IMRPhenomXE: First phenomenological waveform model for eccentric binary black holes with aligned spins

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Iberian gravitational wave meeting
June 10, 2021

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Introduction

- During O1, O2 and O3a, all LIGO and Virgo gravitational wave (GW) signals consistent with quasicircular binaries.

- Some efforts to search for eccentric signatures in GW events, like Romero+2019, Gayathri+2020.

- Current quasicircular waveform models for binary black holes (BBH) in a mature state with several families :
  - Effective One Body (EOB) : Hamiltonian formulation of the two body problem into effective description.
  - NRSurrogate : Interpolation of numerical relativity simulation data.
  - IMRPhenom : Phenomenological description of the GW signal.

- Increasing effort from EOB and NRSurrogate waveform families on modelling eccentric non-precessing binaries with higher order modes: Nagar+2021, Liu+2021, Khalil+2021, Albanesi+2021, Islam+2021

- Focus on latest generation of IMRPhenomX models:
  - IMRPhenomXAS [Pratten+2020]: only (2,±2) modes with non-precessing spins.
  - IMRPhenomXHM [Garcia-Quiros+2020]: higher modes with non-precessing spins.
  - IMRPhenomXP [Pratten+2020]: precessing spins with only (2,±2) modes in co-precessing frame.
  - IMRPhenomXPHM [Pratten+2020]: precessing spins with higher modes in co-precessing frame.

  - IMRPhenomTHM(PHM)[Estelles+2021]: new IMRPhenom family in time domain. See Hector's talk!

- This work → Extension of IMRPhenomXAS to small eccentricities ($e<0.1$).

Introduction to binary black holes

- Focus on BBH systems. GWs from a generic BBH can be described by 17 parameters:
  - Intrinsic parameters: component masses, m_i, dimensionless spin vectors, $\vec{\chi}_i = \vec{S}_i/m_i^2$, eccentricity, e, and argument of periapsis/mean anomaly, l.
  - Extrinsic parameters: luminosity distance, d_L, azimuthal angle, φ, inclination, ι, time of coalescence, t_c, right ascension, θ, declination, ϕ, and polarization angle ψ.

- The GW polarizations can be decomposed in a complex waveform strain:

$$h(t;\Theta) = h_+ - ih_\times = \sum_{l=2}^{\infty} \sum_{m=-l}^{l} Y_{lm}^{-2}(\iota,\varphi) h_{lm}(t;\Theta), \quad \Theta = \{m_i, \vec{\chi}_i, e, \Omega\}_{i=1,2}.$$

Y_{lm}^{-2} : spin-weighted spherical harmonics.

- BBH coalescence can be divided into 3 parts:
  - Inspiral: described by post-Newtonian (PN) theory. Weak field and slow motion compared to c.
  - Merger: speeds comparable to c, non-linear effects become dominant. Numerical relativity (NR) required.
  - Ringdown: Application of perturbation theory to study the remnant perturbed black hole (BH) decaying to a Kerr solution.

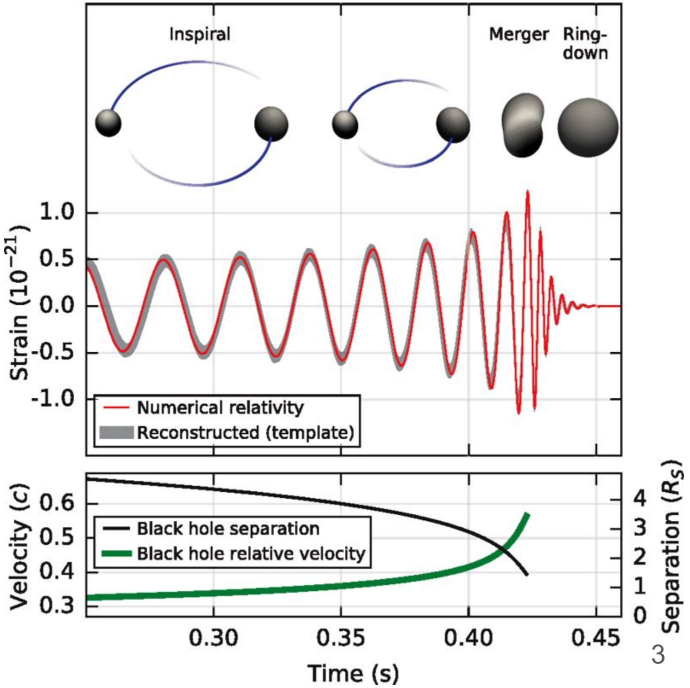


Figure from PhysRevLett.116.061102

IMRPhenomXAS

- Frequency domain model describing (2,2) mode of non-preccesing binary black holes.
- As with previous Phenom models, split modelling of the GW signal into 3 regions.
- Model tuned to 652 NR simulations including test particle limit waveforms.

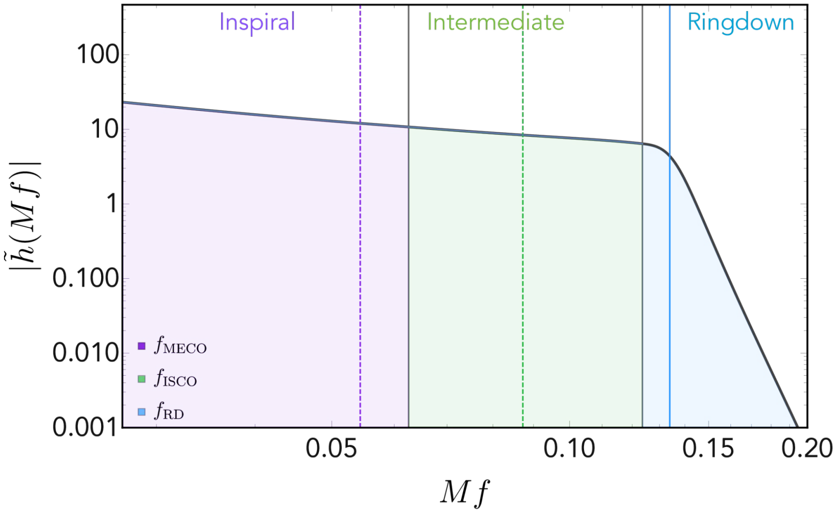


Figure from Phys. Rev. D 102, 064001

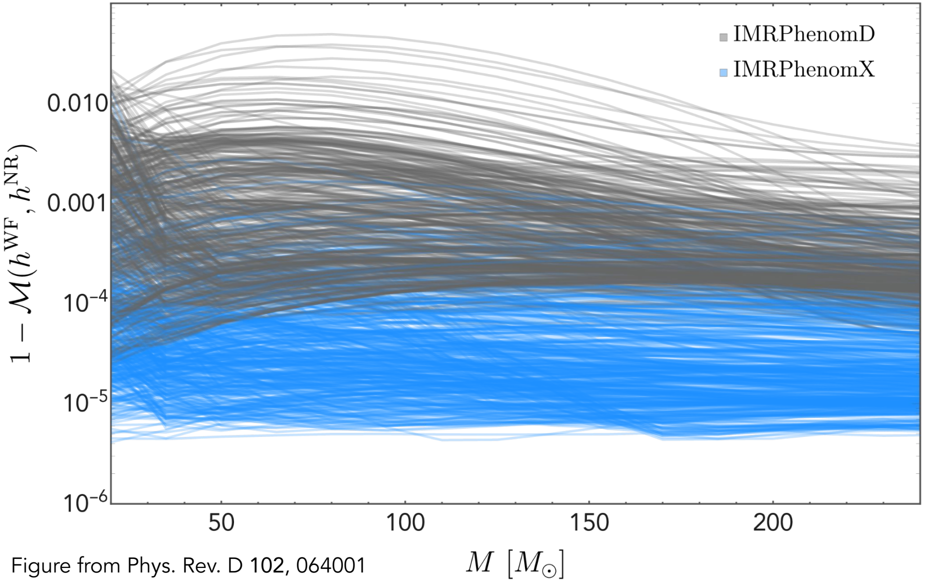


Figure from Phys. Rev. D 102, 064001

- Mismatch noise-weighted inner product between waveforms:
  - Mismatch = 0 : Identical waveforms.
  - Mismatch = 1 : Totally different waveforms.
- Comparison against NR → Order of magnitude accuracy improvement of IMRPhenomXAS.

Eccentric post-Newtonian waveforms I

- Use post-Newtonian eccentric non-spinning waveforms from Tiwari+2019.

- PN expressions derived within the quasiKeplerian parametrization using stationary phase approximation (SPA).

- Application of SPA → Signal decomposition as a sum of harmonics in terms of mean anomaly,

$$h_{+,\times}(t) = -\frac{Gm\eta}{c^2 D_L}\, x \sum_{j=1}^{10} \left[C_{+,\times}^{(j)}(e_t, \iota, \varphi) \cos jl + S_{+,\times}^{(j)}(e_t, \iota, \varphi) \sin jl \right]$$

- Analytical expressions for the polarizations expanded up to $\mathcal{O}(e_t^6)$. → Small eccentricity validity $e_t \lesssim 0.1.$

- The eccentric inspiral waveforms are:
  - 1PN in the amplitude.
  - 3PN in the phase incorporating periastron advance effects.

Eccentric post-Newtonian waveforms II

- Work to adapt Tiwari+ results for the polarizations to PhenomXAS 22-mode,
  - Project polarizations onto Y_{lm}^{-2} and apply the SPA to the resulting modes.
  - Analytical expressions for the (2,2) mode, but also for the higher order modes.
  - Added non-spinning eccentric corrections to amplitude and phase, only quasicircular spinning effects.

- Functional form of the (2,2) mode inspiral waveform,

$$h_{22}(f, \eta, e_t, f_{\rm ref}, f_{\rm cut}) = \sum_n \sum_{j=1}^N A_{n,j} e^{i\Psi_{n,j}} \times \Theta\left[\left(j - \frac{k}{1+k}(j+n)\right) f_{\rm cut} - 2f\right], \qquad n = +2, -2 \rightarrow N = 4, 8.$$

- Heaviside function → harmonics computed only on physically allowed frequency intervals.

- We choose $f_{\rm cut} = f_{\rm MECO}$.

- In the quasicircular limit only the n=-2 j=2 harmonic survives. It reduces to TaylorF2 approximant.

Inclusion of PN eccentric corrections to IMRPhenomXAS

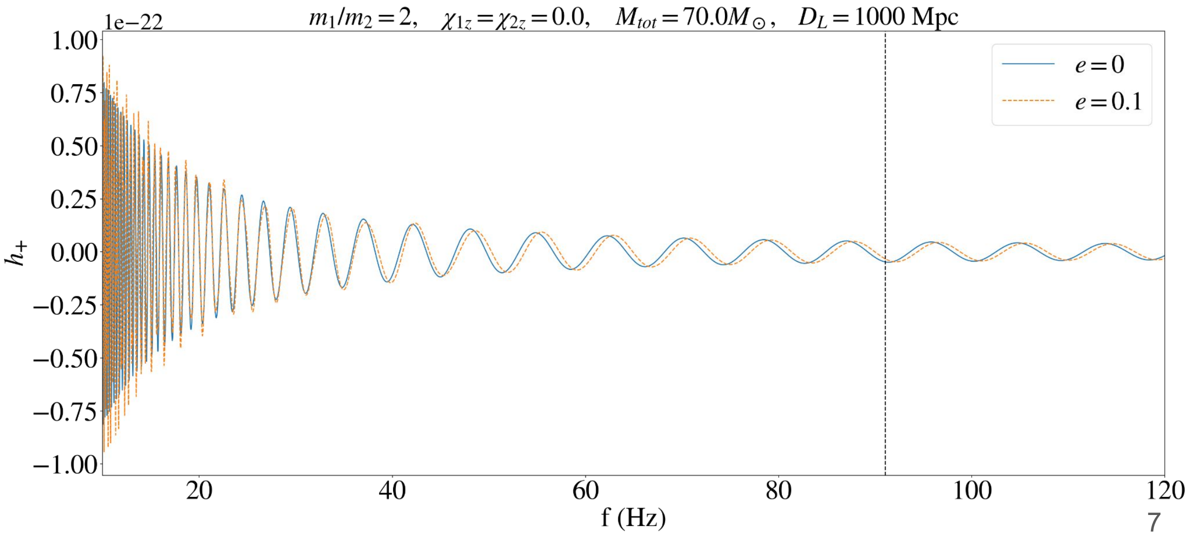
- We want to combine 2 waveforms: $h^{\rm XAS}_{\rm insp} = A^{\rm XAS} e^{i\phi^{\rm XAS}}, \quad h^{PN}_{\rm ecc} = \sum_n \sum_{j=1}^{N} A_{n,j} e^{i\Psi_{n,j}}.$

- Include IMRPhenomXAS amplitude terms to the (n=-2, j=2) harmonic,

$$h^{\rm XE}_{\rm insp} = \left(A_{-2,2} + A^{\rm XAS}\right) e^{i(\Psi_{-2,2}+\phi^{\rm XAS})} + \sum_n \sum_{j=1}^{N} A_{n,j} e^{i(\Psi_{n,j}+\phi^{\rm XAS})}.$$

Rescaled by j, n indices.

- Smoothness in attachment ensured by continuity of amplitude and phase at intermediate region.

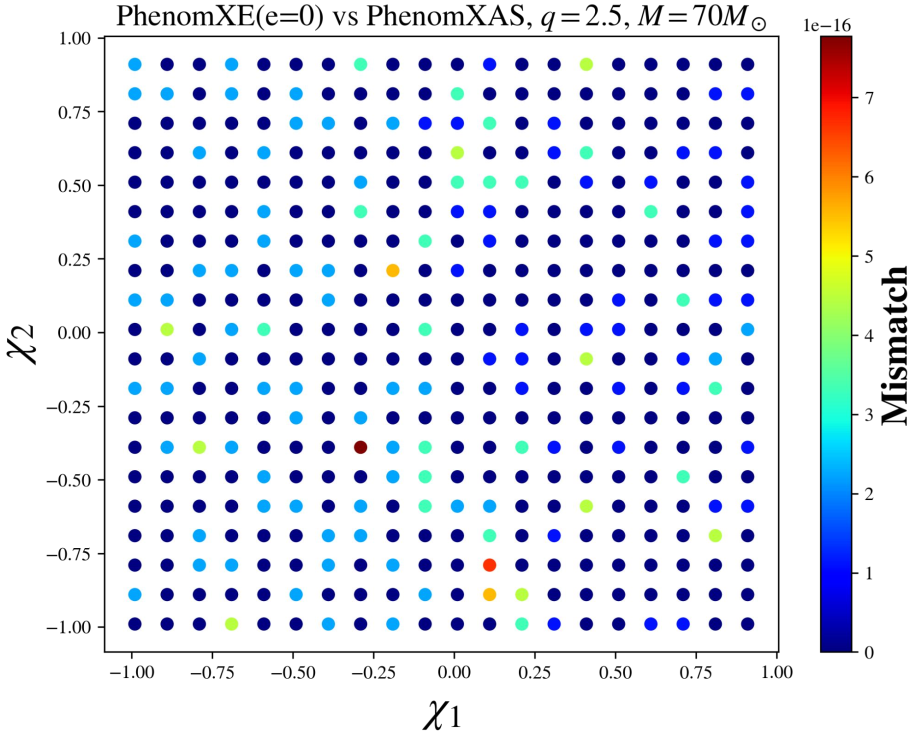


- Example evaluation time:
  - XAS : 0.015s.
  - XE : 0.056s.
  - $T_{XE}/T_{XAS} \sim 3.6$

- Fast model, feasible speed for standard parameter estimation methods.

IMRPhenomXE. Quasicircular limit

- Quasicircular limit: exact recovery of IMRPhenomXAS.

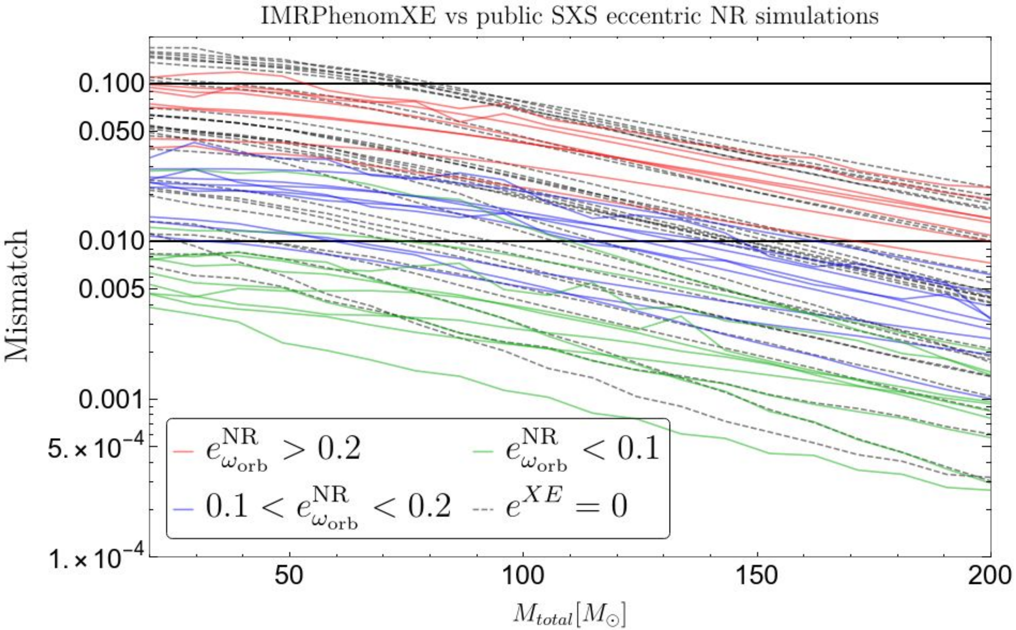


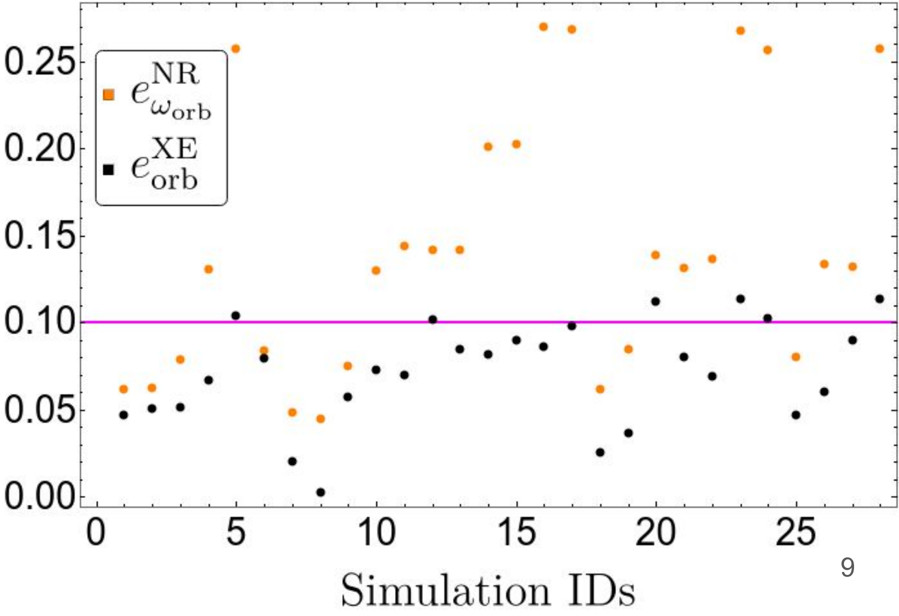
IMRPhenomXE. Comparison against eccentric NR simulations

- Comparison also against 32 public SpEC NR simulations with $0.06 < e_{\omega_{orb}} < 0.3.$
- Maximization over eccentricity and mean anomaly of the model.

$$e_{\omega_{orb}} = \frac{\omega_{\mathrm{orb},p}^{1/2} - \omega_{\mathrm{orb},a}^{1/2}}{\omega_{\mathrm{orb},p}^{1/2} + \omega_{\mathrm{orb},a}^{1/2}}$$

- For e> 0.1 mismatch above 1% due to limitations of the model.
- Eccentricity of the model, e_t , and NR measured one differ (as expected). Currently checking eccentricity bias of the model.





Conclusions

- Presented a frequency domain waveform model, IMRPhenomXE, for non-precessing spins and small eccentricities.
- Already implemented in LALsimulation.
- Quasicircular limit (e=0) → exact recovery of IMRPhenomXAS.
- Comparison against eccentric NR waveforms → low unfaithfulness against NR simulations with e<0.1.
- Fast waveform model, evaluation time ~6 times slower than IMRPhenomXAS.

Ongoing and future work

- Ongoing parameter estimation injections and GW events analysis.
- Include higher order modes (eccentric harmonics already computed).
- Possible extension to higher eccentricities with Pade resummations [Tiwari+2021].
- Include eccentricity-spin interactions.

EXTRA SLIDES

IMRPhenomXE. Eccentricity bias