Off-axis focal shift for rotationally nonsymmetric screens

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We report on an analytical formulation for evaluating the amplitude distribution along any line directed toward the geometrical focus of a spherical wave front that passes through a rotationally nonsymmetric diffracting screen. Our formula consists of two factors. The first factor involves the one-dimensional Fourier transform of the projection of the screen function onto the off-axis line. The second factor depends on the inverse distance to the screen and permits us to recognize the existence of focal shift along off-axis lines.

Nature sometimes uses optical systems with Fresnel numbers less than 10, as, for example, in the case of the fly lens.[1] Man-made optical systems may also operate with low Fresnel numbers.[2] In both cases, the axial irradiance distribution exhibits the focal-shift effect.[3] One may then consider the following: For systems that operate with a low Fresnel number, is there a focal-shift effect along any arbitrarily directed line toward the geometrical focus?

The goal of this Letter is to report on a compact analytical formula for evaluating, along any line directed toward the geometrical focus, the field that is diffracted by a rotationally nonsymmetric screen under converging spherical-wave illumination. The formula consists of two factors. As in McCutchen's formalism,[4] the first factor involves a suitable one-dimensional Fourier transform of the azimuthally averaged amplitude transmittance of the screen around the off-axis line. The second factor is the amplitude representation of the inverse-square law and is responsible for the focal-shift effect along the above-mentioned straight lines.

For our present discussion, we start by considering a spherical monochromatic wave emerging from a rotationally nonsymmetric screen and converging toward the focal point F. According to the Huygens–Fresnel principle, the amplitude of the diffracted field at a point P in the vicinity of F is given by[5]

$$U(P) = \frac{-i\exp(-ikf)}{f}\iint_W \bar{A}(S)\frac{\exp(iks)}{s}\,\mathrm{d}S\,, \quad (1)$$

where $\bar{A}(S)$ is the complex amplitude-transmittance function of the screen stretched over the spherical wave front W, of radius f, and s represents the distance from a typical point Q of the spherical wave front to the point P, as is depicted in Fig. 1.

Since we are interested in evaluating the diffracted field along an arbitrary line directed toward the paraxial focus, it is convenient to center our reference cylindrical-coordinate system at point A, where the selected off-axis line intersects the incoming spherical wave front that fills the aperture. The notation is illustrated in Fig. 1. In this way, using the cosine law, we can write

$$s^2 = f^2 + z^2 - 2fz\cos(\hat{s}) = f^2 + z^2 + 2fz[1 - (r/f)^2]^{1/2}. \quad (2)$$

If we assume that $r \ll f$, then

$$s \approx (f+z) - \frac{zr^2}{2f(f+z)}\cdot \quad (3)$$

Hence, under the paraxial domain, Eq. (1) allows us to express the amplitude distribution $U_A(z)$ along the straight line passing through points A and F as

$$U_A(z) = \frac{-i}{\lambda}\,\frac{\exp(ikz)}{f(f+z)}\int_0^{2\pi}\int_{r_{\min}}^{r_{\max}} A(r,\phi) \times \exp\left[-i\,\frac{k}{2}\,\frac{z}{f(f+z)}\,r^2\right] r\,\mathrm{d}r\,\mathrm{d}\phi\,, \quad (4)$$

where $A(r,\phi)$ is the amplitude transmittance of the stretched screen as seen from point A, expressed in polar coordinates, and $r_{\max}$ and $r_{\min}$ represent,

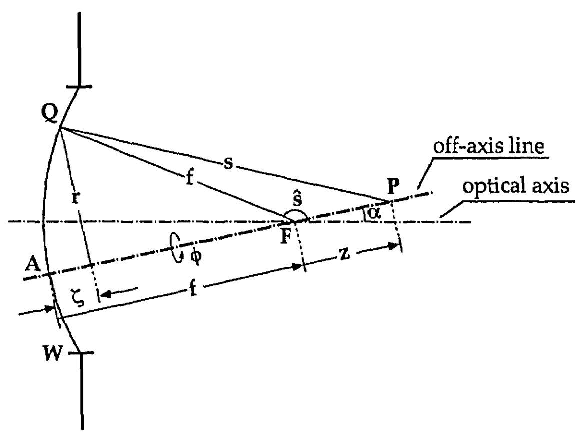

Fig. 1. Geometry used for diffraction investigation under converging spherical-wave illumination. The origin of the cylindrical coordinates, A, coincides with the point where the off-axis line intersects the spherical wave front.

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respectively, the maximum and the minimum radial extent of $A(r, \phi)$. Of course, if the off-axis line passes through the screen, then $r_{\min} = 0$. It is clear that a change in the selection of the straight line passing through F involves a change in the shape of the function $A(r, \phi)$ and, consequently, as Eq. (4) reveals, a change in the profile of the off-axis amplitude distribution.

Now, it is convenient to make the following change of variable:

$$\zeta = \frac{r^2}{2f}, \qquad \mathcal{A}(\zeta, \phi) = A(r, \phi). \tag{5}$$

We recognize that ζ represents the distance from pole A to the projection onto the off-axis line of the points of the stretched screen at radius r, as is shown in Fig. 1. By use of the above change of variable, Eq. (4) can be rewritten as

$$U_A(z) = \frac{-i}{\lambda(f+z)} \exp(ikz) \int_0^{2\pi} \int_{\zeta_{\min}}^{\zeta_{\max}} \mathcal{A}(\zeta, \phi) \times \exp\left[-ik \frac{z}{(f+z)} \zeta\right] \mathrm{d}\zeta \mathrm{d}\phi. \tag{6}$$

If we perform the integration with respect to ϕ, we obtain

$$U_A(z) = \frac{-ik}{f+z} \exp(ikz) \int_{\zeta_{\min}}^{\zeta_{\max}} \mathcal{A}(\zeta) \times \exp\left[-ik \frac{z}{(f+z)} \zeta\right] \mathrm{d}\zeta. \tag{7}$$

where $\mathcal{A}_o(\zeta)$ denotes the azimuthal average of $\mathcal{A}(\zeta, \phi)$ over a ring located at ζ. In other words, $\mathcal{A}_o(\zeta)$ is the projection of the function $A(r, \phi)$ onto the coordinate axis. Equation (7) is valid for any line directed toward the paraxial focus, and it contains as a particular case the equation for describing the amplitude distribution along the optical axis.

From Eq. (7) it is clear that the complex amplitude distribution along each of the above off-axis lines consists of two terms. The first term involves the one-dimensional Fourier transform of $\mathcal{A}_o(\zeta)$. The scale factor of this transformation is $z/\lambda(f + z)$. The amplitude of the second term varies as $1/(f + z)$ and is responsible for the lost of symmetry in the irradiance distribution $I_A(z) = |U_A(z)|^2$, as we discuss next.

The Fresnel number of the noncentered aperture function, i.e., the number of Fresnel zones that are covered by the aperture function centered at point A when viewed from the paraxial focus, is $N_A = (r_{\max}^2 - r_{\min}^2)/\lambda f = 2(\zeta_{\max} - \zeta_{\min})/\lambda$. It is apparent that this parameter represents the length of the projection of the stretched screen onto the off-axis line and is measured in units of half a wavelength. In the case of large Fresnel numbers, i.e., if the axial length of the aperture, $\zeta_{\max} - \zeta_{\min}$, is many wavelengths of light, then the integral over ζ is negligible unless z is small enough that it can be ignored when it appears in $f + z$. In this case, the expression for the amplitude along any line is the one-dimensional Fourier-transform relation of McCutchen,[4] and there is no focal shift. However, when the Fresnel number is small, z cannot be ignored, and the factor $1/(f + z)$ outside the integral shifts the irradiance peak to negative values of z i.e., toward the aperture, resulting in the focal-shift effect.

It is interesting to point out that, as point A moves from the optical axis, the length of the projection onto the off-axis line is increased and the integral in Eq. (7) is then shorter. Although the shape of the function $\mathcal{A}_o$ changes, in general the corresponding off-axis irradiance will have a sharper maximum, which is less displaced by the factor $1/(f + z)$ outside the integral. So we expect that the focal shift will be reduced.

In order to illustrate our result, from Eq. (7) we numerically evaluate the normalized irradiance distribution along different axes for a clear circular aperture with radius R. We start by assessing, as in the usual case, the normalized irradiance distribution along the optical axis. In this case, the mathematical expression for the maximum axial length of the screen ζ_M and for the Fresnel number of the centered apertured N are $\zeta_M = R^2/2f$ and $N = R^2/\lambda f$, respectively, and the function $\mathcal{A}_o(\zeta)$ has a rectangular profile, as is depicted by the solid line in Fig. 2. The corresponding normalized axial irradiance distribution is plotted as the solid line in Fig. 3. In this plot we have assumed that $N = 2$ and that the normalization is such that $I_A(z = 0) = 1$.

Next, the profile of the function $\mathcal{A}_o(\zeta)$ for various off-axis lines, i.e., for various values of the parameter α in Fig. 1, is shown by the dashed curves in Fig. 2. We recognize that, as point A moves from the optical axis, the extension of the function $\mathcal{A}_o(\zeta)$ is gradually increased. The corresponding off-axis irradiance distributions are plotted as the dashed curves in Fig. 3. We note from Fig. 3 that the off-axis point of maximum irradiance still remains without coinciding with the paraxial focal point. As we predicted, the amount of focal shift decreases as the angle α increases. This happens because the Fresnel number becomes larger.

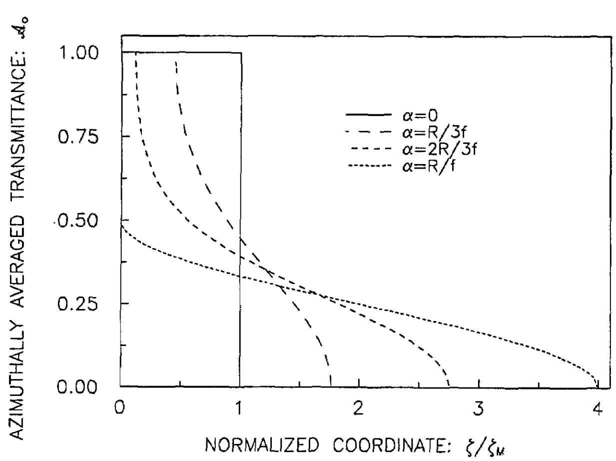

Fig. 2. Profile of the azimuthally averaged transmittance $\mathcal{A}_o(\zeta)$ of a circular aperture versus the normalized axial coordinate, $\zeta/\zeta_M = (r/R)^2$, for four different positions of the origin of the polar coordinates.

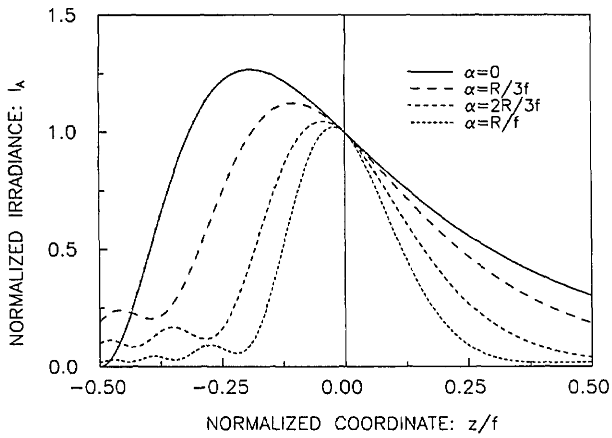

Fig. 3. Normalized irradiance distribution for a clear circular aperture, with $N = 2$, along four different lines directed toward the geometrical focus.

Summarizing, we have described a novel formalism for analytically evaluating, along off-axis straight lines passing through the focus, the irradiance distribution produced by a rotationally nonsymmetric diffracting screen under converging spherical-wave illumination. The cornerstone of our approach is the proper selection of the reference coordinate system. We applied our formalism to recognize that a circular aperture is able to produce off-axis focal shift for low Fresnel numbers. To illustrate our result, we have shown some numerically evaluated examples.

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