Machine Learning forecasts of the distance duality relation with strongly lensed GW events

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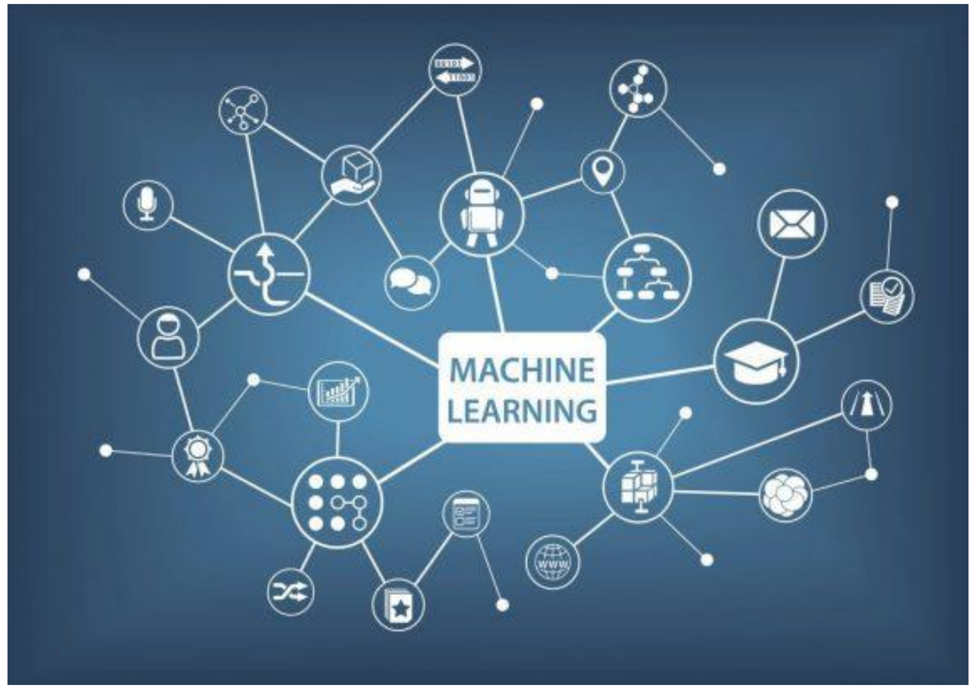
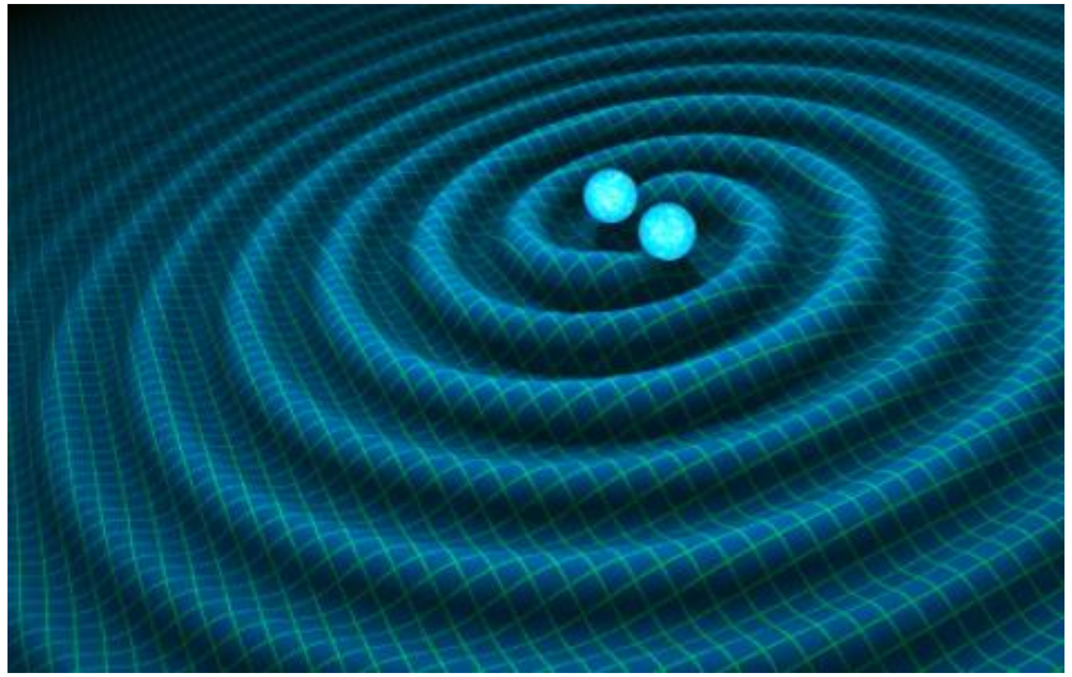
R. Arjona et al. 2001.02718





11th Iberian Gravitational Waves Meeting

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Outline

I. Distance Duality relation

II. Strongly Lensed GW events

III. Machine Learning forecasts

IV. Results

The distance duality relation

Etherington relation: relates the luminosity distance to the angular diameter distance at any redshift z

$$d_L(z) = (1+z)^2 d_A(z)$$

Duality parameter $$\eta(z) \equiv \frac{d_L(z)}{(1+z)^2 d_A(z)}$$

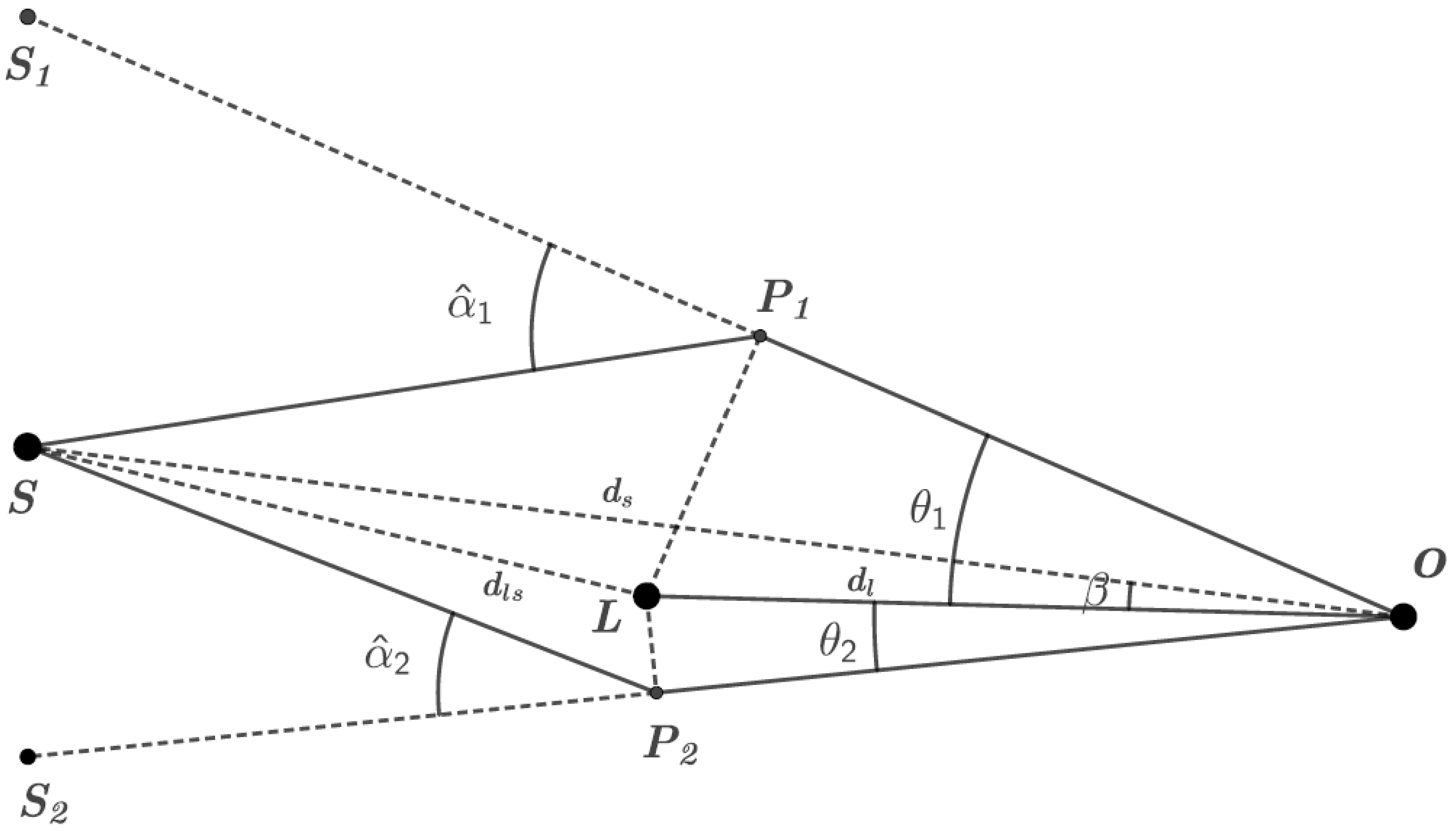
$$\equiv (1+z)^{\epsilon(z)}$$

Hint of new physics $\eta(z) \neq 1$ or $\epsilon_0 \neq 0$

Strongly Lensed GW events: $\mathbf{d_A(z)}$

In order for our method to work, we must independently measure the following observables:

$$\left(z_l,\ z_s,\ \Delta t, \Delta\phi,\ \theta_E,\ \sigma_{\mathrm{SIS}}\right)$$



The geometry of gravitational lensing

Strongly Lensed GW events: $\mathbf{d_A(z)}$

$$\left(z_l,\ z_s,\ \Delta t, \Delta\phi,\ \theta_E,\ \sigma_{\mathrm{SIS}}\right)$$

Time delay between the images $\Delta t = \frac{(1+z_l)}{c}\frac{d_A(z_l)d_A(z_s)}{d_A(z_l,z_s)}\Delta\phi$

Fermat potential difference $\Delta\phi = \frac{(\theta_1-\beta)^2}{2} - \Psi(\theta_1) - \frac{(\theta_2-\beta)^2}{2} + \Psi(\theta_2)$

Einstein radius $\theta_E = |\theta_1 - \theta_2|/2$ $\qquad \theta_E = \frac{4\pi\sigma_{\mathrm{SIS}}^2 d_A(z_l,z_s)}{c^2 d_A(z_s)}$

Distance ratio $\qquad R_A \equiv \frac{d_A(z_l,z_s)}{d_A(z_s)} = \frac{c^2\theta_E}{4\pi\sigma_{\mathrm{SIS}}^2}$

Strongly Lensed GW events: **$d_A(z)$**

Then we can solve uniquely for the angular diameter distance

$$d_A(z_s) = \frac{1+z_l}{1+z_s}\frac{R_A D_{\Delta t}}{1-R_A}$$

$$D_{\Delta t} \equiv \frac{d_A(z_l) d_A(z_s)}{d_A(z_l, z_s)} = \frac{c}{1+z_l}\frac{\Delta t}{\Delta \phi}$$

$$\frac{\delta d_A(z_s)}{d_A(z_s)} = \sqrt{\left(\frac{\delta R_A}{R_A(1-R_A)}\right)^2 + \left(\frac{\delta D_{\Delta t}}{D_{\Delta t}}\right)^2}$$

Luminosity distance from GW signals

The luminosity distance to the source can be directly obtained by matching the GW signals to the GW templates.

$$h(t) = F_+(\theta,\varphi,\psi)h_+(t) + F_\times(\theta,\varphi,\psi)h_\times(t)$$

$$\mathcal{H}(f) = \mathcal{A} f^{-7/6} \exp[i(2\pi f t_0 - \pi/4 + 2\psi(f/2) - \varphi_{(2,0)})]$$

$$\mathcal{A} = \frac{1}{d_L}\sqrt{F_+^2(1+\cos^2\iota)^2 + 4F_\times^2\cos^2\iota}\sqrt{\frac{5\pi}{96}}\pi^{-7/6}\mathcal{M}_c^{5/6}$$

Einstein Telescope

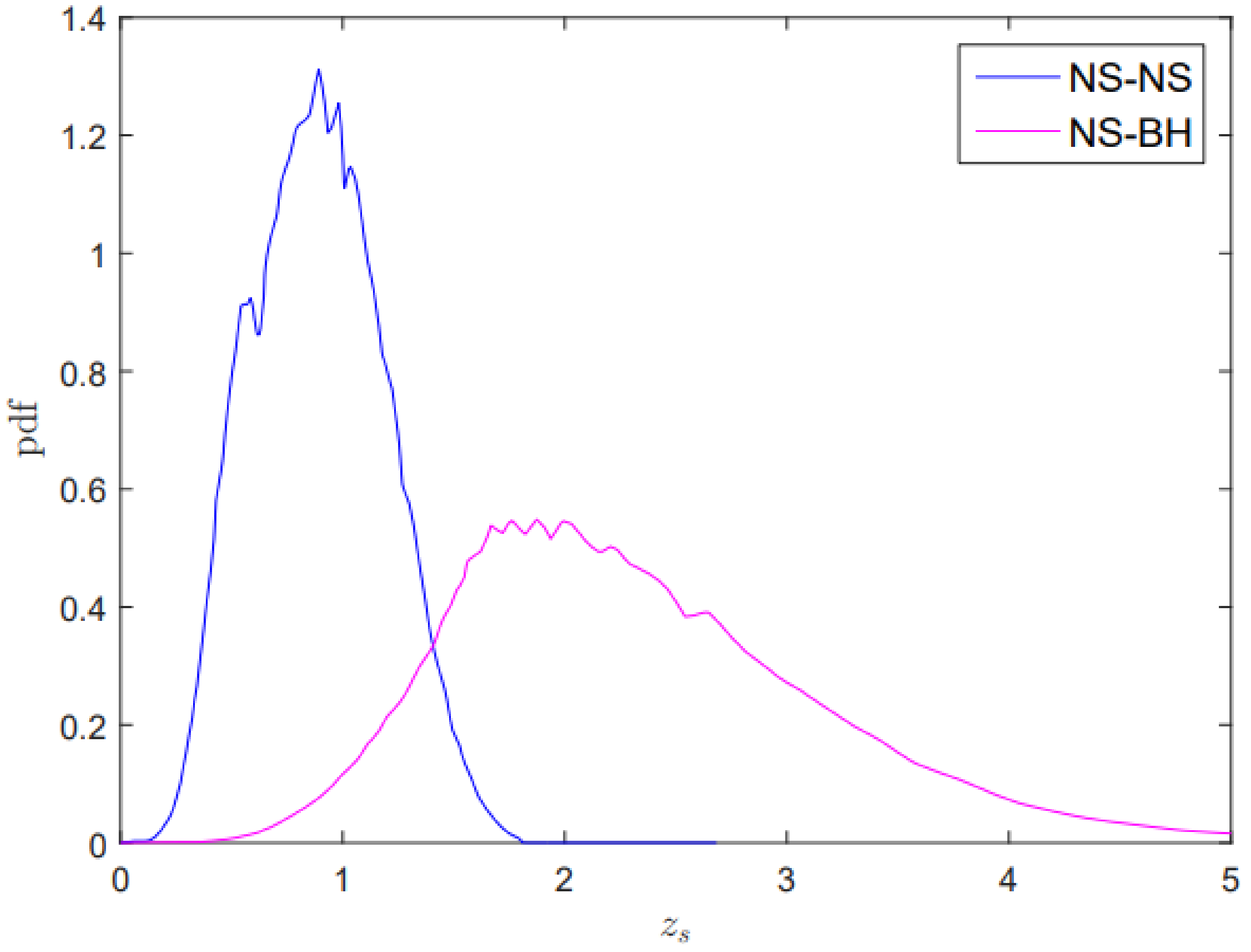
$$S_h(f) = 10^{-50}(2.39\times 10^{-27}x^{-15.64} + 0.349x^{-2.145} + 1.76x^{-0.12} + 0.409x^{1.1})^2\ \mathrm{Hz}^{-1}$$

Our method to measure the duality relation requires direct measurements of source redhsift, achievable only for NS-NS and NS-BH mergers.

Fiducial cosmological distances

Forecast direct measurements of the duality parameter from the ET

We use a MCMC approach to create mock samples.



Fiducial Cosmology ΛCDM

$$H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$$

$$\Omega_{m,0} = 0.3$$

$$\Omega_k = 0$$

Redshift distribution of SL GW sources based on the initial configuration of ET.

Fiducial cosmological distances

Modification of the luminosity distance

$$d_{L,\rm obs}(z) = (1+z)^{\epsilon(z)} d_{L,\rm bare}(z) \longrightarrow \eta(z) \equiv \frac{d_L(z)}{(1+z)^2 d_A(z)} \equiv (1+z)^{\epsilon(z)}$$

$$\epsilon_0 = (0.01,\ 0.05,\ 0.1)$$

Mock sample: at each redshift we create mock distances $d_A(z_s)$ and $d_L(z_s)$

$$(D_{i,\rm mock}, \sigma_{i,\rm mock}) \to \mathcal{N}(D_{i,\rm fid}, \sigma_{i,\rm fid})$$

Mock DDR data points

MCMC-like approach to obtain the mean values and the errors of the data points as follows:

1. Using the mock distances at each redshift $D_{i,\mathrm{mock}}$ we draw 10,000 random samples from the assumed distribution for $D_{i,\mathrm{mock}}$.

2. We then estimate $\eta(z_i)$ at each redshift z_i for each of the 10,000 random points using Eq. (2) to obtain 10,000 realisations of the distribution of $\eta(z_i)$.

$$\eta(z) \equiv \frac{d_L(z)}{(1+z)^2 d_A(z)} \equiv (1+z)^{\epsilon(z)}$$

3. We estimate the mean and standard deviation of $\log_{10}\eta(z_i)$ at each redshift point to create our final mock sample.

$$-2\ln\mathcal{L} = \sum_{i=1}^{N_{\mathrm{lens}}} \left(\frac{\log_{10}\eta(z_i) - \log_{10}\eta^{\mathrm{th}}(z_i)}{\sigma_{\log_{10}\eta(z_i)}} \right)^2$$

Machine Learning

Machine learning (**ML**) is the study of computer algorithms that improve automatically through experience

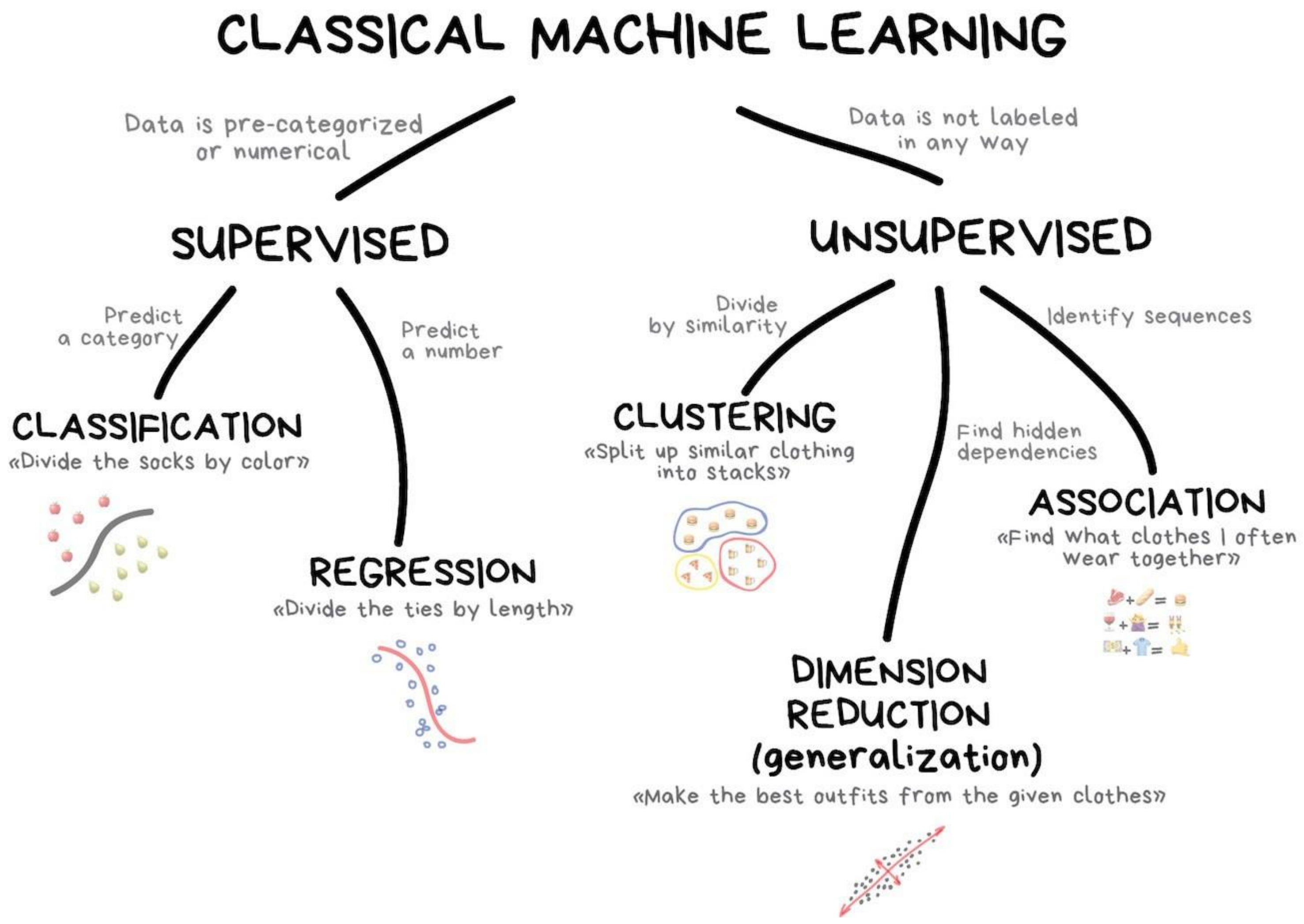
-Can **remove biases** due to a priori chosen models.

-Will play a big role in **testing** accurately the ΛCDM model.

-Search for new physics or **systematics** in the data.

-Search for tensions in the data by placing **tighter constraints** on parameters.

-Applied to **reconstruct** consistency tests of ΛCDM.



Genetic Algorithms (GA)

-The GA is a stochastic optimization and reconstruction ML approach, not very different from MCMC.

-The model itself is evolving as the code runs, since we are exploring the functional space.

-It's not biased by a priori selected models or other assumptions.

-Strong mathematical foundations with several rigorous theorems on convergence, selection, etc.

-Well tested and simple to implement.

Genetic Algorithms

Reconstruct data without assumptions on the theoretical model

-Stochastic search approach

-Start with a set of functions and grammar

$$(\sin(x), 1 + x + x^2, e^x, \log(x), \ldots)(+, -, \times, \div, \wedge)$$

-Two basic operations:

*Mutation

$$f_1(x) = 1 + x + x^2 \;\Downarrow\; f_1(x) = 1+2x+x^2$$

$$f_2(x) = \sin(x) + \cos(x) \;\Downarrow\; f_2(x) = \sin(x^2)+\cos(x)$$

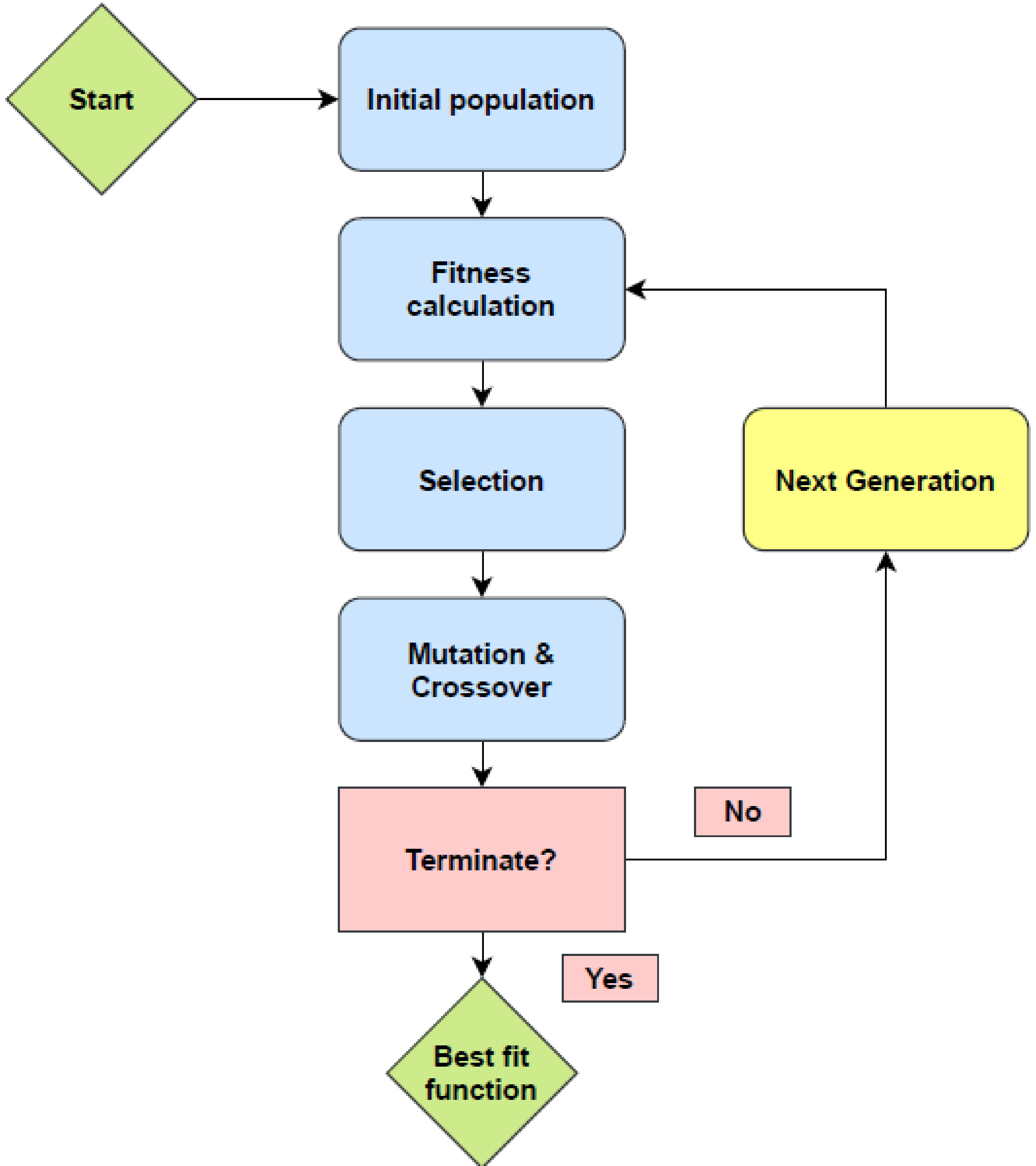
*Crossover

$$\widetilde{f}_1(x) = 1 + 2x + \cos(x)$$

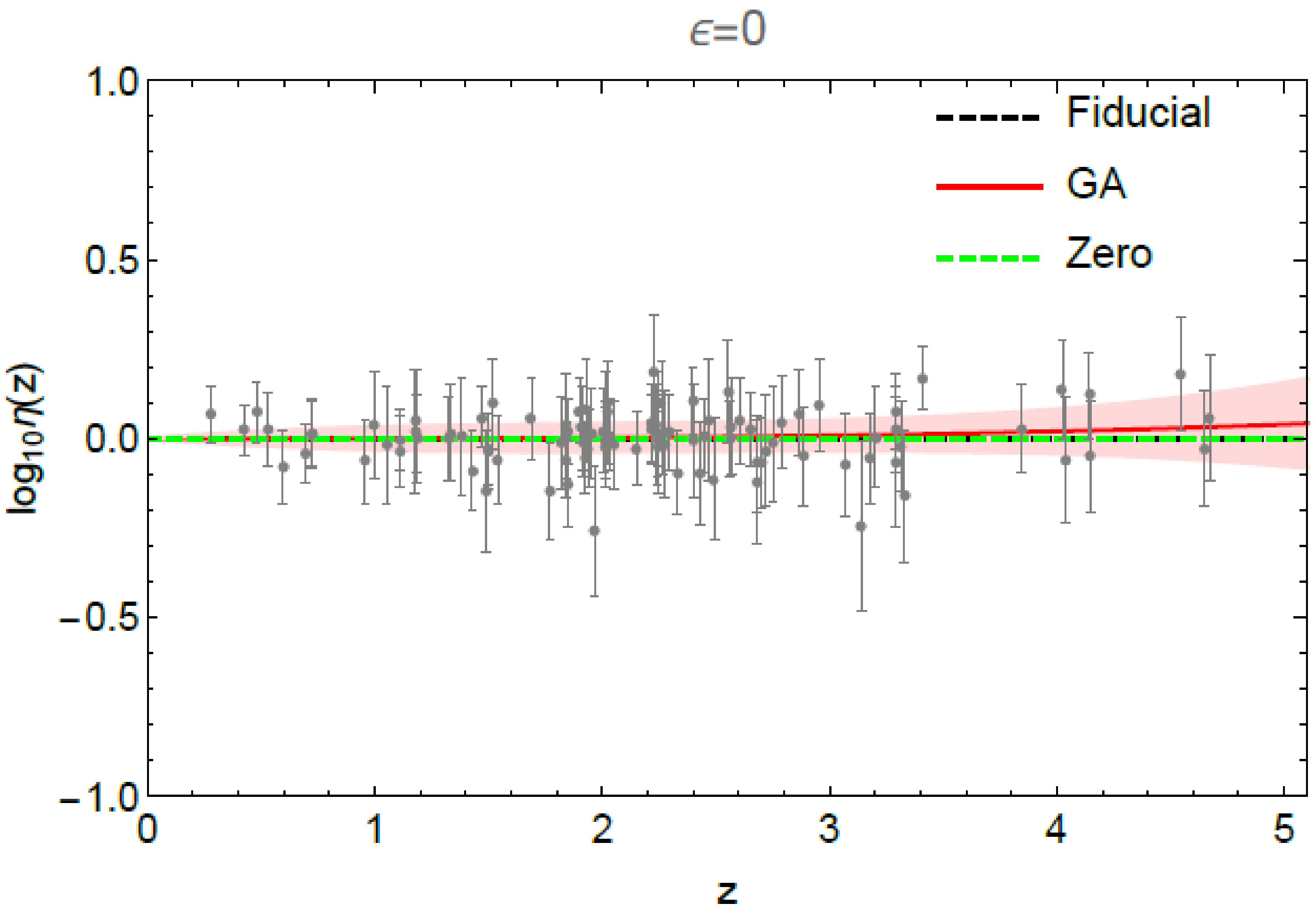
$$\widetilde{f}_2(x) = x^2 + \sin(x^2)$$

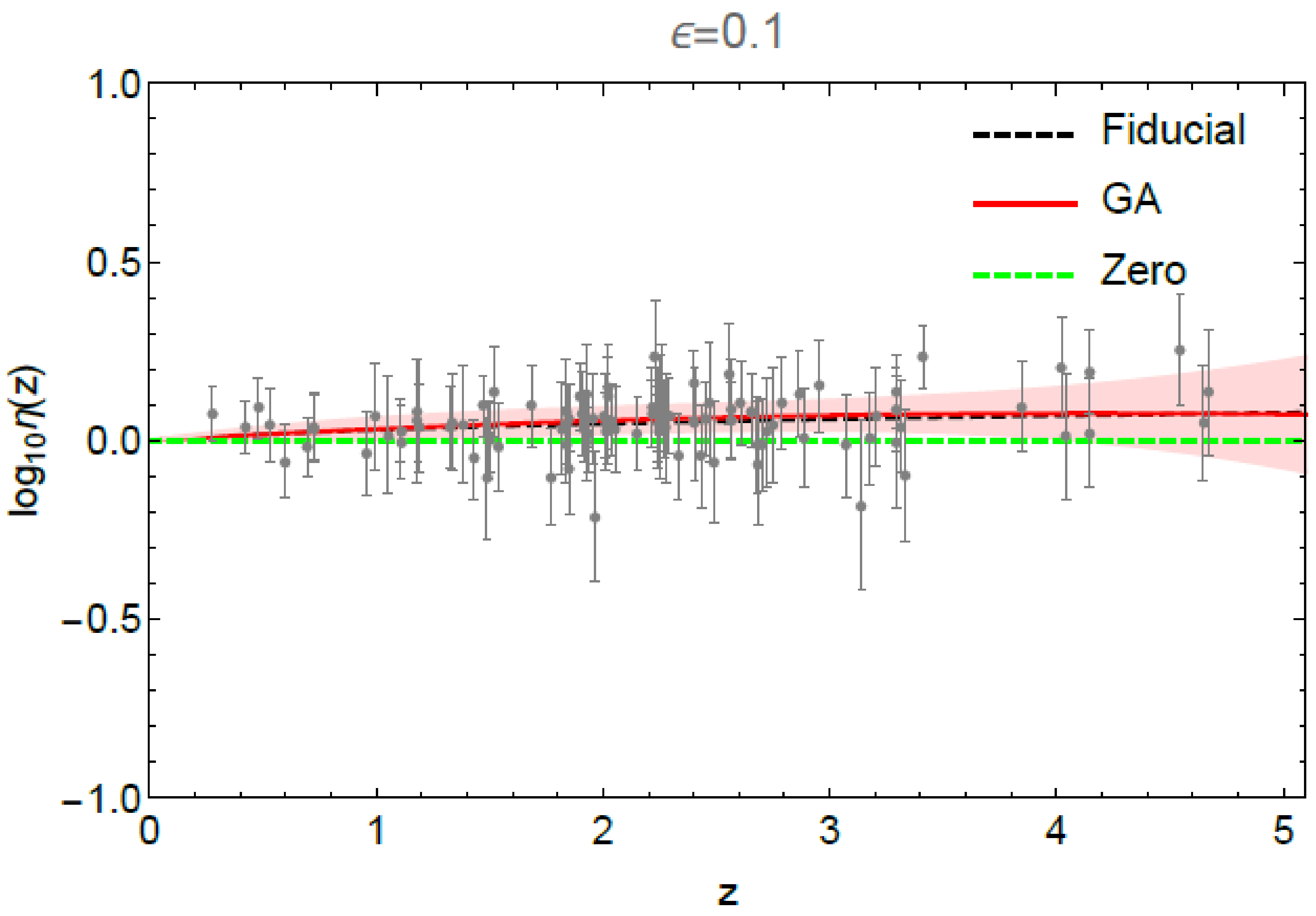
-Fitness Function

$$\chi^2(f) \equiv \sum_{i=1}^{N} \left(\frac{y_i - f(x_i)}{\sigma_i} \right)^2$$



Results


ϵ=0
Fiducial
GA
Zero
1.0
0.5
0.0
−0.5
−1.0
0
1
2
3
4
5
z
log10η(z)


ϵ=0.1
Fiducial
GA
Zero
1.0
0.5
0.0
−0.5
−1.0
0
1
2
3
4
5
z
log10η(z)

Conclusions

I. **Test** of **fundamental physics** with strong GW lensing

II. Methodology to create direct duality relation **mocks**

III. A **Machine Learning** approach to reconstruct the duality relation

Back-up slides