Application of a hierarchical MCMC follow-up to Advanced LIGO continuous gravitational-wave candidates

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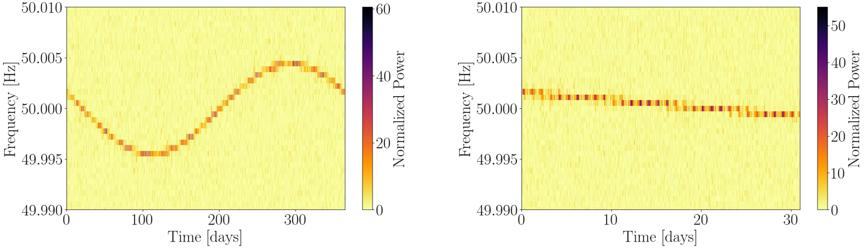
Introduction

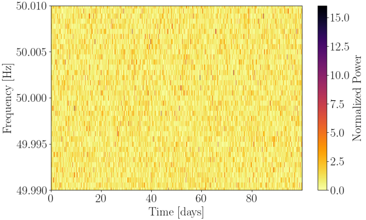
- Continuous waves (CWs) are long-duration quasi-monochromatic gravitational waves (GWs). No direct detection up to date.
- Expected sources are Neutron Stars (NS) presenting a non-axisymmetry (crust deformations, r-modes, free precession).
- CWs are described by amplitude $\mathcal{A}$ and phase-evolution λ parameters:
  - $\mathcal{A} \rightarrow$ source orientation $(\cos\iota, \psi, \phi_0)$, nominal GW amplitude h_0.
  - $\lambda \rightarrow$ source spinup/spindown $(f_0, f_1, \dots)$, Doppler modulation due to the Earth's motion (α, δ), Doppler modulation due to a binary companion.



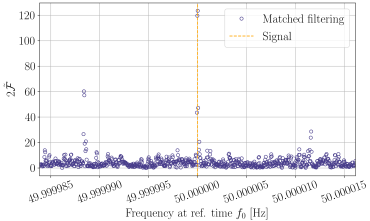
Introduction

Simple example of a CW search

- Set up a template bank covering the parameter space of interest (Note: $\mathcal{A}$ can be maximized out analytically, so only look for λ).
- Evaluate a statistic (e. g. matched filter) $2\tilde{\mathcal{F}}$ on each template.
- Retrieve outliers for further inspection.



$\xrightarrow{\text{Search } f_0}$

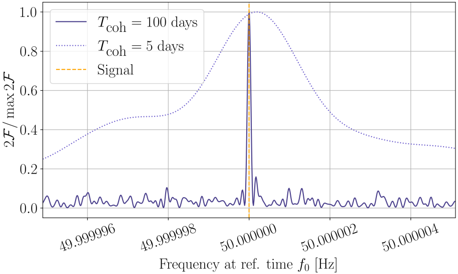


The number of required templates (computing cost) in order not to lose a signal scales with a strong power of the observing time. Matched filtering is unfeasible for wide parameter-space searches.

Semicoherent CW searches

Semicoherent $\mathcal{F}$-statistic

- Divide the data stream into N_{seg} segments with a duration of T_{coh}.
- Sum the result of filtering the data stream x against a signal template $h(\lambda)$ in each of these segments $2\hat{\mathcal{F}}(\lambda; x) = \sum_{s=1,\ \ldots,\ N_{\mathrm{seg}}} 2\tilde{\mathcal{F}}(\lambda; x(s))$.
- Shorter T_{coh} $\rightarrow$ Less constraining templates $\rightarrow$ Wider maxima.
- Semicoherent statistics allow to use coarser template banks (less computing cost), but are more susceptible to false alarms.

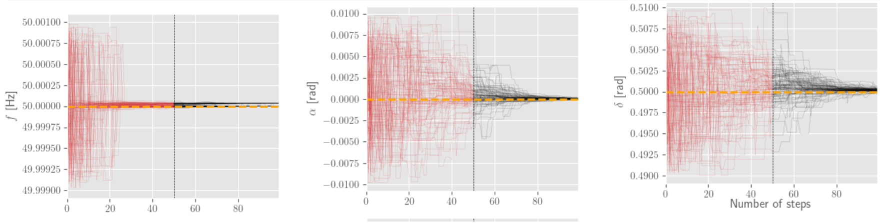


Hierarchical approach:

- Sweep wide regions with low coherence time.
- Focus on concrete outliers using longer coherence times.

A Bayesian perspective

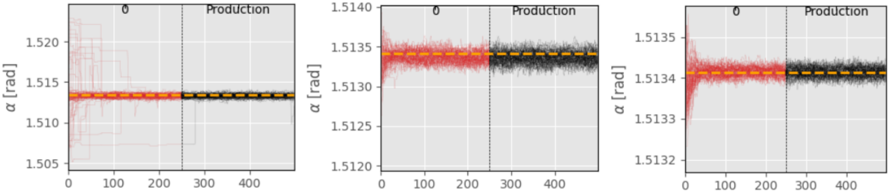
- $\hat{\mathcal{F}}(\lambda; x)$ is similar to a Bayes factor updating our prior knowledge on λ.
- Sampling the posterior $\mathrm{P}(\lambda|x) \propto \hat{\mathcal{F}}(\lambda; x)\ \mathrm{P}(\lambda)$ is equivalent to an adaptive grid search.
- `PyFstat` [D. Keitel + J. Open Source Softw. (2021)] implements this procedure using the `ptemcee` sampler [W. Vousden + MNRAS Vol. 455 1919-1937 (2016)]



Using an MCMC sampler simplifies the overall setup: its effectiveness essentially depends on the number of templates within the prior volume.

Hierarchical follow-up

- After running a search using $T_{\mathrm{coh}}^{(0)}$, a follow-up can be run using $T_{\mathrm{coh}}^{(1)} > T_{\mathrm{coh}}^{(0)}$, refining the effective parameter-space resolution.
- Coherence time can be increased as the MCMC sampler narrows down the outlier parameters until a fully-coherent follow-up is reached.



A Hierarchical MCMC follow-up [Ashton & Prix Phys. Rev. D 97, 103020 (2018)]

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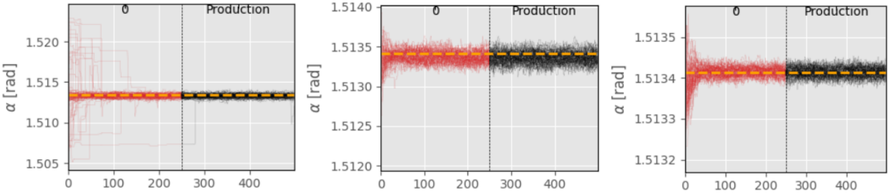


So...?

What can we conclude about a set of follow-up stages with different $T_{\rm coh}$?

Evaluating the hierarchical follow-up with a Bayes factor

Bayes factor

Suppose we run a complete MCMC follow-up and the loudest candidate returns an $\mathcal{F}$-statistic value of $2\hat{\mathcal{F}}^*$ from a semicoherent stage and the final fully-coherent $2\tilde{\mathcal{F}}^*$ stage.

- ($\mathcal{H}_\mathrm{N}$) If there is no signal, $2\tilde{\mathcal{F}}^*$ is the maximum of $\mathcal{N}$ random variables (one for each template sampled).
- ($\mathcal{H}_\mathrm{S}$) If there is a signal, the retrieved value of $2\tilde{\mathcal{F}}^*$ has to be consistent with the value implied by $2\hat{\mathcal{F}}^*$.

$$\mathcal{B}^*_{\mathrm{S/N}} = \frac{\mathrm{P}(2\tilde{\mathcal{F}}^* | \mathcal{H}_\mathrm{S})}{\mathrm{P}(2\tilde{\mathcal{F}}^* | \mathcal{H}_\mathrm{N})}$$

$\mathcal{B}^*_{\mathrm{S/N}}$ can be easily evaluated to distinguish interesting candidates.

Noise Hypothesis

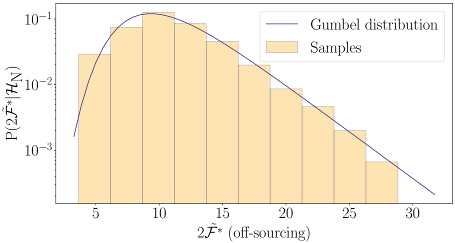
- We are interested in the distribution of $2\tilde{\mathcal{F}}^* = \max_{\mathcal{N}} 2\tilde{\mathcal{F}}$.
- Under the noise hypothesis, $2\tilde{\mathcal{F}} \sim \chi^2_4$, which means

$$\mathrm{P}(2\tilde{\mathcal{F}}^* | \mathcal{H}_{\mathrm{N}}) = \mathcal{N} \cdot \chi^2_4(2\tilde{\mathcal{F}}^*) \cdot \left[\int_0^{2\tilde{\mathcal{F}}^*} \mathrm{d}\xi \; \chi^2_4(\xi) \right]^{\mathcal{N}-1}$$

- This approach, however, only works if the $\mathcal{N}$ templates are completely independent. By construction, templates are correlated to ensure a good parameter-space coverage.
- A usual work around is to evaluate a template bank on Gaussian noise and fit the distribution using an *effective* number of templates $\mathcal{N}'$, but this is known to give inaccurate results (see Appendix A of arXiv:2105.13860 [gr-qc]).

Noise Hypothesis using Extreme Value Theory

- The distribution of $\max_{\mathcal{N}} 2\tilde{\mathcal{F}}^*$ for $\mathcal{N} \gg 1$ is $\mathrm{Gumbel}(\mu_{\mathrm{N}}, \sigma_{\mathrm{N}})$.
- Background noise samples can be generated by evaluating the template bank on sky positions away from that of the outlier ("off-sourcing" M. Isi + Phys. Rev. D 102, 123027 (2020)).
- $\mathrm{P}(2\tilde{\mathcal{F}}^* | \mathcal{H}_{\mathrm{N}})$ is constructed by fitting a Gumbel distribution to the maximum of off-sourced evaluations.



Signal Hypothesis [R. Prix LIGO-T1700236]

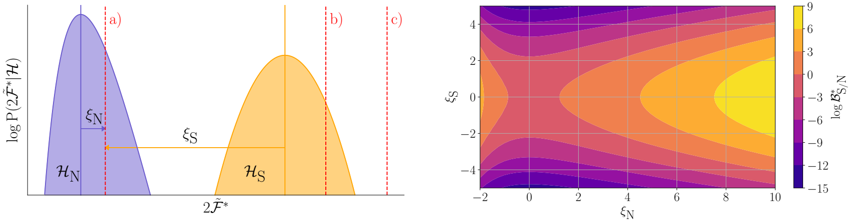
- The presence of a signal with (squared) SNR ρ^2 shifts the $\mathcal{F}$-statistic distribution to a non-central χ^2 distribution $2\hat{\mathcal{F}} \sim \chi^2_{4N_{\mathrm{seg}}}(\rho^2)$.
- The basic idea to construct a probability distribution is $2\hat{\mathcal{F}} \to \rho^2 \to 2\tilde{\mathcal{F}}$, which translates to

$$\mathrm{P}(2\tilde{\mathcal{F}}|\mathcal{H}_{\mathrm{S}}) \propto \int_0^\infty \mathrm{d}\rho^2 \; \mathrm{P}(2\tilde{\mathcal{F}}|\rho^2)\; \mathrm{P}(2\hat{\mathcal{F}}|\rho^2, N_{\mathrm{seg}})\; \mathrm{P}(\rho^2)\;.$$

- This integral can be numerically evaluated as all the involved probabilities are analytically known.
- In the strong signal regime $\rho^2 \gg 1$ this expression simplifies to a Gaussian distribution

$$\mathrm{P}(2\tilde{\mathcal{F}}^*|\mathcal{H}_{\mathrm{S}}) = \mathrm{Gauss}(2\tilde{\mathcal{F}}^*; \mu_{\mathrm{S}}, \sigma_{\mathrm{S}})$$

Bayes factor: Qualitative description



Basic behavior of $\ln \mathcal{B}^*_{\mathrm{S/N}}$

a) $\xi_{\mathrm{N}} < \xi_{\mathrm{S}} \rightarrow \ln \mathcal{B}^*_{\mathrm{S/N}} < 0$ and the signal hypothesis is disfavoured.

b) $\xi_{\mathrm{S}} \sim 0 \rightarrow \ln \mathcal{B}^*_{\mathrm{S/N}} \propto \xi_{\mathrm{N}}$ and the signal hypothesis favours with SNR.

c) $\xi_{\mathrm{S}} \gg 0 \rightarrow \ln \mathcal{B}^*_{\mathrm{S/N}} \simeq -\frac{1}{2}\xi_{\mathrm{S}}^2 + \xi_{\mathrm{N}}$ *disfavouring* signal hypothesis.

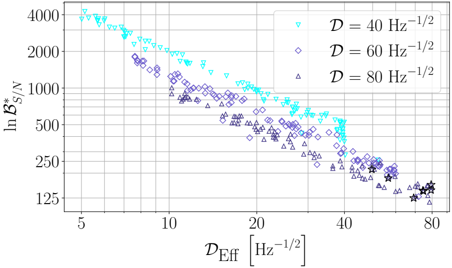
Bayes factor

O2-like injection campaign

- 100 CW signals were injected into 9 months of Gaussian noise at three depth values $\mathcal{D} = (40,\ 60,\ 80) 1/\sqrt{\mathrm{Hz}}$.

Stage	0	1	2	3	4
N_{seg}	500	250	55	5	1
T_{coh} [days]	0.5	1	5	55	270

Hyperparameter	Value
Parallel chains	3
Walkers per chain	100
Burn-in + Production	250 + 250

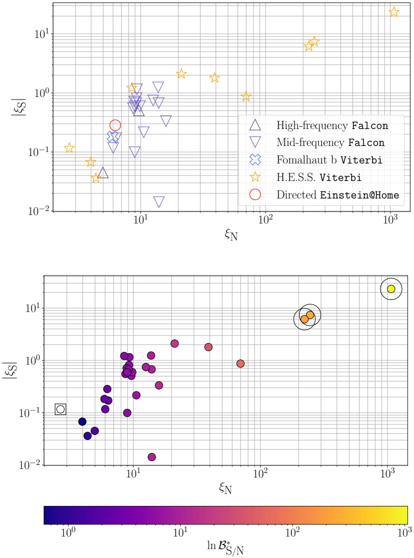


$$\mathcal{D}_{\mathrm{Eff}} = \frac{\sqrt{S_{\mathrm{n}}}/h_0}{\sqrt{\cos^4 \iota + 6\cos^2 \iota + 1}}$$

Outliers in O2 LIGO data

- We apply our new follow-up method to 30 outliers from open data searches on O2 LIGO data, using the same configuration as in the injection campaign.
- O2 Searches:
  - Mid-frequency Falcon search Dergachev & Papa Phys. Rev. Lett. 125, 171101 (2020).
  - High-frequency Falcon search Dergachev & Papa Phys. Rev. D 103, 063019 (2021).
  - Directed Einstein@Home search M. A. Papa + ApJ 897 22 (2020).
  - Fomalhaut b Viterbi search D. Jones & L. Sun Phys. Rev. D 103, 023020 (2021).
  - H.E.S.S. sources Viterbi search D. Beniwal + Phys.Rev.D 103 083009 (2021).
- Falcon & E@H are wide-parameter space searches reporting outliers as parameter-space points (with uncertainty regions).
- Viterbi searches are directed towards a particular sky position and use a Hidden Markov Model to model the frequency evolution of a CW.

Outliers in O2 LIGO data



- Safe threshold at $\ln \mathcal{B}^*_{S/N} = 90$ from the injection campaign.
- All but three H.E.S.S. Viterbi outliers are below threshold.
  - Outlier at $f_0 \simeq 20$ Hz corresponds to a known instrumental comb.
  - Outlier at $f_0 \simeq 26.34$ Hz due to a very strong hardware injection.
  - Outlier at $f_0 \simeq 15.4$ Hz was manually inspected and is consistent with an instrumental artifact.
- The next two outliers, also from H.E.S.S. Viterbi, can be related to a 1 Hz comb.

Conclusion

- We present the first complete framework to conduct hierarchical MCMC follow-ups on outliers stemming from generic CW searches.
- The results are evaluated using a novel Bayes factor. The probability distribution of the signal hypothesis is derived from first principles; the noise hypothesis' is constructed using extreme value theory.
- The follow-up is applied to a set of 30 outliers produced by several CW searches on open O2 LIGO Data.
- Only three of the outliers end up above threshold: One of them is due to an identified instrumental artifact, another one is produced by a strong hardware injection. Manual inspection suggests an instrumental origin for the third one as well, but we could not identify the cause.
- This new tool allows for the routine use of long coherence-time follow-ups on general CW searches, vastly simplifying the setup of post-processing steps.