Machine Learning forecasts of the distance duality relation with strongly lensed GW events

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I. Distance Duality relation

II. Strongly Lensed GW events

III. Machine Learning forecasts

IV. Results
The distance duality relation

Etherington relation: relates the luminosity distance to the angular diameter distance at any redshift $z$

$$d_L(z) = (1 + z)^2 d_A(z)$$

Duality parameter

$$\eta(z) \equiv \frac{d_L(z)}{(1 + z)^2 d_A(z)}$$

$$\equiv (1 + z)^{\epsilon(z)}$$

Hint of new physics

$$\eta(z) \neq 1 \text{ or } \epsilon_0 \neq 0$$
Strongly Lensed GW events: $d_A(z)$

In order for our method to work, we must independently measure the following observables:

$$(z_l, z_s, \Delta t, \Delta \phi, \theta_E, \sigma_{\text{SIS}})$$

The geometry of gravitational lensing
Strongly Lensed GW events: $d_A(z)$

$$(z_l, z_s, \Delta t, \Delta \phi, \theta_E, \sigma_{SIS})$$

Time delay between the images
$$\Delta t = \frac{(1 + z_l)}{c} \frac{d_A(z_l) d_A(z_s)}{d_A(z_l, z_s)} \Delta \phi$$

Fermat potential difference
$$\Delta \phi = \frac{(\theta_1 - \beta)^2}{2} - \Psi(\theta_1) - \frac{(\theta_2 - \beta)^2}{2} + \Psi(\theta_2)$$

Einstein radius
$$\theta_E = \frac{|\theta_1 - \theta_2|}{2} \quad \theta_E = \frac{4\pi \sigma_{SIS}^2 d_A(z_l, z_s)}{c^2 d_A(z_s)}$$

Distance ratio
$$R_A \equiv \frac{d_A(z_l, z_s)}{d_A(z_s)} = \frac{c^2 \theta_E}{4\pi \sigma_{SIS}^2}$$
Strongly Lensed GW events: $d_A(z)$

Then we can solve uniquely for the angular diameter distance

$$d_A(z_s) = \frac{1 + z_l}{1 + z_s} \frac{R_A D_{\Delta t}}{1 - R_A}$$

$$D_{\Delta t} \equiv \frac{d_A(z_l) d_A(z_s)}{d_A(z_l, z_s)} = \frac{c}{1 + z_l} \frac{\Delta t}{\Delta \phi}$$

$$\frac{\delta d_A(z_s)}{d_A(z_s)} = \sqrt{\left(\frac{\delta R_A}{R_A(1 - R_A)}\right)^2 + \left(\frac{\delta D_{\Delta t}}{D_{\Delta t}}\right)^2}$$
The luminosity distance to the source can be directly obtained by matching the GW signals to the GW templates.

\[ h(t) = F_+(\theta, \varphi, \psi)h_+(t) + F_\times(\theta, \varphi, \psi)h_\times(t) \]

\[ \mathcal{H}(f) = Af^{-7/6} \exp[i(2\pi ft_0 - \pi/4 + 2\psi(f/2) - \varphi(2,0))] \]

\[ A = \frac{1}{d_L} \sqrt{F_+^2(1 + \cos^2 \iota)^2 + 4F_\times^2 \cos^2 \iota} \sqrt{\frac{5\pi}{96}} \pi^{-7/6} M_c^{5/6} \]

\[ S_h(f) = 10^{-50}(2.39 \times 10^{-27} x^{-15.64} + 0.349 x^{-2.145} + 1.76 x^{-0.12} + 0.409 x^{1.1})^2 \text{ Hz}^{-1} \]

Our method to measure the duality relation requires direct measurements of source redshift, achievable only for NS-NS and NS-BH mergers.
Fiducial cosmological distances

Forecast direct measurements of the duality parameter from the ET

We use a MCMC approach to create mock samples.

![Graph showing redshift distribution](image)

**Fiducial Cosmology $\Lambda$CDM**

\[
H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}
\]

\[
\Omega_{m,0} = 0.3
\]

\[
\Omega_k = 0
\]

Redshift distribution of SL GW sources based on the initial configuration of ET.
Fiducial cosmological distances

Modification of the luminosity distance

\[ d_{L,\text{obs}}(z) = (1 + z)^\varepsilon(z) d_{L,\text{bare}}(z) \]

\[ \eta(z) \equiv \frac{d_L(z)}{(1 + z)^2 d_A(z)} \equiv (1 + z)^\varepsilon(z) \]

\[ \varepsilon_0 = (0.01, 0.05, 0.1) \]

Mock sample: at each redshift we create mock distances \( d_A(z_s) \) and \( d_L(z_s) \)

\[ (D_{i,\text{mock}}, \sigma_{i,\text{mock}}) \rightarrow \mathcal{N}(D_{i,\text{fid}}, \sigma_{i,\text{fid}}) \]
Mock DDR data points

MCMC-like approach to obtain the mean values and the errors of the data points as follows:

1. Using the mock distances at each redshift \( D_{i,\text{mock}} \) we draw 10,000 random samples from the assumed distribution for \( D_{i,\text{mock}} \).

2. We then estimate \( \eta(z_i) \) at each redshift \( z_i \) for each of the 10,000 random points using Eq. (2) to obtain 10,000 realisations of the distribution of \( \eta(z_i) \).

3. We estimate the mean and standard deviation of \( \log_{10} \eta(z_i) \) at each redshift point to create our final mock sample.

\[
-2 \ln \mathcal{L} = \sum_{i=1}^{N_{\text{lens}}} \left( \frac{\log_{10} \eta(z_i) - \log_{10} \eta^{\text{th}}(z_i)}{\sigma_{\log_{10} \eta(z_i)}} \right)^2
\]
Machine Learning

**Machine learning (ML)** is the study of computer algorithms that improve automatically through experience.

- Can **remove biases** due to a priori chosen models.

- Will play a big role in **testing** accurately the ΛCDM model.

- Search for new physics or **systematics** in the data.

- Search for tensions in the data by placing **tighter constraints** on parameters.

- Applied to **reconstruct** consistency tests of ΛCDM.
Genetic Algorithms (GA)

-The GA is a stochastic optimization and reconstruction ML approach, not very different from MCMC.

-The model itself is evolving as the code runs, since we are exploring the functional space.

-It’s not biased by a priori selected models or other assumptions.

-Strong mathematical foundations with several rigorous theorems on convergence, selection, etc.

-Well tested and simple to implement.
Genetic Algorithms

**Reconstruct data** without assumptions on the theoretical model

- Stochastic search approach

- Start with a set of functions and grammar
  \[(\sin(x), 1 + x + x^2, e^x, \log(x), \ldots) (\pm, -, \times, \div, \lor)\]

- Two basic operations:

  **Mutation**
  
  \[f_1(x) = 1 + x + x^2 \quad \Rightarrow \quad f_1(x) = 1 + 2x + x^2\]
  
  \[f_2(x) = \sin(x) + \cos(x) \quad \Rightarrow \quad f_2(x) = \sin(x^2) + \cos(x)\]

  **Crossover**
  
  \[\tilde{f}_1(x) = 1 + 2x + \cos(x)\]
  
  \[\tilde{f}_2(x) = x^2 + \sin(x^2)\]

- Fitness Function
  
  \[\chi^2(f) = \sum_{i=1}^{N} \left( \frac{y_i - f(x_i)}{\sigma_i} \right)^2\]
Results
Conclusions

I. Test of fundamental physics with strong GW lensing

II. Methodology to create direct duality relation mocks

III. A Machine Learning approach to reconstruct the duality relation
Back-up slides