Technical Note

Hybrid optical–digital method for local-displacement analysis by use of a phase-space representation

Walter D. Furlan, Genaro Saavedra, and Enrique Tajahuerce

A method for evaluating the local deformation or displacement of an object in speckle metrology is described. The local displacements of the object in one direction are digitally coded in a one-dimensional specklegram. By optically performing the local spectrum of this pattern, one simultaneously achieves information about the local displacement and its spatial position. The good performance of this technique is demonstrated with computer-generated test signals.

Double-exposure speckle photography is a technique that has been widely used in optical metrology for measuring deformations or displacements of objects. With this technique useful information is provided by the Fourier transform of the specklegram in the form of an interference fringe pattern. This operation can be performed either with the whole pattern or locally as pointwise processing. Each approach has inherent disadvantages. With the first, local displacements are hidden by the global character of the transform. On the other hand, pointwise processing is time consuming and is also affected by error in determining the maxima of Young's fringes. To overcome these drawbacks, different approaches have been suggested.[1,2] Also the use of phase-space representations is an interesting solution to these problems. In particular, the Wigner distribution function has been used recently in the analysis of speckle patterns[3]; however, this approach leads to a rather poor signal-to-noise ratio.

In this Note we propose a hybrid optical–digital method for the measurement of local deformations in diffuse objects. It is based on a more versatile phase-space function, the local spectrum (LS).

W. D. Furlan and G. Saavedra are with the Departamento de Óptica, Universitat de València, 46100 Burjassot, Spain. E. Tajahuerce is with the Departamento de Ciencias Experimentales, Universitat Jaume I, 12080 Castellón, Spain.

Received 12 April 1994; revised manuscript received 22 November 1994.
0003-6935/95/224713-04$06.00/0.

The LS behaves like a multichannel spectrum analyzer. It describes the spatial-frequency content of a signal, depending on position. For a one-dimensional (1-D) input function $f(x)$, it is defined as

$$L_f(x', \nu) = \left| \int_{-\infty}^{\infty} f(x) g(x - x') \exp(-i2\pi\nu x) \mathrm{d}x \right|^2, \qquad (1)$$

where $g(x)$ is a window function that scans the input function at different values of the spatial variable x'.

If $f(x)$ is a double-exposed speckle pattern, the magnitude of the local deformations produced on the specimen being studied can be obtained directly from $L_f(x', \nu)$. In mathematical terms a 1-D binary specklegram can be expressed as

$$f(x) = \sum_i \delta(x - x_i) + \sum_i \delta(x - x_i'), \qquad (2)$$

where δ denotes Dirac's delta function and x_i and x_i' are the positions of the ith speckle point at the first and second exposures, respectively. For the sake of simplicity, let us consider the case of only two double-exposed speckles and a constant shift d between exposures. In this case $f(x)$ is reduced to

$$\begin{aligned} f(x) &= \delta(x - x_0) + \delta(x - x_1) + \delta(x - x_0') + \delta(x - x_1') \\ &= \delta(x - x_0) + \delta(x - x_1) \\ &\quad + \delta[x - (x_0 - d)] + \delta[x - (x_1 - d)]. \end{aligned} \qquad (3)$$

As a window function $g(x)$ we use a simple rectangular function of width b, so that $b > |x_1 - x_0| + d$.

When all the nonzero values of $f(x)$ are within the scanning window, the LS results:

$$L_f(x', \nu) = \cos^2(\pi\nu d)\cos^2[\pi\nu(x_1 - x_0)]. \quad (4)$$

The final result in Eq. (4) reveals an interferencelike pattern consisting of two factors. In the first, the distance between the maxima of the fringes provides information about the magnitude of the local deformation. In the second, a noise term arises from the cross interference that occurs between the noncorresponding speckles within the window. If the condition $|x_1 - x_0| > d$ is fulfilled, the noise term becomes the higher-frequency factor, being modulated by the one containing the information of interest. As the width of the window function decreases, being always $b > d$, the spatial localization of the deformation becomes more accurate and the influence of the noise is reduced. The noise term disappears if the condition $d \leq b \leq |x_1 - x_0|$ is satisfied.

When the analysis above is applied to a more general case, in which more speckles at random distances are present within the window, it leads to a similar result for the LS in Eq. (4) but contains more high-frequency factors owing to the cross interference between the speckles at x_2, x_3, etc. As in the previous case, if the mean distance between the speckles is controlled to be greater than the magnitude of the local deformation, the term of interest becomes the low-frequency envelope of the noise terms.

As can be seen from the definition, the LS involves a sequential scanning of the entire input by the window function. Alternatively, we can profit from the two-dimensional (2-D) nature of optical systems to avoid this sequential operation.[4,5] To obtain the LS of a 1-D specklegram, we used the optical setup shown in Fig. 1. We obtained the input pattern $f(x)$ starting from two 2-D speckle patterns of the test object captured before and after the deformation (e.g., by a CCD camera). If one follows the numerical algorithm proposed by Widjaja *et al.*,[3] the 1-D specklegram is obtained in four steps: (a) selecting a strip area in the 2-D specklegrams, (b) clipping, (c) superimposing, and (d) spreading in a direction normal to the strip. Using an image processor board controlled by a computer, we can perform the previous process and display the result on a spatial light modulator, e.g., a liquid-crystal television.

The window function is placed just behind the input and in-plane rotated by an angle θ. In mathematical terms this rotation is expressed as the change of variables $x' = y \tan\theta$ in Eq. (1). The spherocylindrical lens was used to perform the 1-D Fourier transform in the x direction simultaneously for all values of x'. In this way the intensity distribution at the output plane of the optical setup provides the LS of the 1-D clipped specklegram. The result is a mixed spatial–spatial frequency representation in which the periods of the modulating fringes are inversely proportional to the magnitude of the local displacement.

The proposed method was tested with an arbitrary speckle pattern that was digitally preprocessed to simulate two kinds of deformation. In the first case the original pattern was subdivided into three regions, simulating in each one a lateral displacement of a different amount following the ratio 5:2:4. In the second case a linearly increasing displacement from our stretching the original pattern is produced. In both cases the selected strip area was parallel to the displacement.

A simple rectangular function was used as a window function in the optical setup in Fig. 1. This window was rotated at an angle of $\theta = 45°$ with respect to the input in order to cover the whole object. The width of this slit was ~20 times the lowest simulated displacement.

The resulting LS for the first test function is shown in Fig. 2. Figure 3 shows the intensity profiles along the ν axis for the values of x' represented as A, B, and C in Fig. 2. From the spatial frequencies of the cosinusoidal envelopes, the ratio among the local

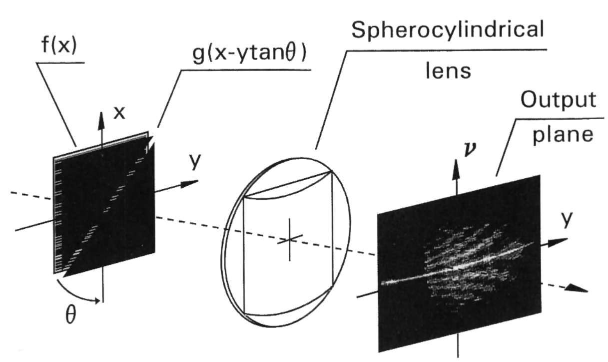

Fig. 1. Optical setup for obtaining the LS. A spherocylindrical lens was used to perform the 1-D Fourier transform in the x direction and an image was used in the y direction.

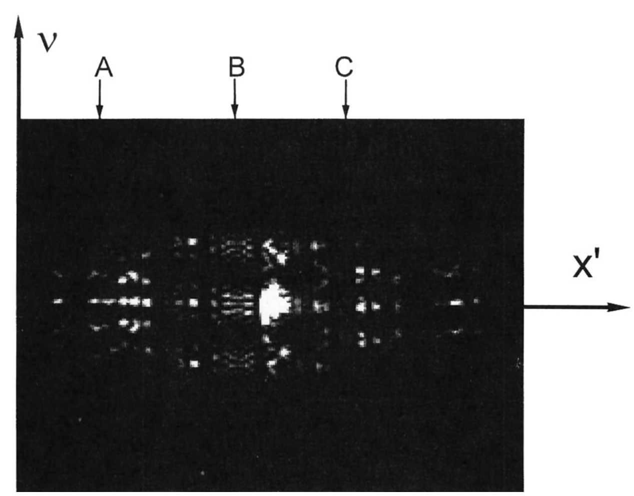

Fig. 2. LS of a test signal composed of three regions with a different constant deformation between exposures, following the ratio 5:2:4.

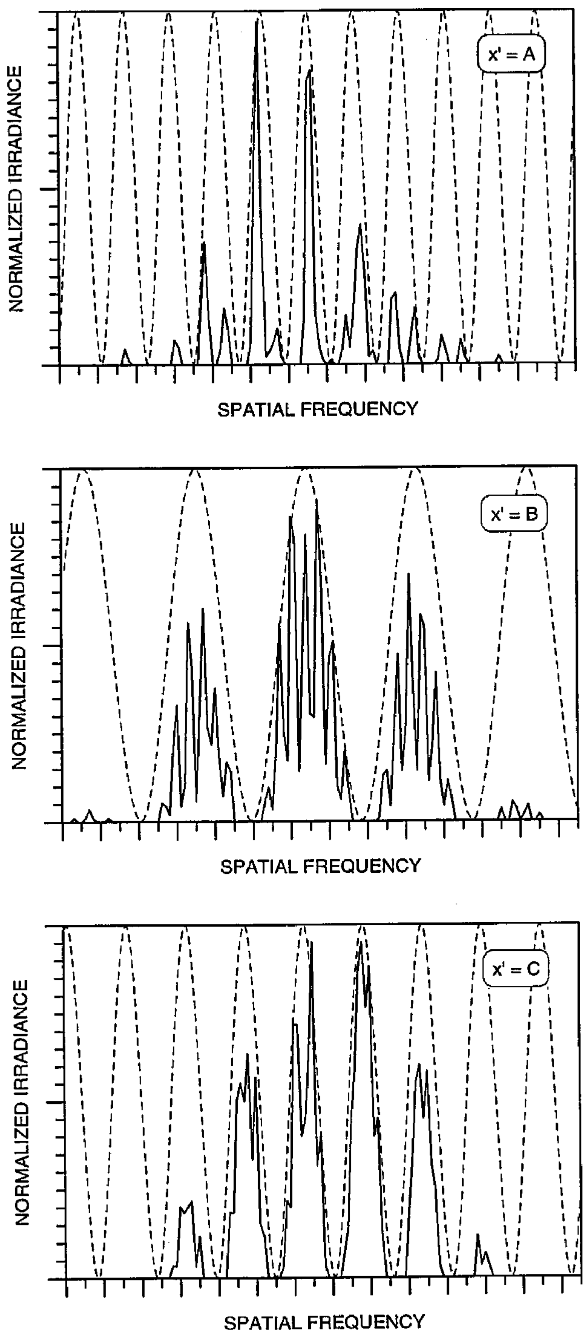

Fig. 3. Intensity profiles along the spatial frequency axis for positions $x' = A, B, C$ of the LS in Fig. 4. The dashed curve represents the theoretical cosinusoidal envelope; the distance between the maxima is inversely proportional to the local displacement.

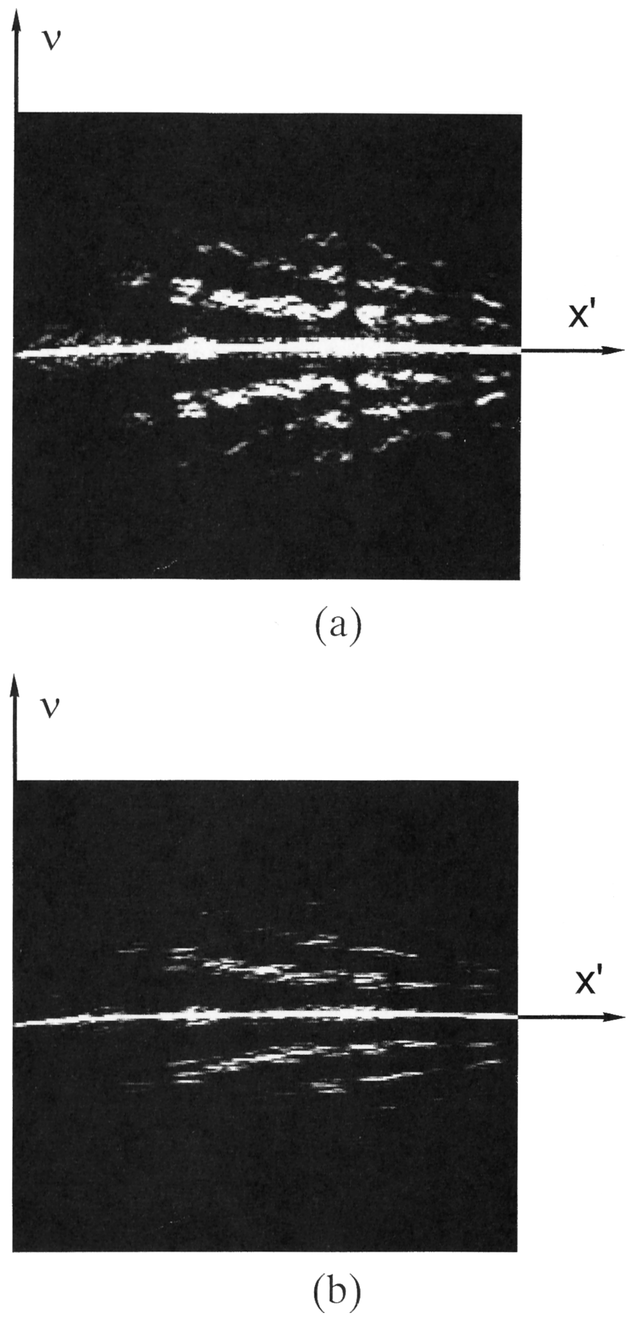

Fig. 4. LS's of a test signal with linearly increasing deformation (stretching) between exposures. Two different widths for the window function were used; the narrower one (a) corresponds to half of the wider one (b).

displacements that were obtained is just 5:2:4. Figure 4 shows the LS for the second test function. Two different window-function widths were used in this case, the narrower corresponding to Fig. 4(a). It is clear that the wider the window function is, the better will be the determination of the amount of local deformation to the detriment of the precision of its spatial localization and conversely.

The hybrid optical–digital processing system that we propose combines the versatility of the digital systems in preprocessing the sample and the speed and simplicity of the optical systems in obtaining the LS. For example, an artificial shift between the clipped patterns in the numerical algorithm can be useful either for measuring displacements away from the range of those directly measurable by the system or for establishing the displacement sign. The technique is especially interesting for detecting nonuniform deformations such as stretching or compression. The free choice of shape and width of the window is one of the most important advantages of this method. This versatile selection, which cannot be achieved with any other phase-space function, is useful for counterbalancing, within the limit of the uncertainty principle, accuracy in localizing conjugate space-phase variables. Moreover the use of the LS reduces the number of noise cross-interference terms compared with those appearing when the Wigner distribution function is used,[3] improving in this way the signal-to-noise ratio. The use of liquid-crystal televisions allows our method to be applied to fully automated systems and consequently permits a real-time technique to be implemented.[6]

This research was supported by Universitat Jaume I-Fundació Caixa Castelló agreement (grant CE.25. 017/92), Spain. E. Tajahuerce and G. Saavedra were supported by the Conselleria d'Educació i Ciència de la Generalitat Valenciana, Spain. Partial support from the Universitat de València (Projectes Precompetitius d'Investigació 1994) is also acknowledged.

References

1. C. Joenathan and R. S. Sirohi, "Elimination of error in speckle photography," Appl. Opt. **25,** 1791–1794 (1986).
2. M. Huntley, H. T. Goldrein, and L. R. Benckert, "Parallel processing system for rapid analysis of speckle-photography and particle-image-velocimetry data," Appl. Opt. **32,** 3152–3155 (1993).
3. J. Widjaja, J. Uozumi, and T. Asakura, "Real time evaluation of local displacement of objects by means of the Wigner distribution function," J. Opt. **23,** 13–18 (1992).
4. H. O. Bartelt, K.-H. Brenner, and A. W. Lohmann, "The Wigner distribution function and its optical production," Opt. Commun. **32,** 32–38 (1980).
5. S. I. Grosz, W. D. Furlan, C. Colautti, and M. Garavaglia, "Display of phase-space functions by elliptical zone plates," Opt. Commun. **73,** 7–11 (1989).
6. B. Bates, P. C. Miller, and W. Luchuan, "Liquid-crystal television optical gates applied to real time speckle metrology," J. Mod. Opt. **36,** 317–322 (1989).