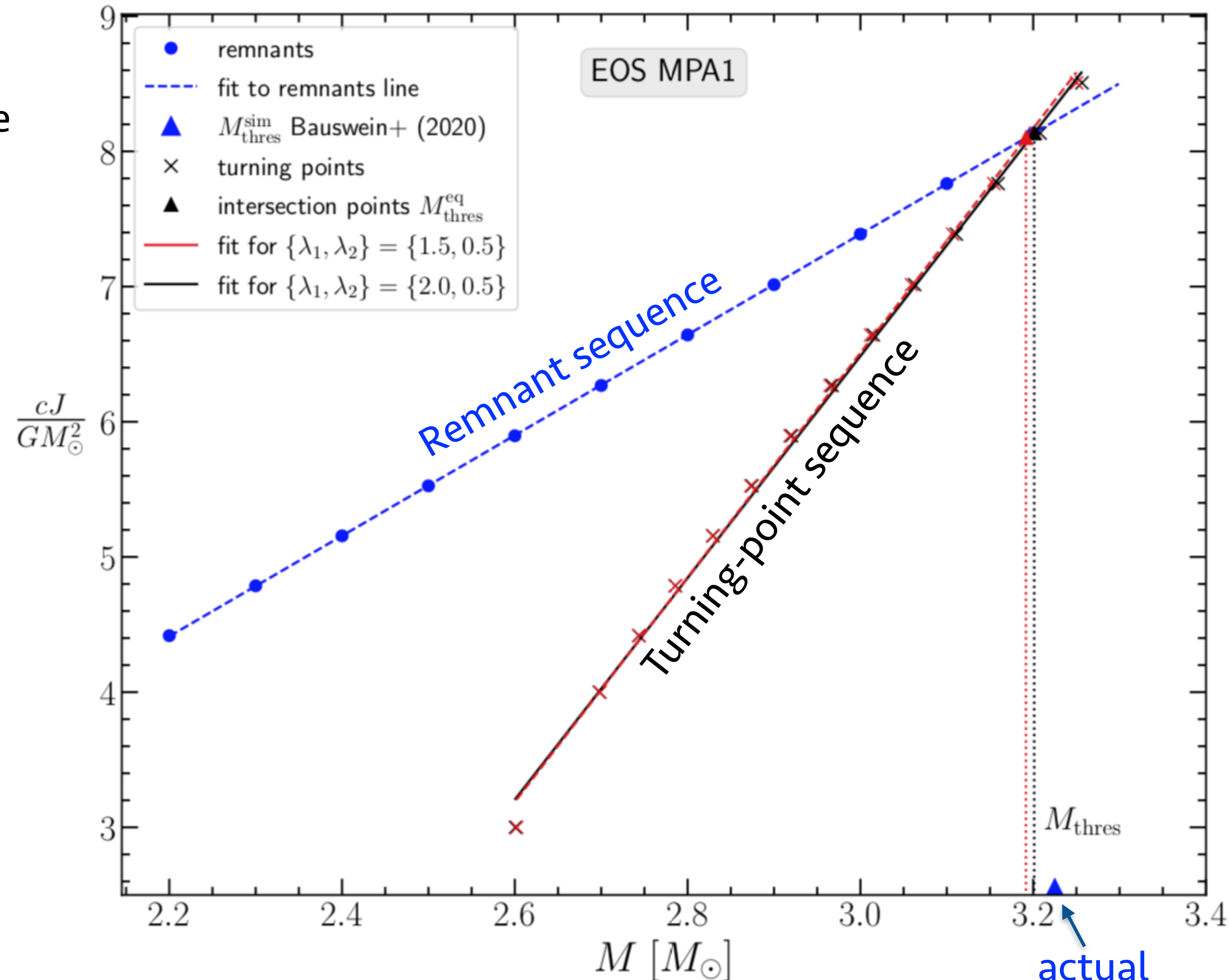
- We consider 3 hadronic EOS with radii between 11km and 13km for typical NS.



REPRODUCTION OF THE THRESHOLD MASS

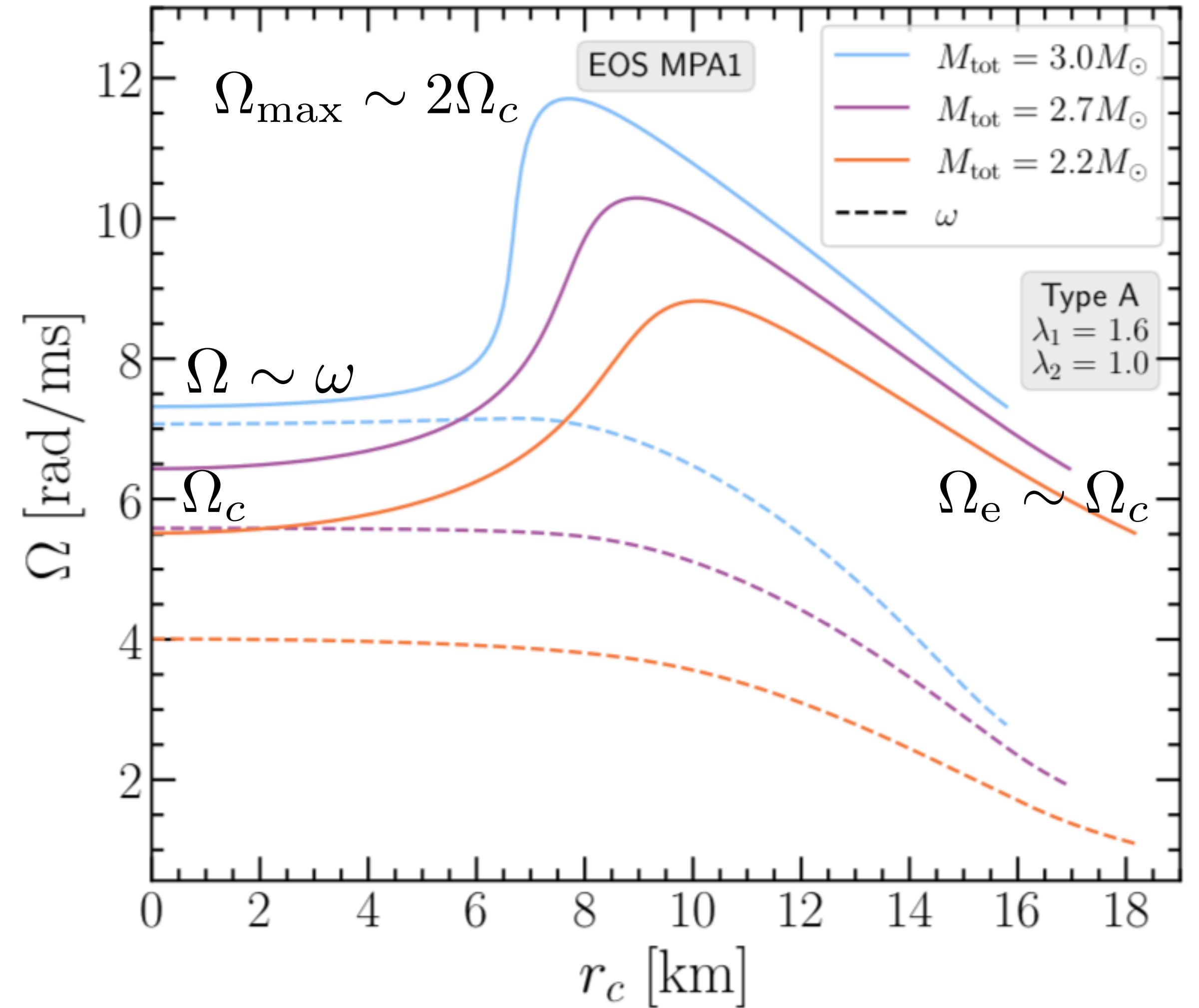
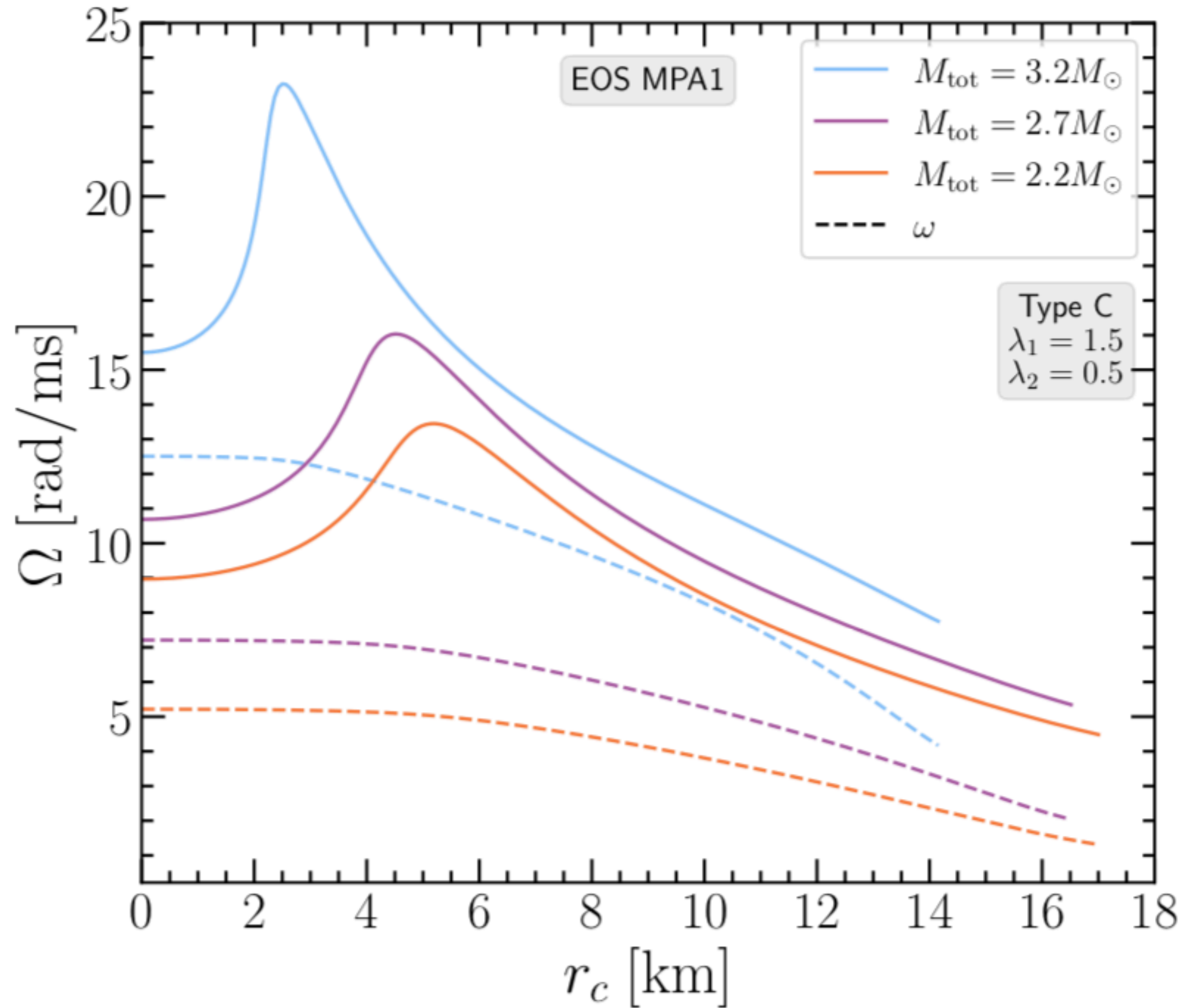
The intersection between the sequence of merger remnants and the turning-point line determines a threshold mass that agrees remarkably well for all 3 EOS.

EOS	$M_{\text{thres}}^{\text{eq}}$	$J_{\text{thres}}^{\text{eq}}$	$M_{\text{thres}}^{\text{sim}}$	δM_{thres}
$\{\lambda_1, \lambda_2\}$	$[M_{\odot}]$	$[\frac{GM_{\odot}^2}{c}]$	$[M_{\odot}]$	$[\%]$
APR			2.825	
{2.0, 0.5}	2.851	6.524		0.92
{1.5, 0.5}	2.842	6.492		0.60
DD2			3.325	
{2.0, 0.5}	3.302	8.766		0.69
{1.5, 0.5}	3.285	8.697		1.20
MPA1			3.225	
{2.0, 0.5}	3.201	8.136		0.74
{1.5, 0.5}	3.192	8.100		1.02



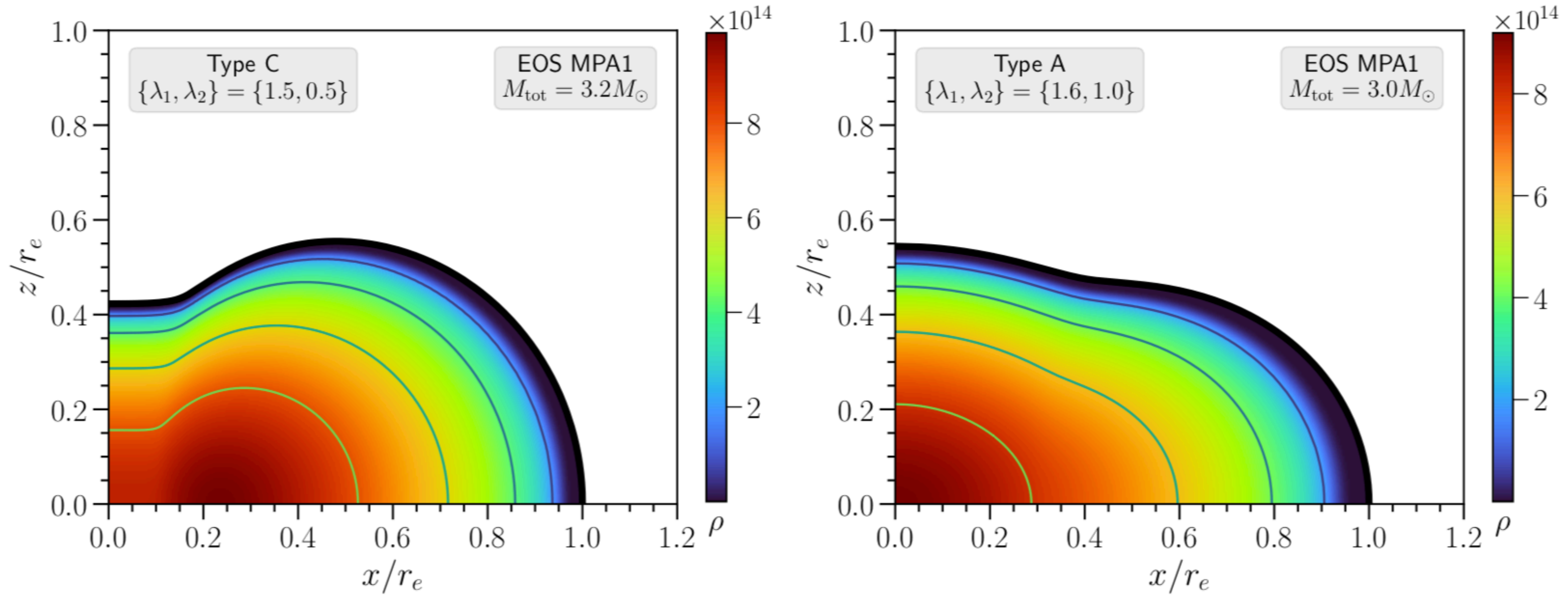
ROTATION PROFILES

The rotation profiles show a qualitative agreement with those extracted from simulations.



DENSITY DISTRIBUTION OF REMNANT MODELS

We find both quasi-toroidal (Type C) and quasi-spherical (Type A) models.



EQUATORIAL COMPACTNESS AT PROMPT COLLAPSE

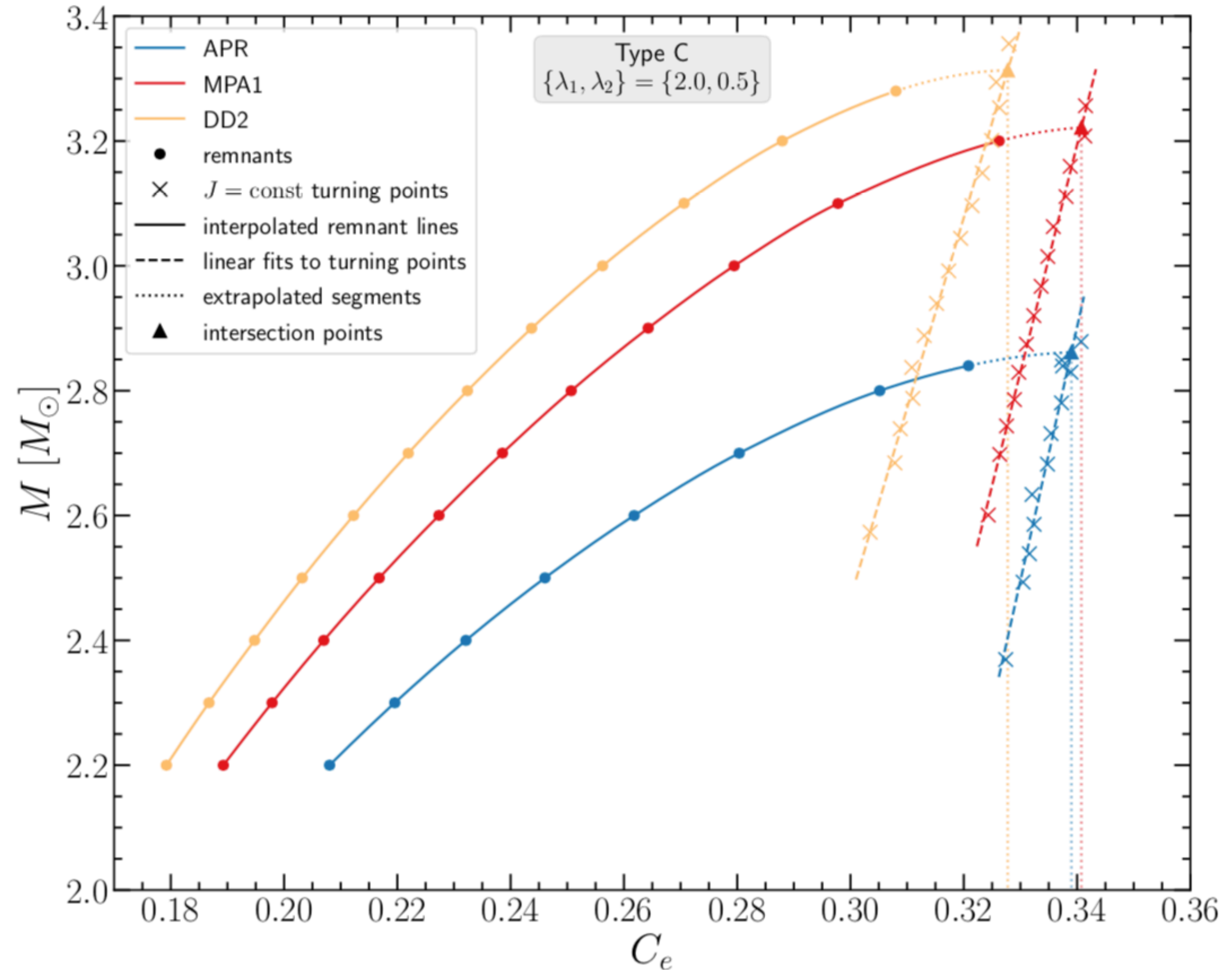
We define the “equatorial compactness” as

$$C_e = M/R_e$$

and find that the models at the threshold mass have

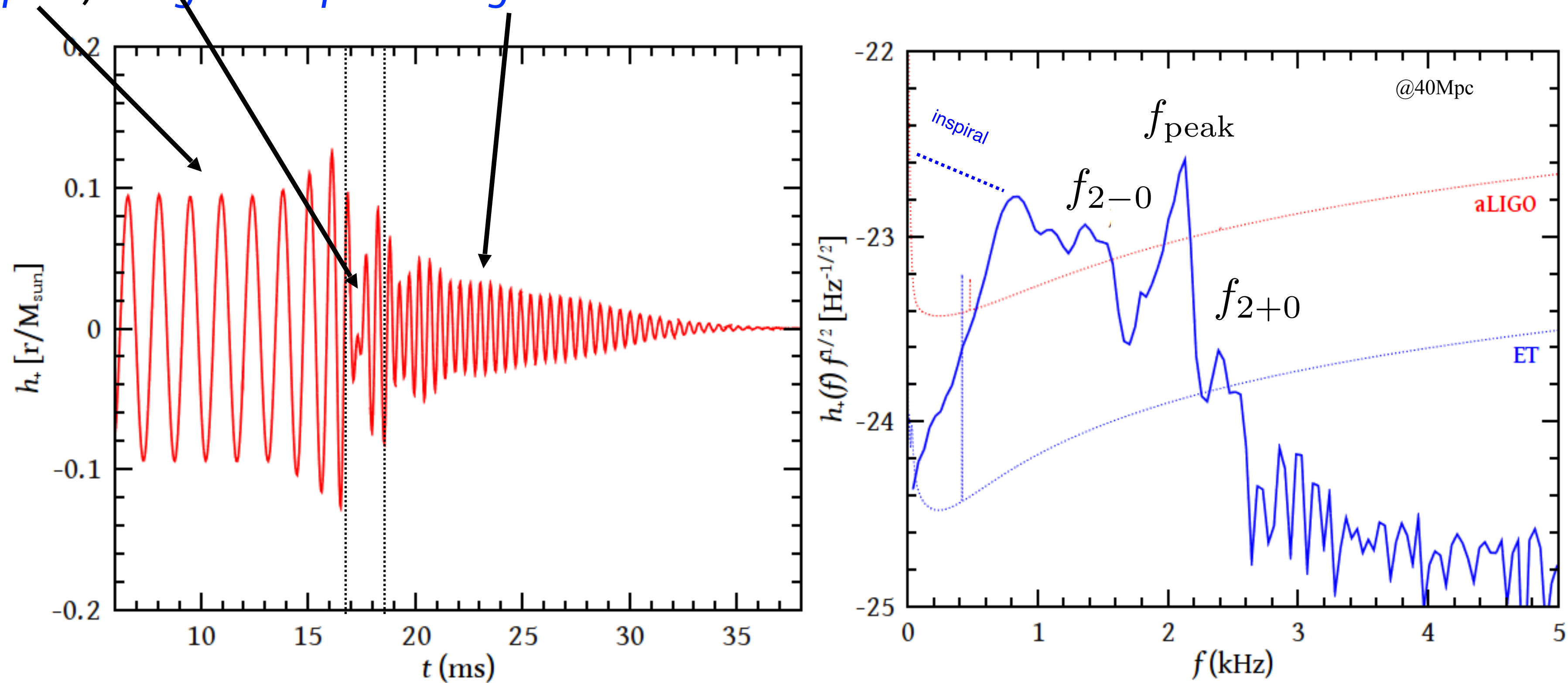
$$C_e \sim 0.33$$

This supports the conjecture by Kastaun, Ohme (2021) that remnants collapse when their slowly rotating, cold core has a compactness comparable to C_{\max}^{TOV} .



POST-MERGER PHASE IN BNS MERGERS

The GW signal can be divided into three distinct phases: *inspiral*, *merger* and *post-merger oscillations*.



$$f_{\text{peak}} = f_2$$

$$f_{2-0} = f_2 - f_0$$

$$f_{2+0} = f_2 + f_0$$

is due to the fundamental $l=m=2$ *f-mode* oscillation

are quasi-linear combination tones

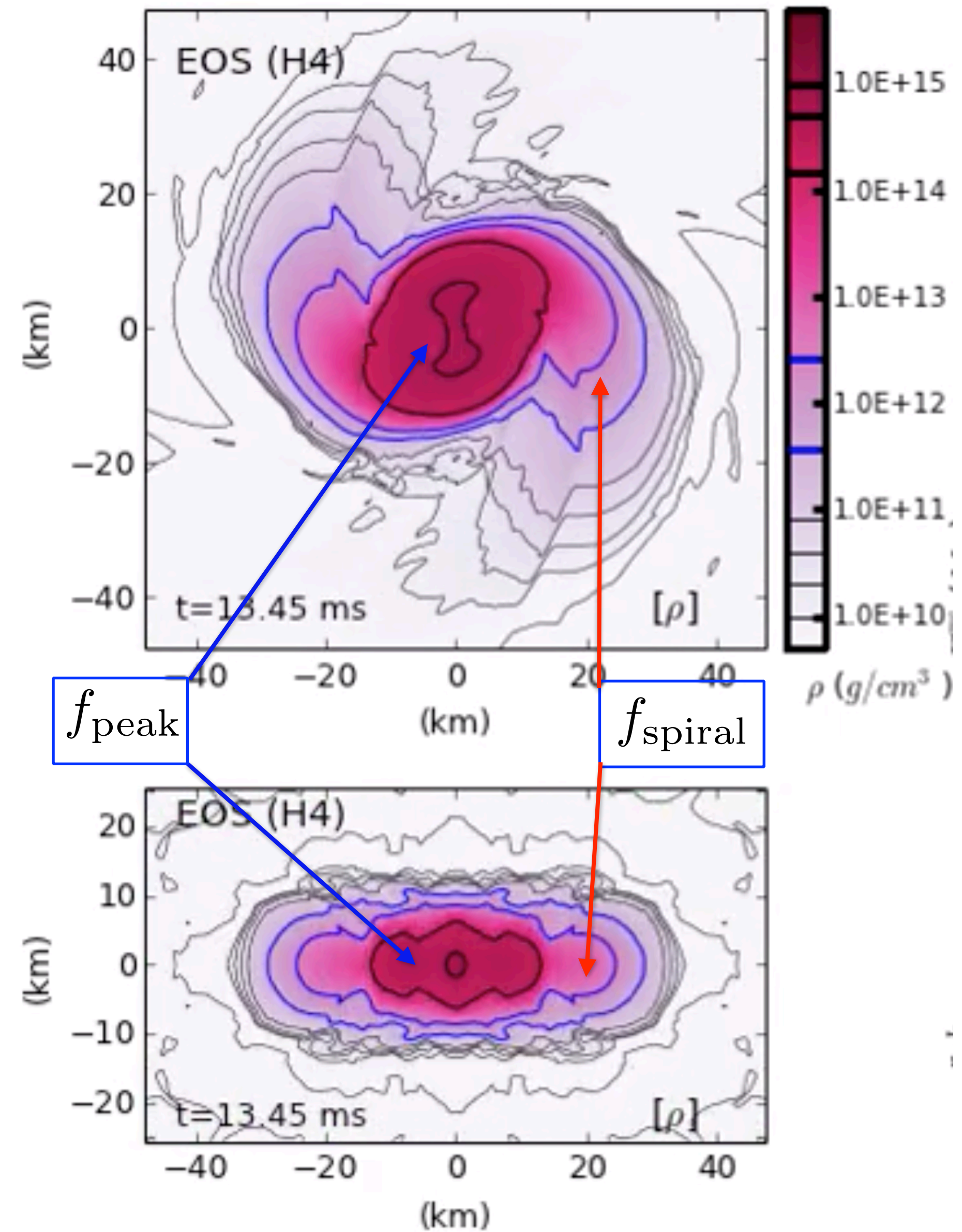
Stergioulas et al. (2011)

POST-MERGER PHASE IN BNS MERGERS

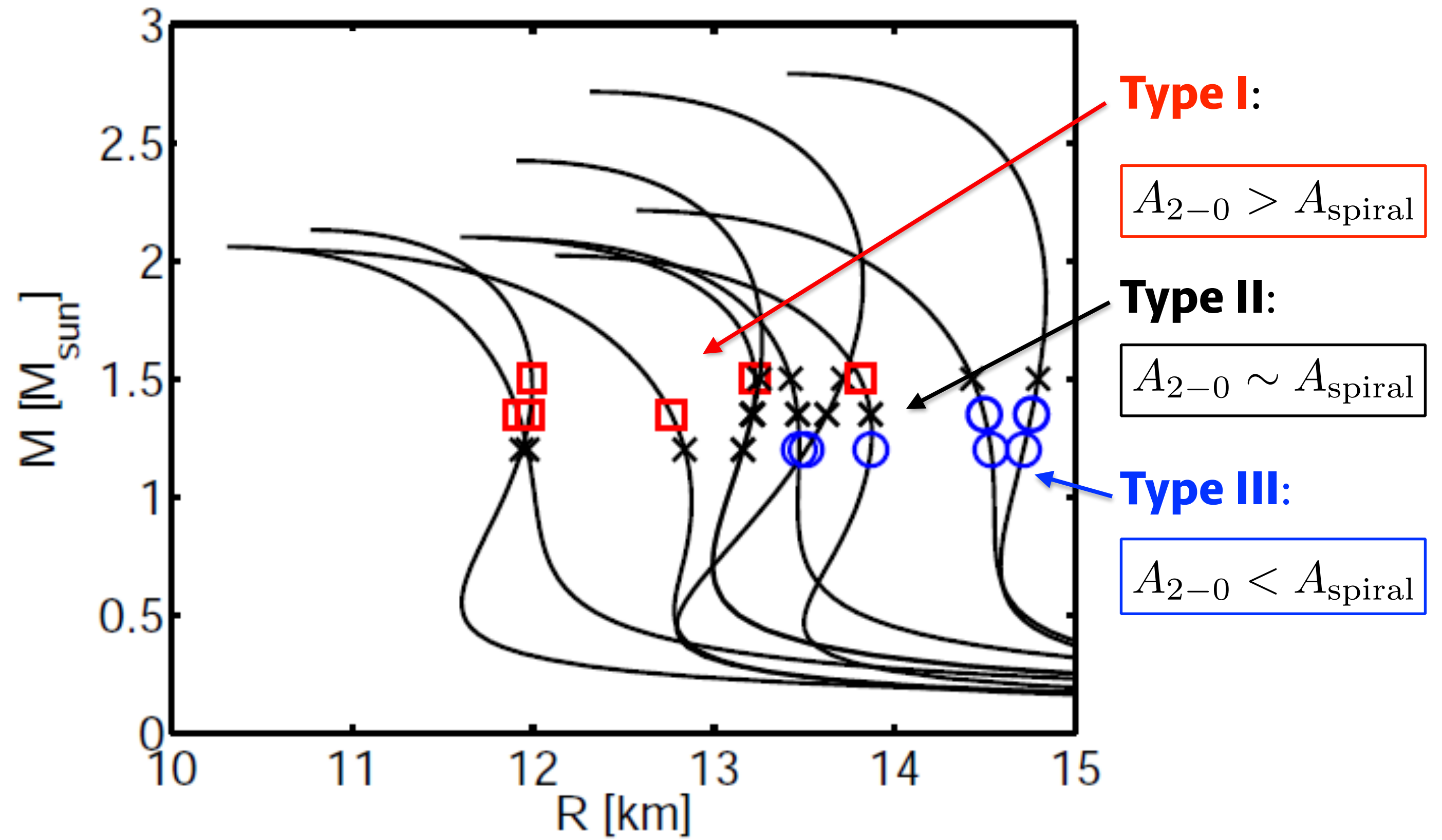
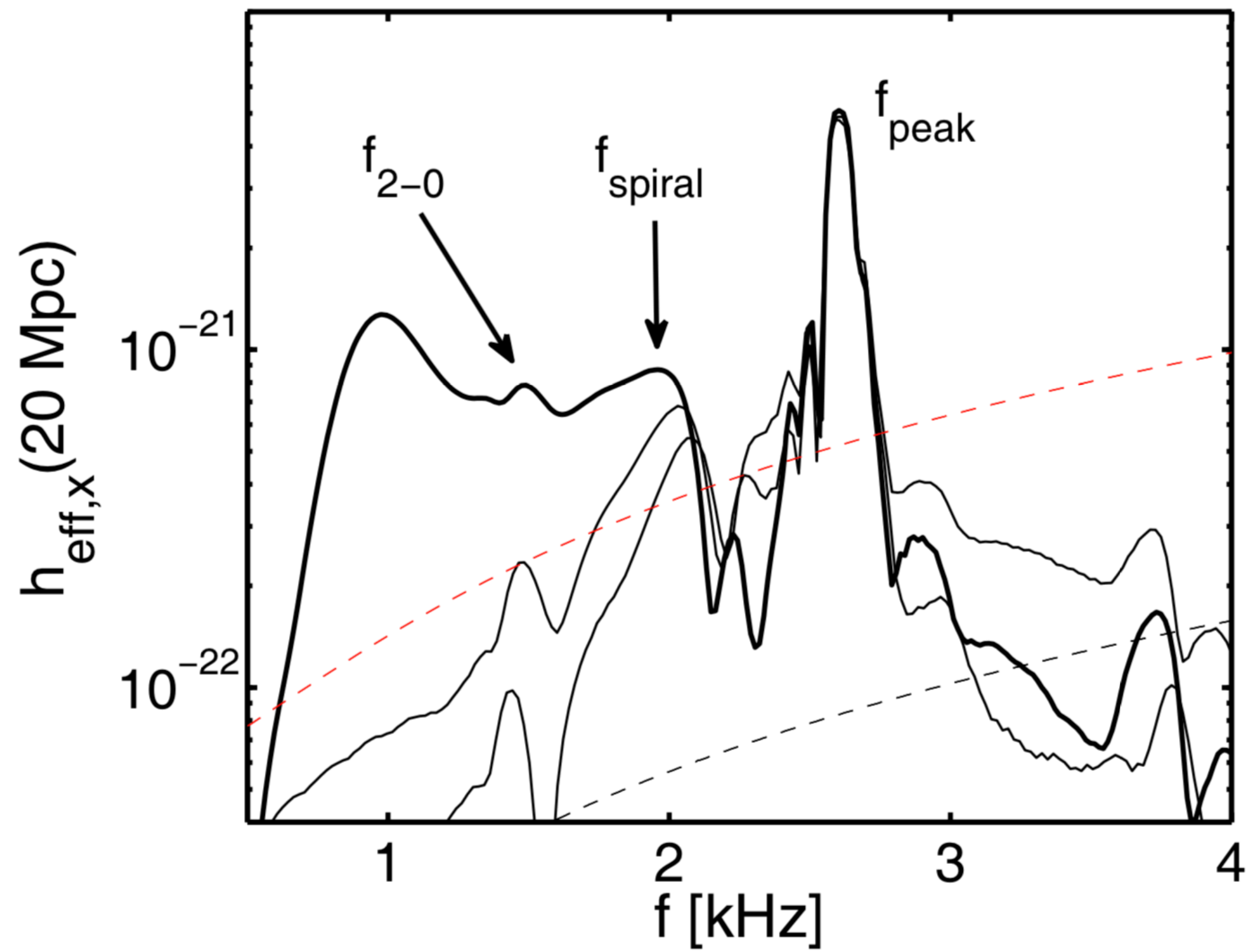
Orbiting spiral arms also lead
to a distinct frequency

$$f_{\text{spiral}}$$

Bauswein & Stergioulas (2015)

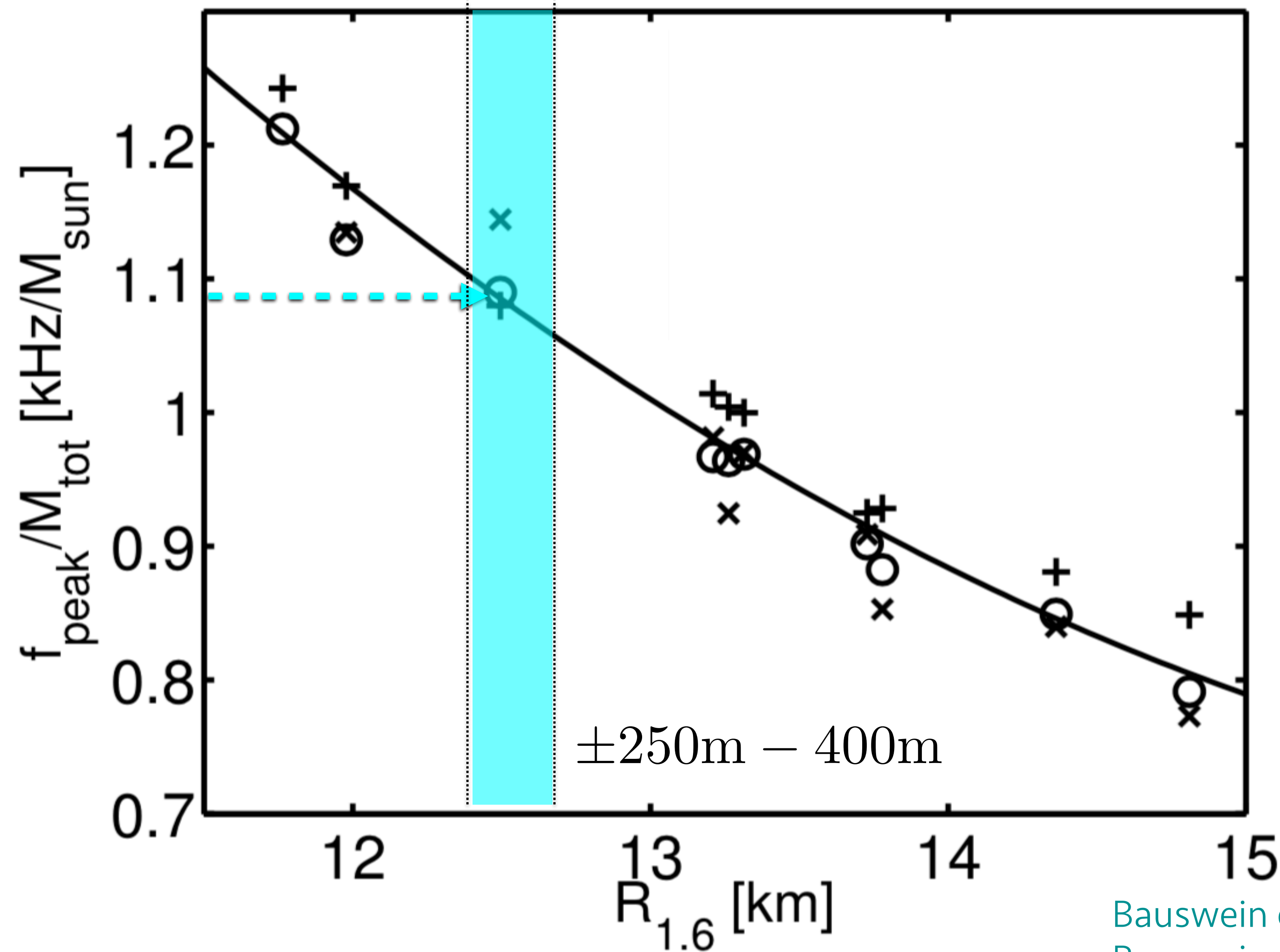


SPECTRAL CLASSIFICATION OF POST-MERGER GW EMISSION



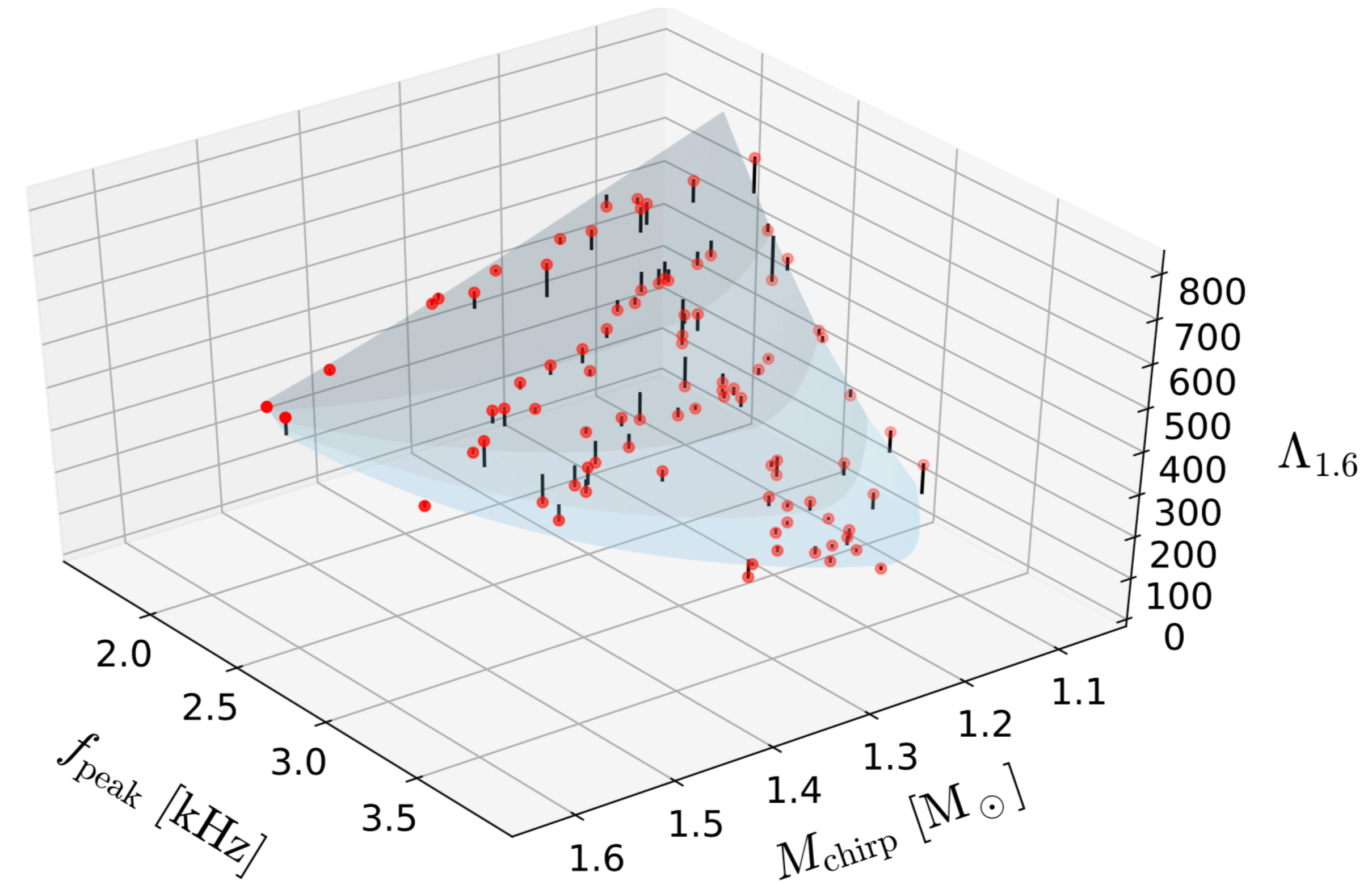
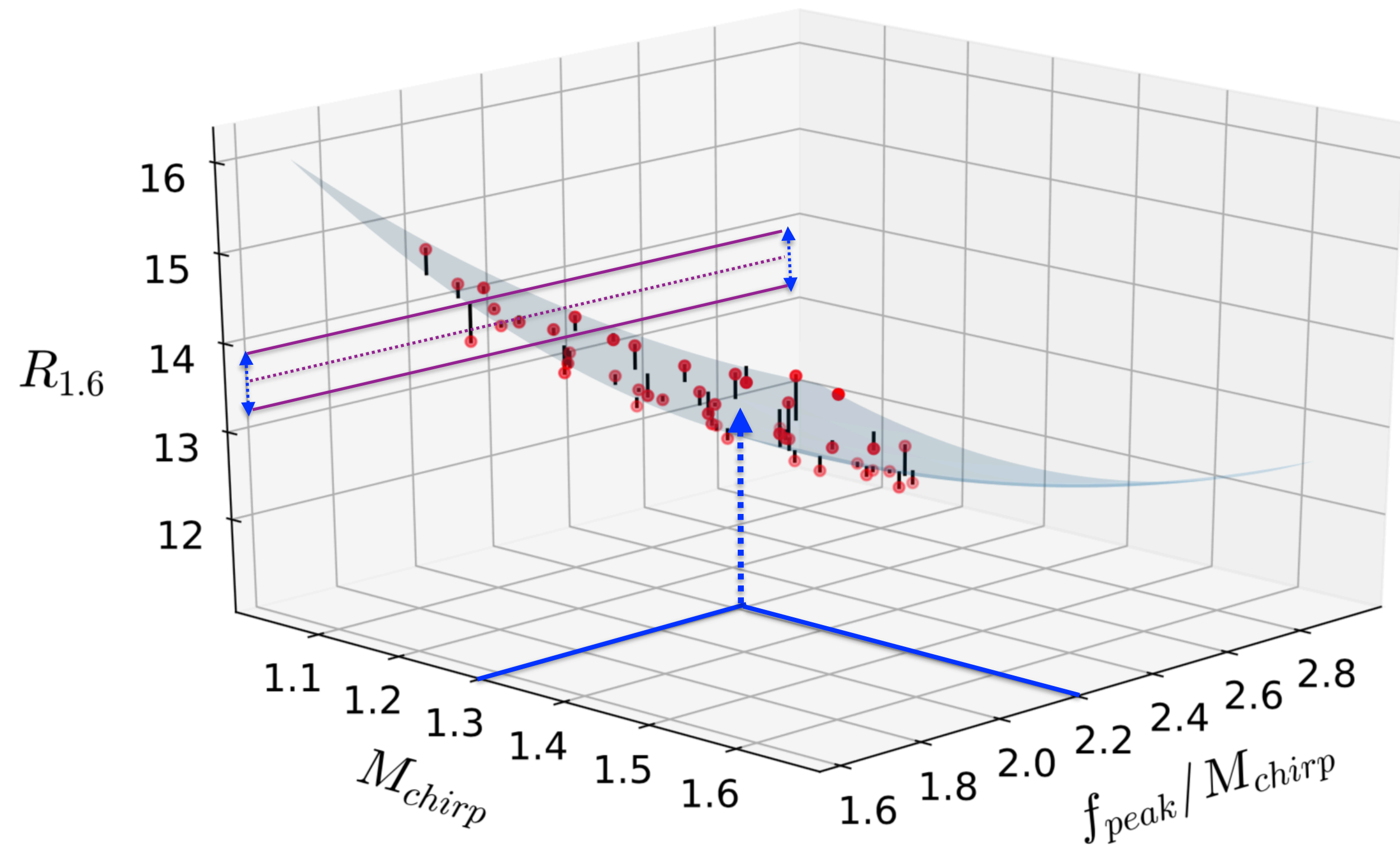
Bauswein & Stergioulas (2015)

RADIUS DETERMINATION THROUGH POST-MERGER GWs



Bauswein et al. (2012)
Bauswein, Stergioulas, Janka (2016)

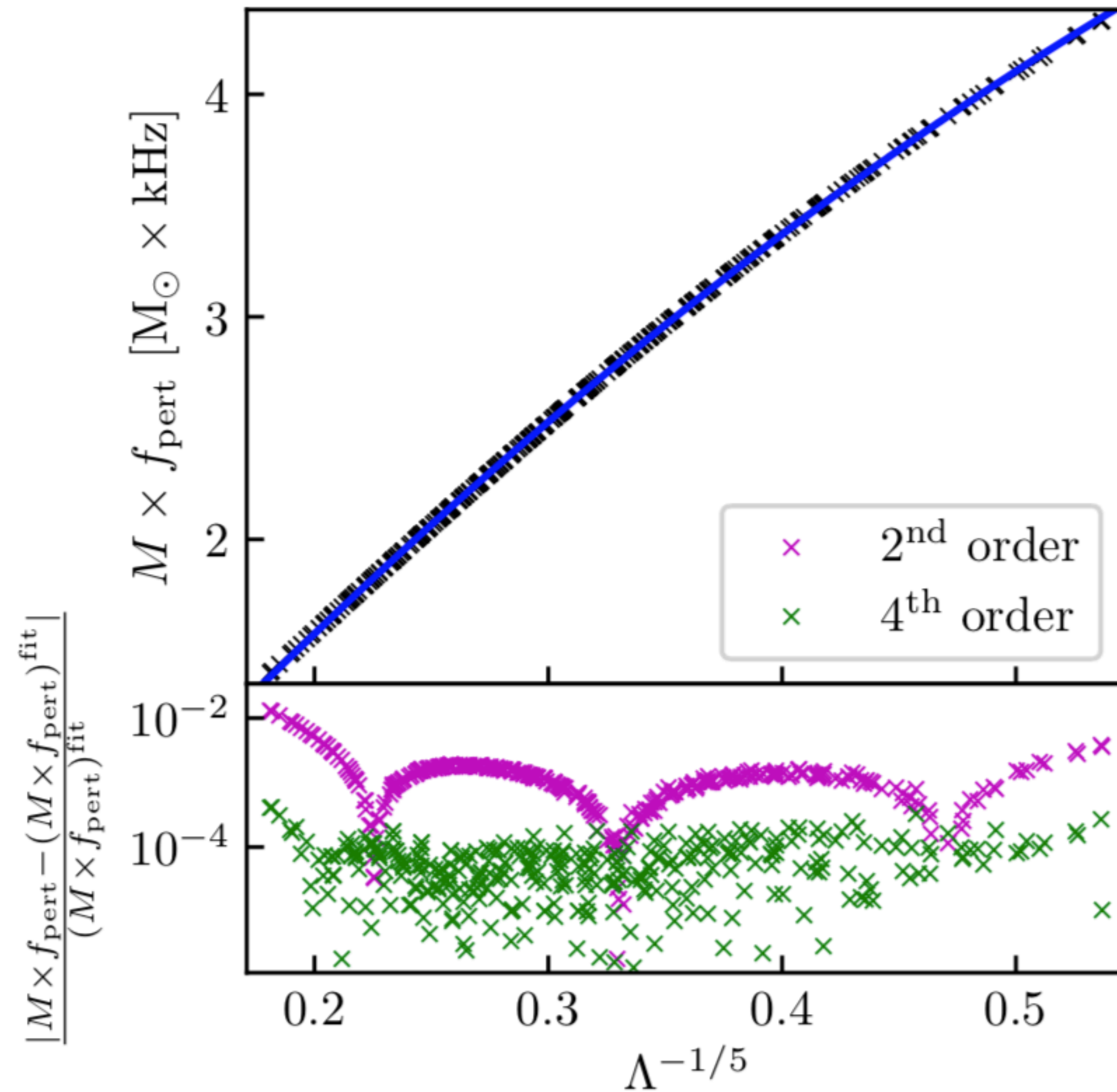
EMPIRICAL RELATIONS FOR GW ASTEROSEISMOLOGY OF BNS MERGERS



Vretinaris, Stergioulas & Bauswein (2020)

HIGHLY ACCURATE UNIVERSAL RELATION FOR NONROTATING STARS

l=2 mode in
nonrotating NS



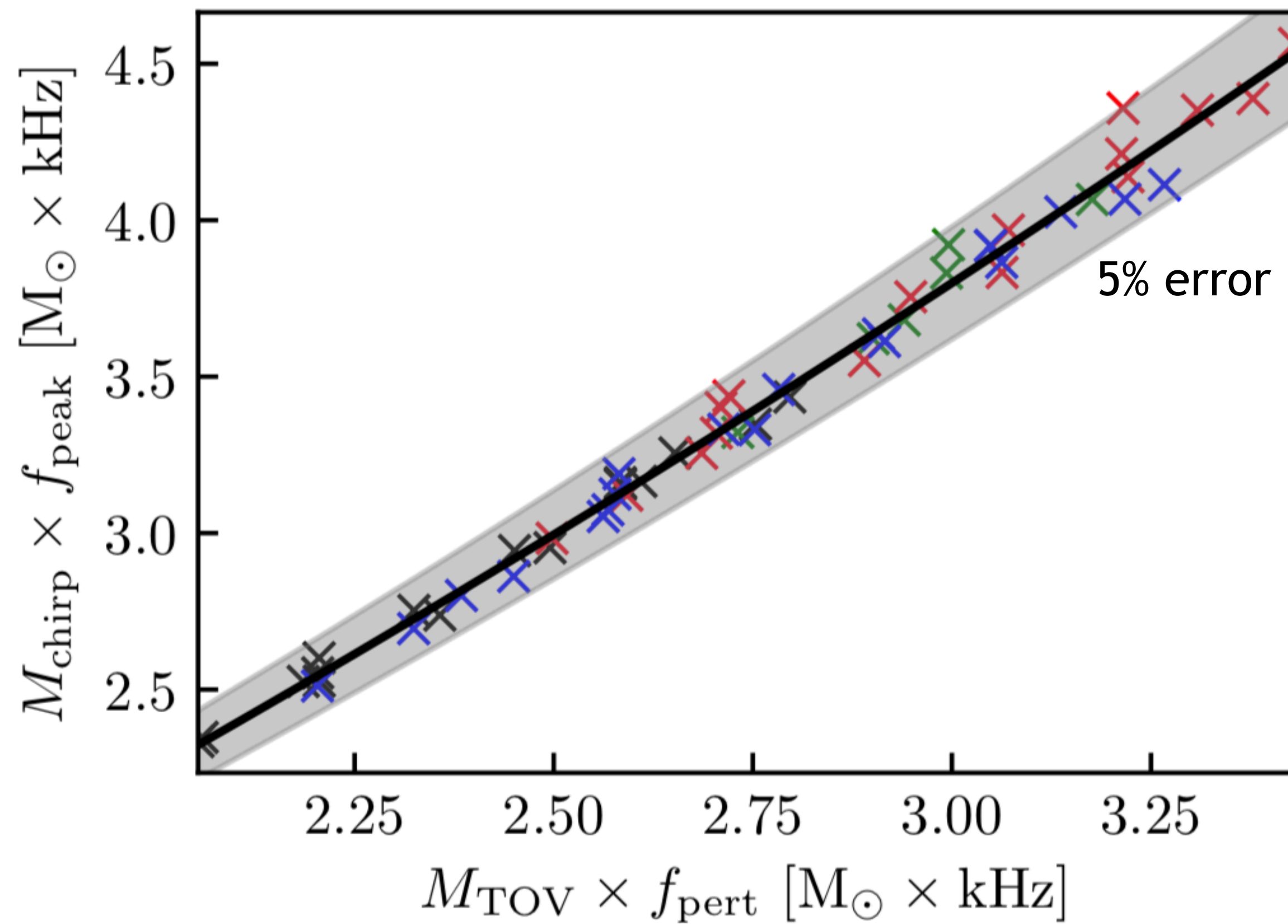
Lioutas, Bauswein, Stergioulas (2021)

$$M f_{\text{pert}} = -0.24 + 7.726\Lambda^{-1/5} + 11.88\Lambda^{-2/5} - 27.65\Lambda^{-3/5} + 15.39\Lambda^{-4/5}$$

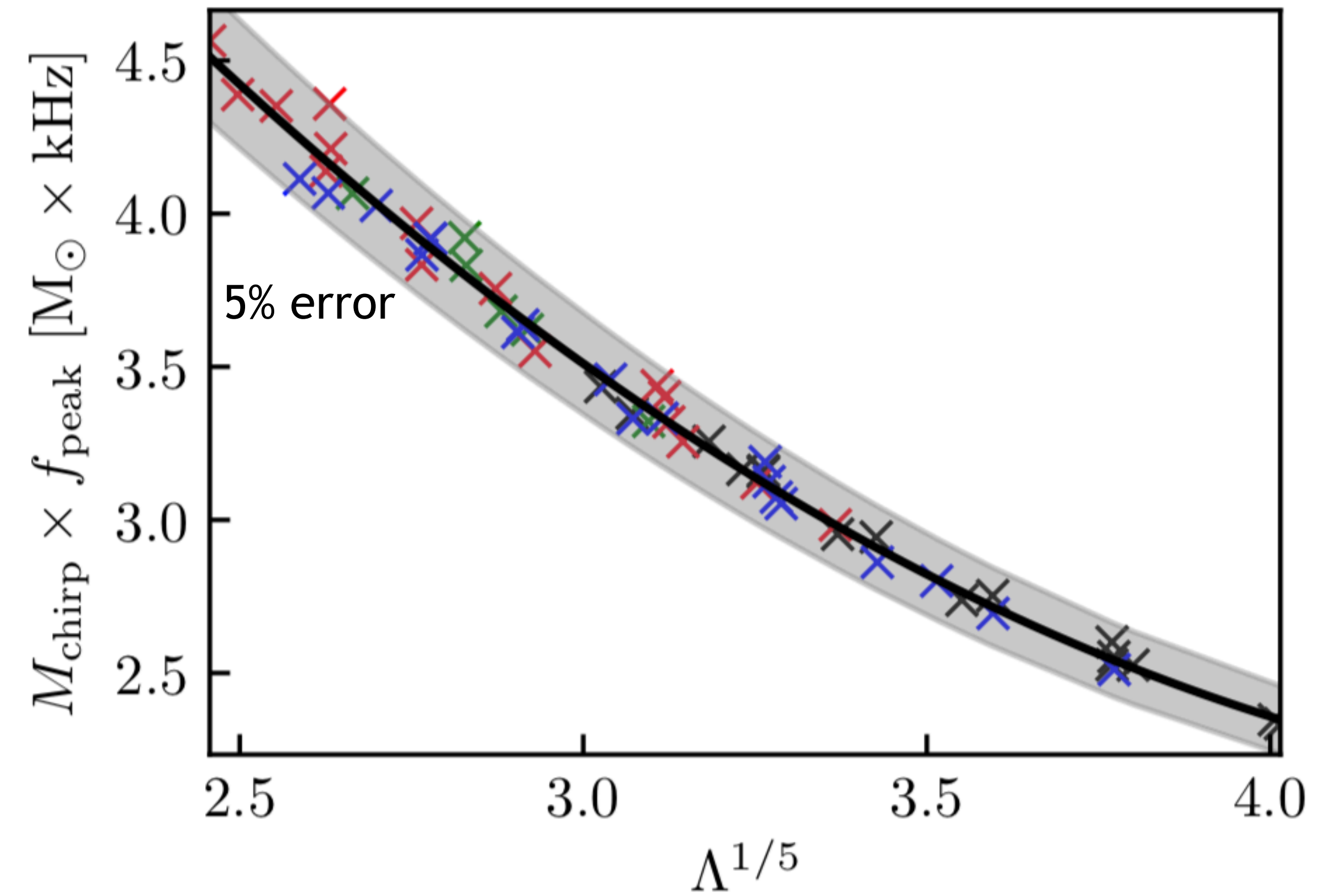
NEW UNIVERSAL RELATIONS BETWEEN REMNANTS AND NONROTATING STARS

- Using the correspondence $M_{\text{TOV}} = \sqrt[5]{2} \times M_{\text{tot}}/2$

f_{peak} (post-merger) vs. f_{pert} (nonrotating star)



f_{peak} (post-merger) vs. $\Lambda^{1/5}$ (nonrotating star)



DETECTABILITY OF POST-MERGER PHASE

Single-detector detectability ($S/N > 5$, optimal orientation)

Instrument	SNR_{full}	SNR_{post}	D_{hor} [Mpc]	$\dot{\mathcal{N}}_{\text{det}}$ [year $^{-1}$]
aLIGO	2.99 ^{3.86} _{2.37}	1.48 ^{1.86} _{1.13}	29.89 ^{38.57} _{23.76}	0.01 ^{0.03} _{0.01}
A+	7.89 ^{10.16} _{6.25}	4.19 ^{5.35} _{3.26}	78.89 ^{101.67} _{62.52}	0.13 ^{0.20} _{0.10}
LV	14.06 ^{18.13} _{11.16}	7.28 ^{9.30} _{5.64}	140.56 ^{181.29} _{111.60}	0.41 ^{0.88} _{0.21}
ET-D	26.65 ^{34.28} _{20.81}	12.16 ^{15.31} _{9.34}	266.52 ^{342.80} _{208.06}	2.81 ^{5.98} _{1.33}
CE	41.50 ^{53.52} _{32.99}	20.52 ^{25.83} _{15.72}	414.62 ^{535.22} _{329.88}	10.59 ^{22.78} _{5.33}

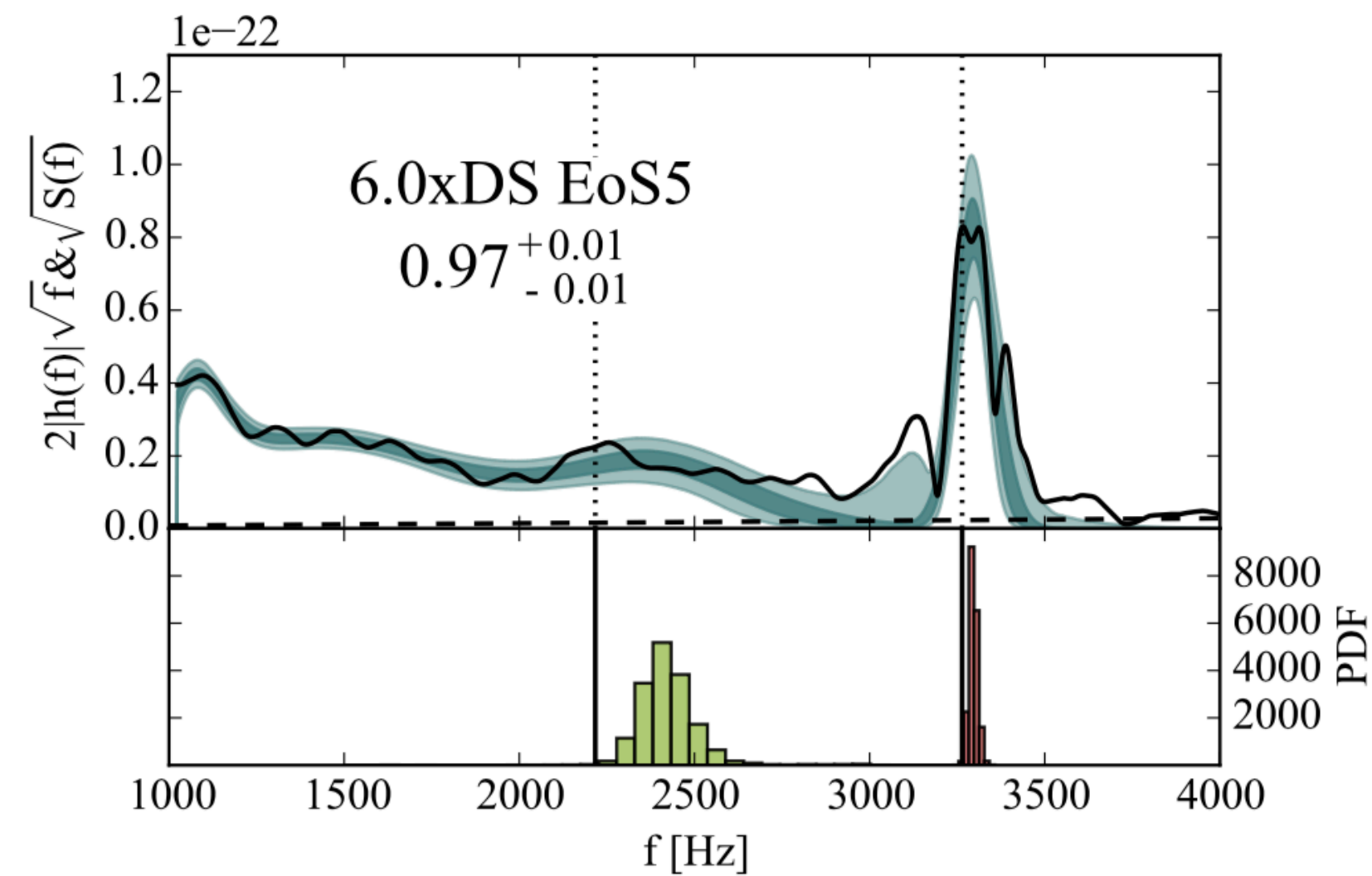
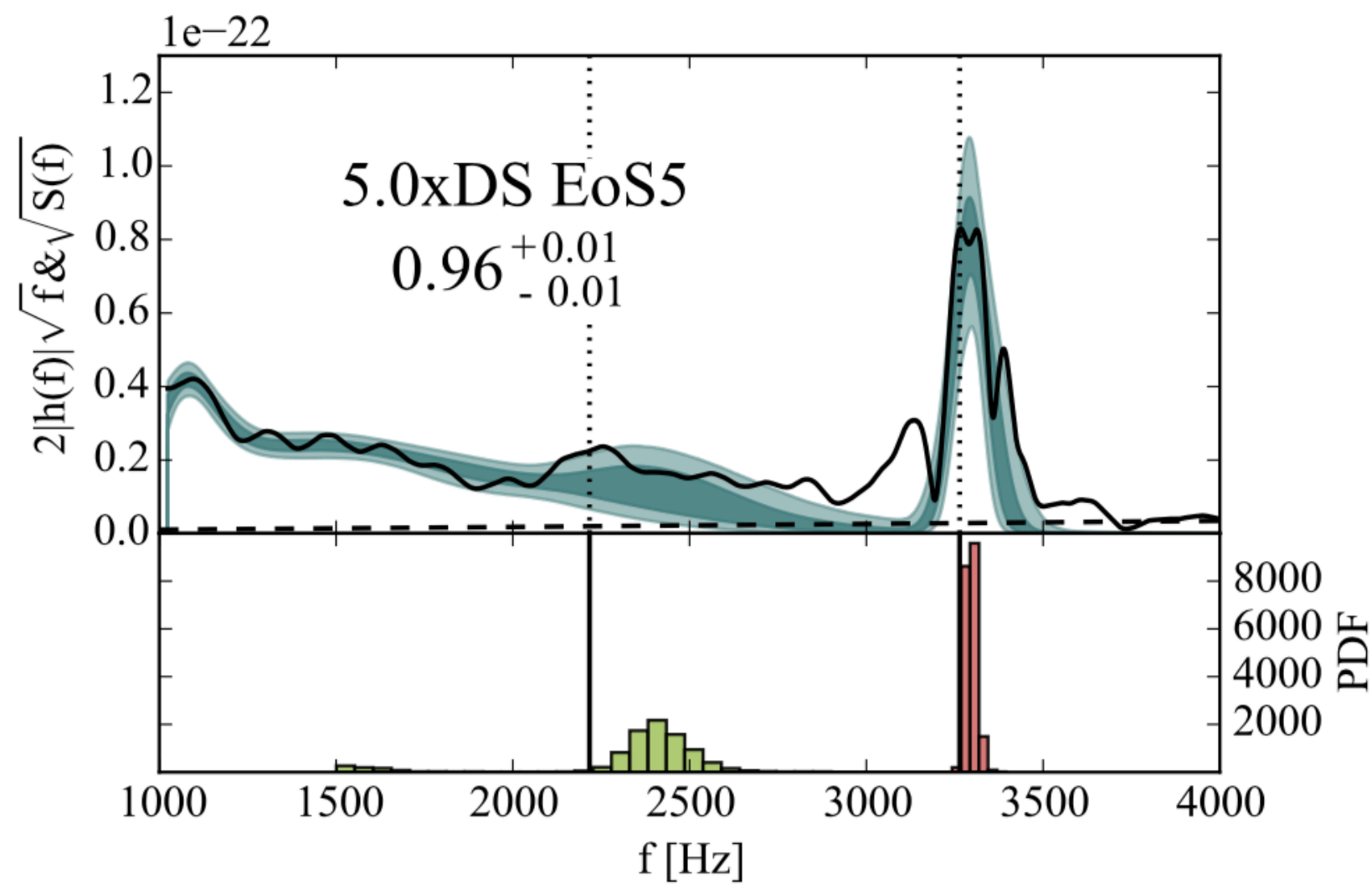
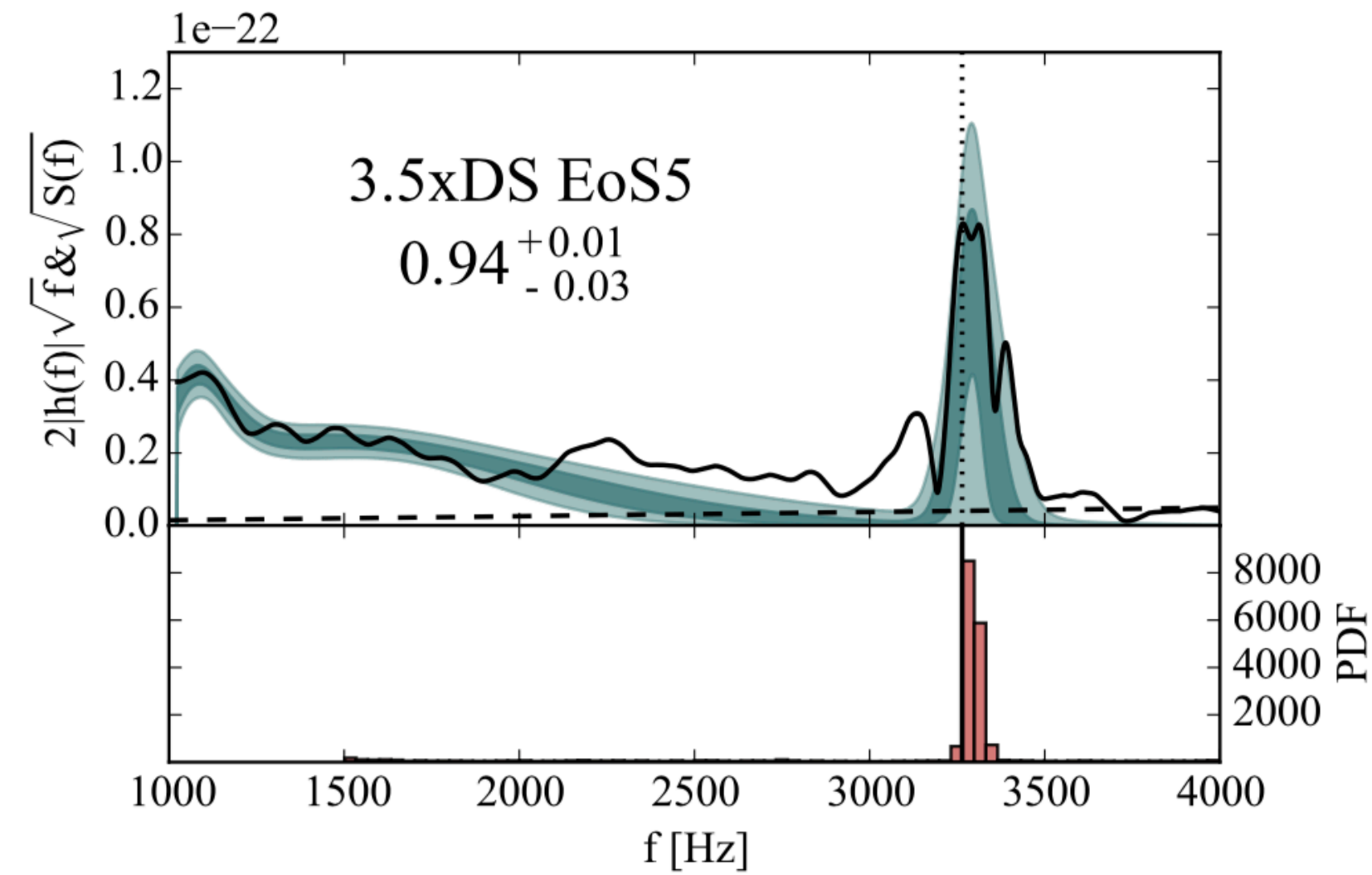
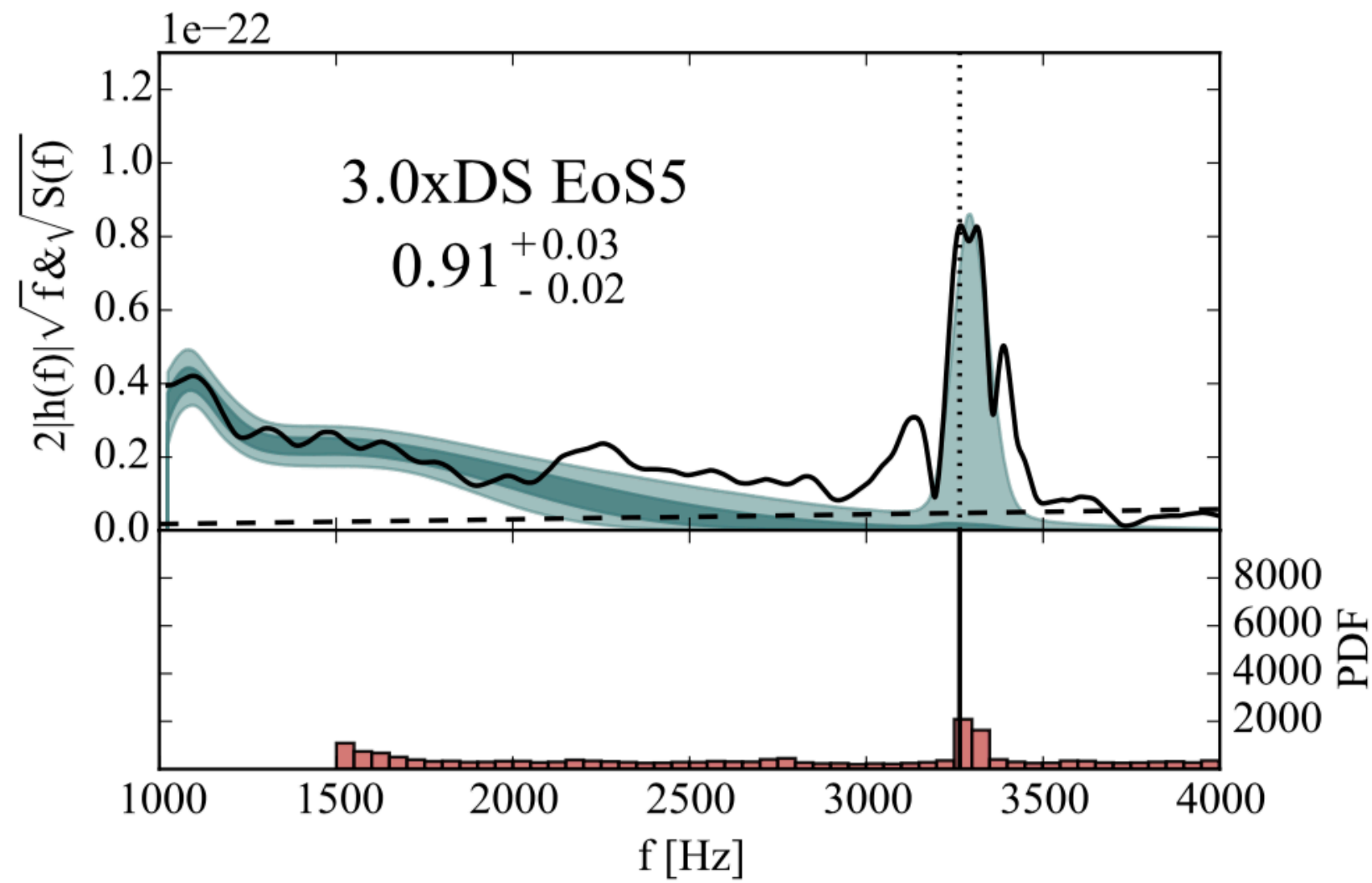
Clark, Bauswein, Stergioulas, Shoemaker (2016)

Possible Improvements:

- Network of 5 detectors
- Stacking of several detections
- Improved templates

DETECTABILITY OF POST-MERGER PHASE

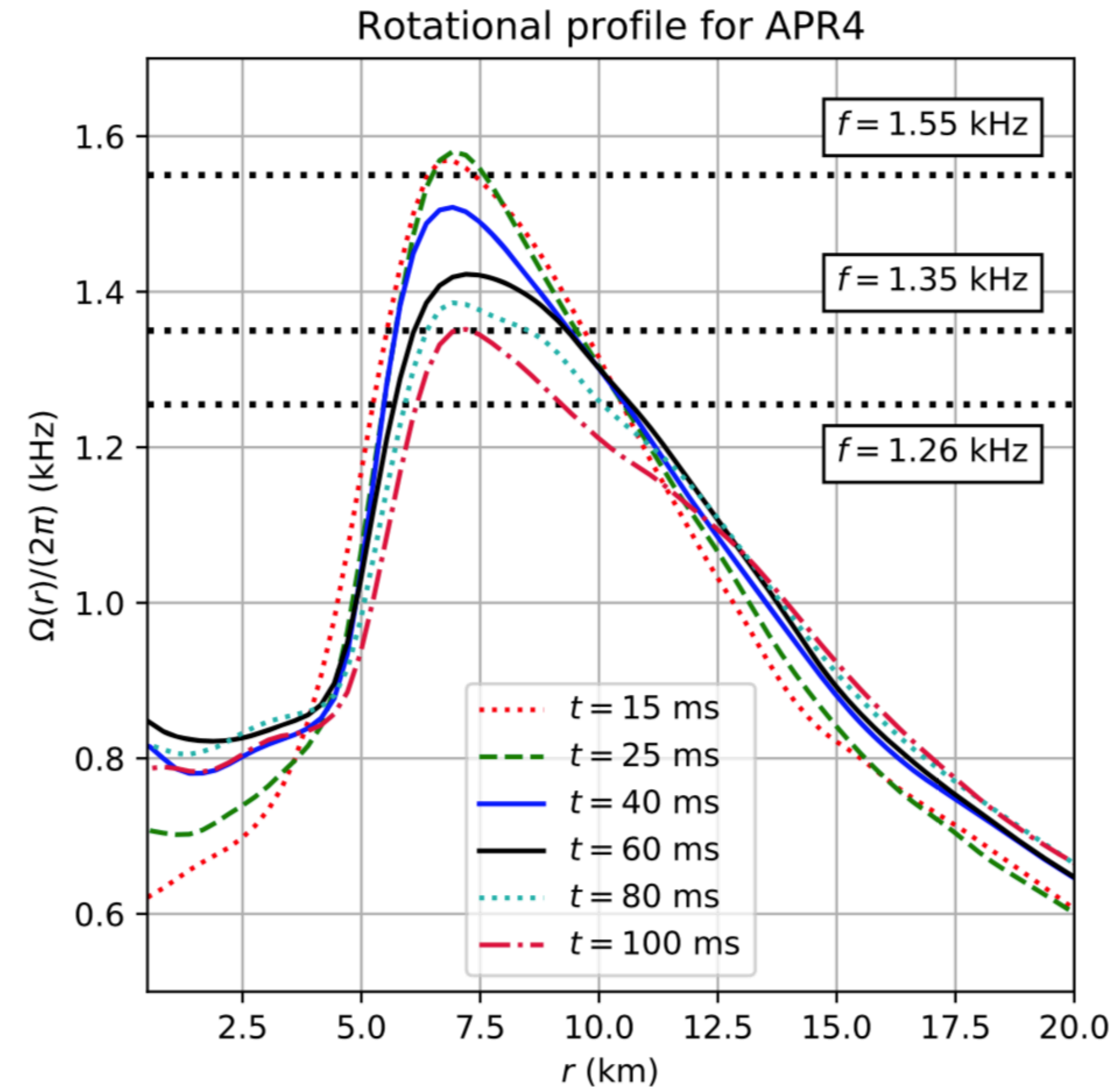
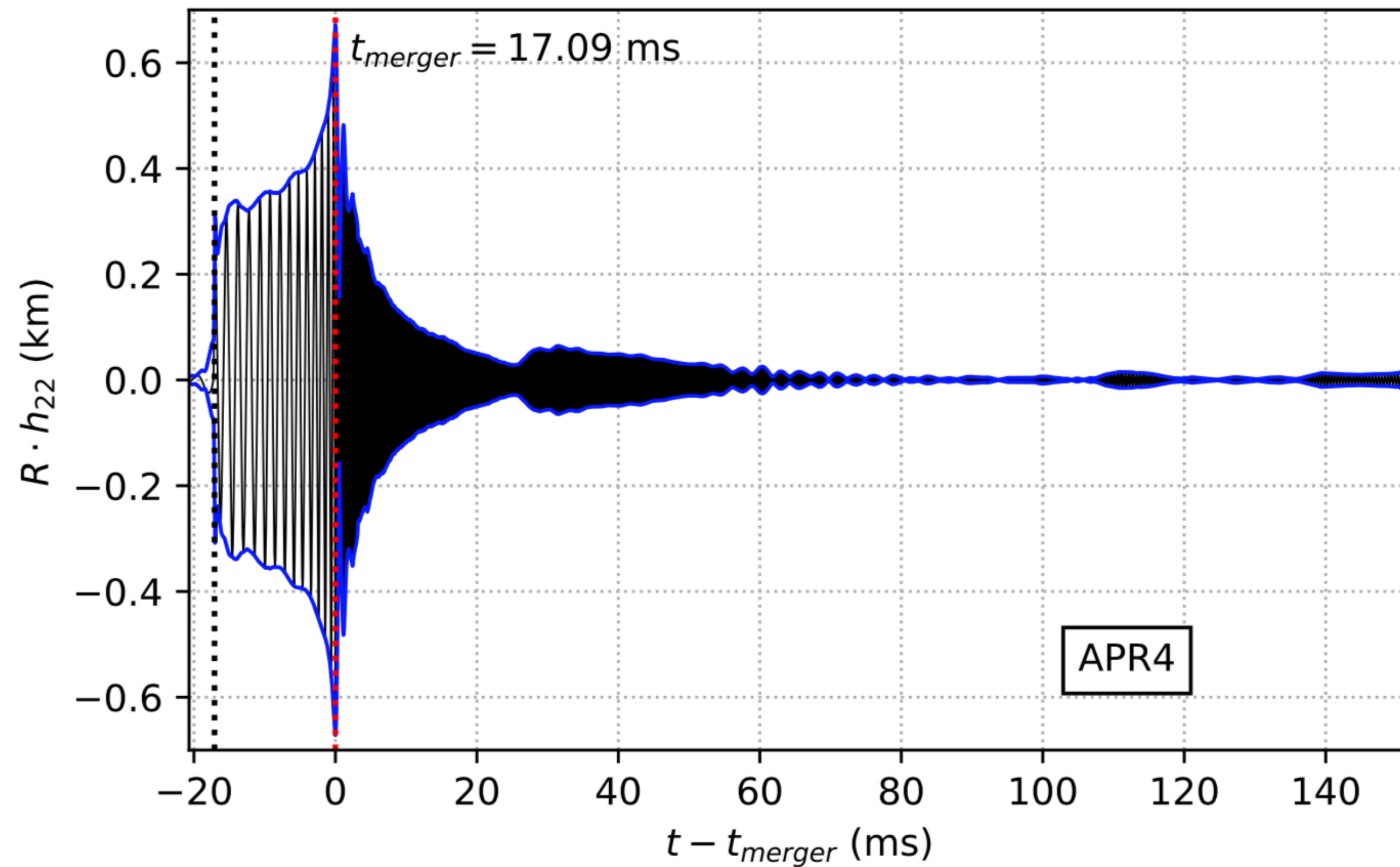
Wavelet-based reconstruction algorithm **BayesWave**



Torres-Rivas, Chatziioannou, Bauswein, Clark (2019)

LOW $|T/W|$ INSTABILITIES IN POST-MERGER REMNANTS

Revival of the $m=2$ mode at late times, due to a low $|T/W|$ shear instability, triggered by corotation points.



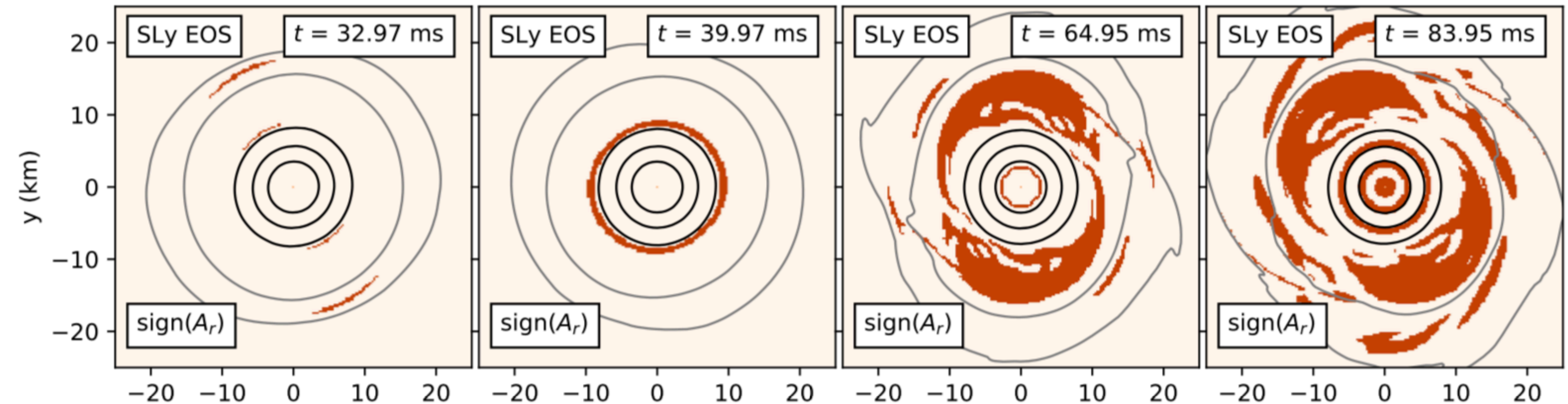
De Pietri et al. (2020)

see also Passamonti, Andersson (2020); Xie et al. (2020)

CONVECTIVE INSTABILITIES AND INERTIAL MODES IN POST-MERGER REMNANTS

At late times, convective instabilities trigger (gravito)-inertial oscillations.

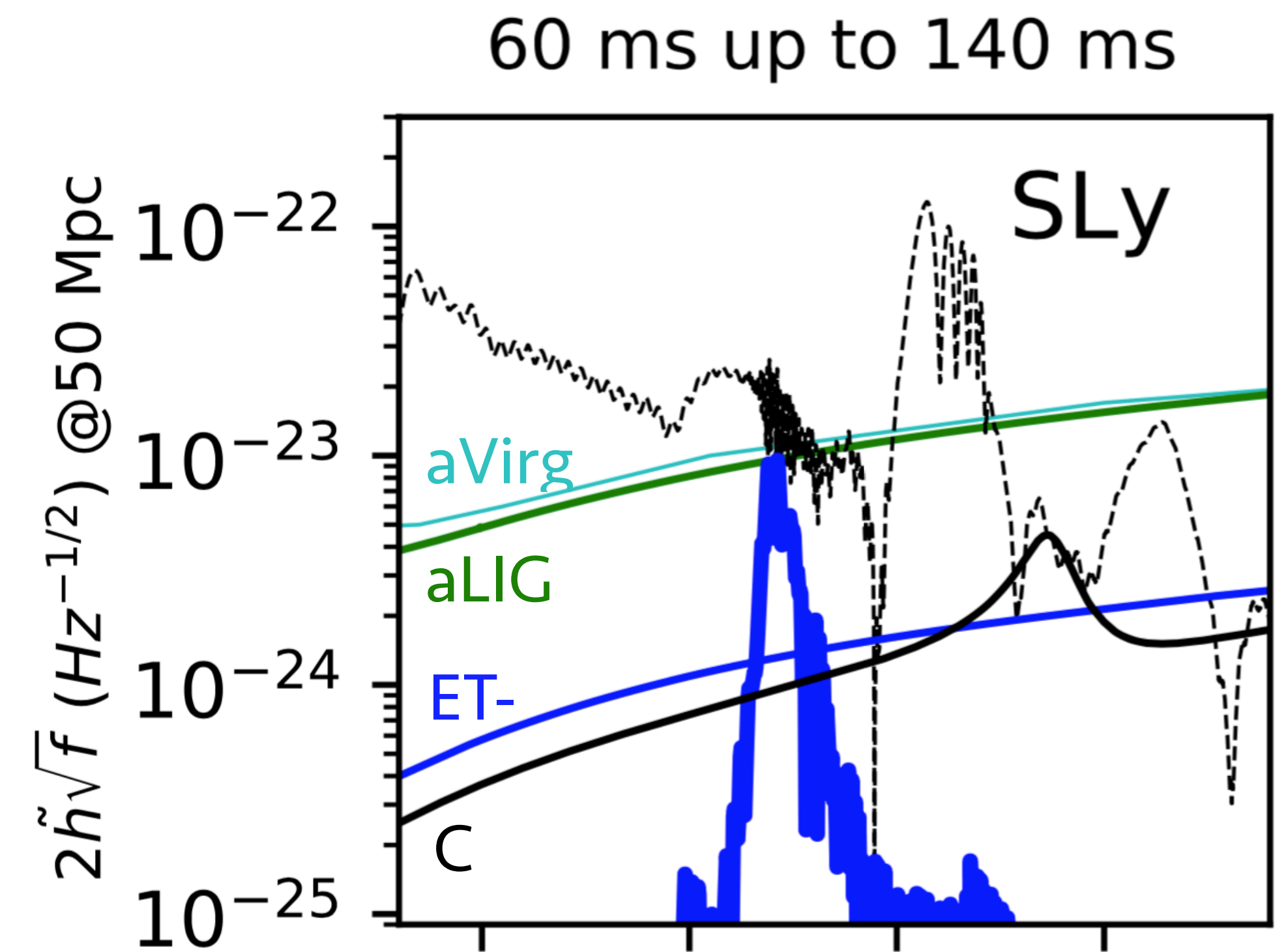
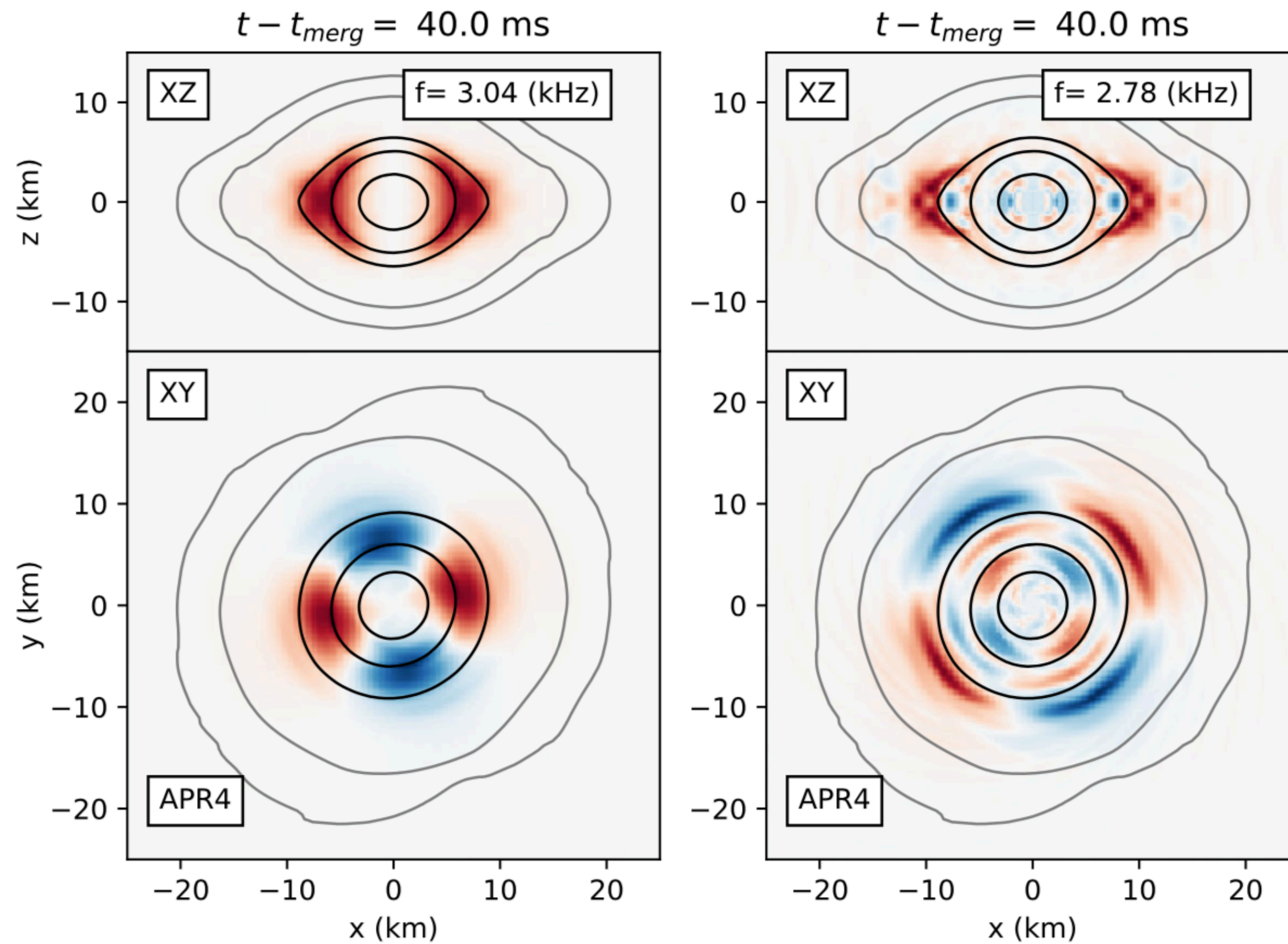
Sign of Schwarzschild discriminant in equatorial plane:



De Pietri et al. (2018) ; De Pietri et al. (2020)

CONVECTIVE INSTABILITIES AND INERTIAL MODES IN POST-MERGER REMNANTS

Potentially detectable with 3G detectors, unless suppressed by strong effective viscosity (e.g. due to MRI).



De Pietri et al. (2018) ; De Pietri et al. (2020)

CONCLUSIONS

- 1) The post-merger phase has rich GW phenomenology and good prospects for constraining EOS
- 2) The frequency of the main post-merger GW peak shows a tight correlation with the frequency of the fundamental quadrupole oscillation of isolated neutron stars.
- 3) We construct accurate empirical relations for the threshold mass M_{thres} , including asymmetric binaries.
- 4) We construct equilibrium models of post-merger remnants with realistic rotation profiles.
- 5) Using the equilibrium models, we can reproduce the threshold mass to collapse with remarkable accuracy.