a reprint from Applied Optics

Zero axial irradiance by annular screens with angular variation

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For optical alignment, it may be convenient to use a three-dimensional diffraction pattern with zero irradiance along the optical axis. This pattern is created here by using annular screens in the form of a phase daisy, a daisy flower, or a pie, with an even number of slices of an equal central angle and with every other slice with a phase retardation of 180°. We recognize this form of angular variation as a particular solution of a wider set of functions that are able to produce zero axial irradiance.

Key words: Optical alignment, apodization.

1. Introduction

High-precision alignment is important in some branches of optical and mechanical engineering. For this purpose, it is convenient to employ screens that have Fresnel diffraction patterns with concentric rings whose central spots remain practically invariant in out-of-focus planes. This yields a test for straightness; for example, in the motion of a lathe tool.

Several classical methods serve the above purpose; the so-called axicon (commonly a conical lens or mirror),[1] circular diffraction gratings,[2] and certain zone plates.[3] All of these methods use screens that have circular symmetry with only radial variation.

Here we present a scheme that generates a diffraction pattern with a dark spot at its center. Thus the scheme uses a null, rather than a maximum, in the irradiance distribution in order to locate its center and create a much greater (ideally infinite) proportional change of a signal for a particular degree of miscentering. But the signal itself near, but not at, the axis is much smaller than with patterns with a central maximum. The usefulness of our scheme must be tested in real circumstances of optical alignment before deciding whether a minimum is better than a maximum.

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Received 3 December 1990.
0003-6935/92/224600-03$05.00/0.

For describing our scheme, in Section 2 we indicate one possible manner of creating a central dark spot. In Section 3, we give the diffraction theory that sets our method in the context of the general condition for creating zero irradiance on the axis. In Section 4, we relate our design with square-wave functions that are employed in signal processing.

2. Practical Realization

An alignment system must have a large depth of focus. This can be achieved by the use of a ring aperture. Previous designs for optical alignment sometimes exploit apertures with more than one ring.

If circular symmetry is abandoned, screens can be produced that give zero irradiance everywhere on the axis. One example is a screen in the form of a phase daisy, a daisy flower, or a pie, with an even number of slices of an equal central angle and every other slice with a phase retardation of 180°. The irradiance is zero anywhere on the axis because any point of the axis receives equal contributions that are 180° out of phase with each other. Likewise, by symmetry, the irradiance is zero on planes about which the screen has reflection antisymmetry, i.e., on planes that are defined by the axis and the edges of the pieces of pie. Elsewhere, the in-phase and out-of-phase contributions do not cancel, and the irradiance is not zero. The resulting diffraction pattern is crossed by dark spokes that meet at a dark hub; the number of spokes is equal to the number of pieces in the pattern.

For experimentally verifying the above statements, phase-daisy apertures were obtained by first creating a binary pattern, which was photographically recorded on a film. A bleaching process was then performed to obtain a binary phase structure with a phase delay of π for a wavelength $\lambda = 457.9$ nm. In

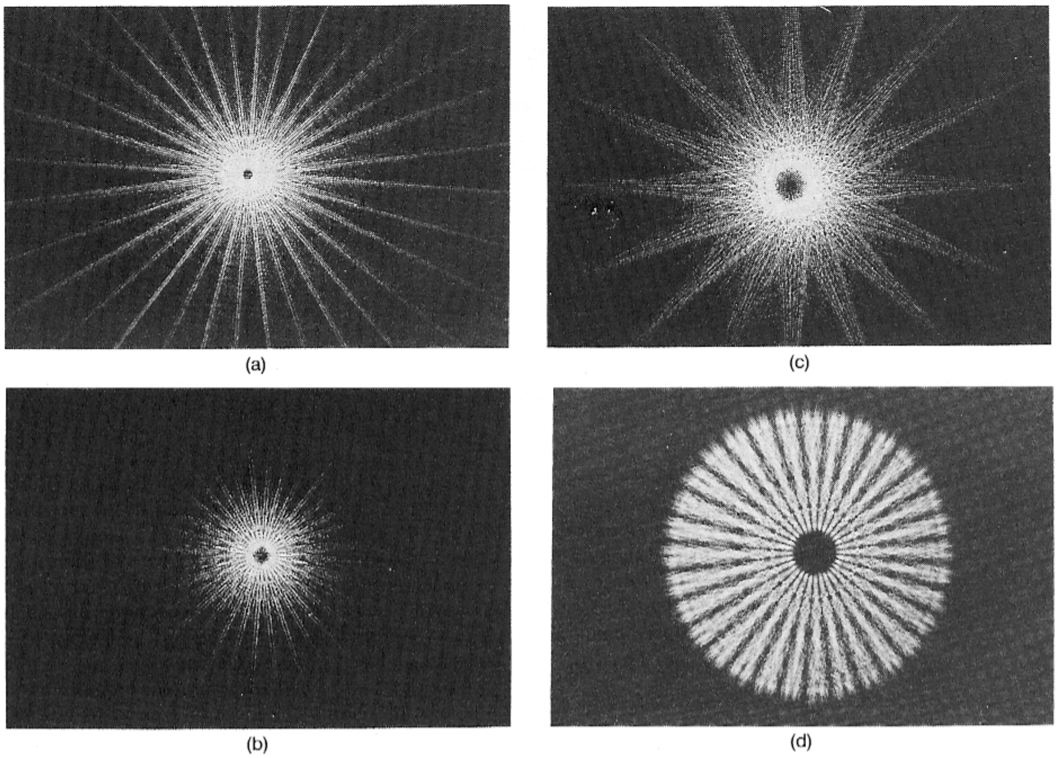

Fig. 1. Image irradiance distribution at several parallel planes beyond the pupil plane when using 32 petals. The z coordinate, as defined in Fig. 2, takes the following values: (a) $z = 1$ m (paraxial focus plane), (b) $z = 0.95$ m, (c) $z = 0.85$ m, (d) $z = 0.60$ m.

Fig. 1 we show some arbitrary out-of-focus images produced by a screen obtained by the above method, using an optical setup similar to that shown in Fig. 2.

As can be appreciated from Fig. 1 the irradiance along the optical axis remains equal to zero for various out-of-focus planes.

3. Diffraction Theory

We start by considering the complex amplitude at out-of-focus planes in the image of a point source, as

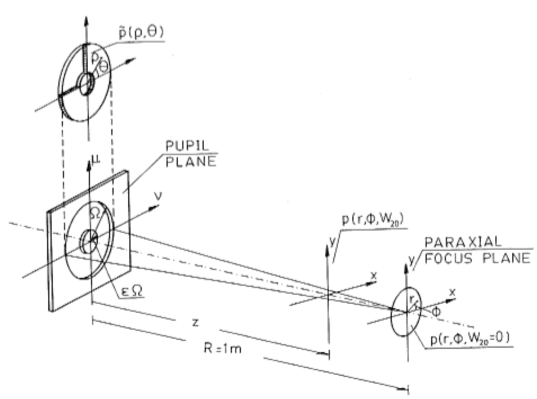

Fig. 2. Schematic diagram of the optical system that forms the image of a point source.

produced by an optical system with an annular pupil (see Fig. 2). For this case,

$$p(r, \phi; W_{20}) = \int_0^{2\pi} \int_{\epsilon\Omega}^{\Omega} \tilde{p}(\rho, \theta) \times \exp\{i2\pi[W_{20}(\rho/\Omega)^2 + r\rho\cos(\phi - \theta)]\}\, \rho d\rho d\theta. \quad (1)$$

In Eq. (1), r is the radius and ϕ is the azimuth in the image; W_{20} specifies the amount of defocus in units of wavelength; ρ is the radial frequency and θ is azimuth in the pupil. The maximum value of ρ is Ω and ϵ is the central obscuration ratio of the annular aperture. If the aperture is clear, $\epsilon = 0$.

The complex amplitude along the optical axis is obtained by setting $r = 0$, in Eq. (1), to give

$$p(W_{20}) = \int_0^{2\pi} \int_{\epsilon\Omega}^{\Omega} \tilde{p}(\rho, \theta)\exp[i2\pi W_{20}(\rho/\Omega)^2]\rho d\rho d\theta. \quad (2)$$

The integration of Eq. (2) over θ gives

$$p(W_{20}) = 2\pi \int_{\epsilon\Omega}^{\Omega} \tilde{p}_{av}(\rho)\exp[i2\pi W_{20}(\rho/\Omega)^2]\rho d\rho, \quad (3)$$

where $\tilde{p}_{av}(\rho)$ is $\tilde{p}(\rho, \theta)$ averaged over a ring at radius ρ. As long as $\tilde{p}_{av}(\rho) = 0$ for all ρ within the interval of integration, $p(W_{20}) = 0$. One can view this as a trivial consequence of McCutchen's theorem.[4] If the projection of the pupil onto the axis is zero every-

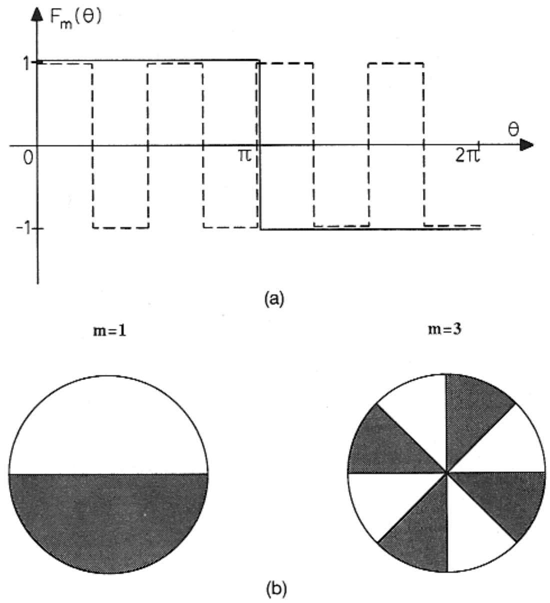

Fig. 3. Polar variation of the complex amplitude in the pupil plane: (a) one-dimensional representation. The two curves shown are the Rademacher function $m = 1$, solid curve, and $m = 3$, dashed curve; (b) actual two-dimensional representation. The shaded portions have a transmission of -1.

where, its Fourier transform is zero everywhere, and so is the amplitude on the axis in the image.

4. The Phase-Daisy Screen

The phase-daisy pupil used in our experiments is described by $\bar{p}(\rho, \theta) = R(\rho)A(\theta)$, where $A(\theta)$ is one of the Rademacher functions[5]:

$$A(\theta) = F_m(\theta) = \mathrm{sign}[\sin(2^{m-1}\theta)], \tag{4}$$

with $m = 1, 2, 3, \ldots$. In Fig. 3(a), we show the one-dimensional shape of two of these functions, while in Fig. 3(b) we display the same functions in their actual two-dimensional form. These functions are angular versions of the so-called Ronchi rulings, in which the zero amplitude is substituted by a transmittance that is equal to -1.

Note that $R(\rho)$ can be any function. In our case, $R(\rho) = 1$ inside the radial interval $\epsilon\Omega \leq \rho \leq \Omega$; elsewhere, $R(\rho) = 0$.

5. Conclusions

If a lens pupil is such that its transmission averaged over any axially symmetric ring is zero, then the diffraction image it creates when illuminated by a point source will have zero amplitude everywhere along the axis. One pupil that satisfies this requirement is a phase daisy, a pupil in the form of a daisy flower with an even number of petals and a 180° phase shift between adjacent petals. The amplitude is a Rademacher function along any ring. Our experiments show that the pupils produce the expected zero amplitude on the axis. The central zero may make the pattern useful in aligning devices.

Part of this work was presented in the 15th Congress of the International Comission for Optics held in Garmish-Partenkirchen, Germany, in August 1990. The authors are most indebted to the referees for their fine and useful suggestions. The authors thank J. A. Quintana and P. Boj (University of Alicante, Spain) for the assistance provided in the elaboration of the phase pupils. The work was supported by the Dirección General de Investigación Científica y Técnica (grant PB87-0617), Ministerio de Educación y Ciencia, Spain.

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