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Three-dimensional superresolution by annular binary filters

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Abstract

We present a new family of annular binary filters for improving the three-dimensional resolving power of optical systems. The filters, whose most important feature is their simplicity, permit to achieve a significant reduction, both in the transverse and in the axial direction, of the central lobe width of the irradiance point spread function of the system. The filters can be used for applications such as optical data storage or confocal scanning microscopy. © 1999 Elsevier Science B.V. All rights reserved.

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1. Introduction

The design of pupil filters to overcome the limits in resolution imposed by diffraction in imaging systems has long been the aim of many research efforts. The attention of these efforts has been chiefly centered in improving the resolving capacity of imaging systems in the transverse direction [1–9]. A lower number of publications has been addressed to the design of filters for achieving superresolution along the optical axis [7,9–12], which is of great interest when dealing with imaging systems in which optical sectioning is important. What is highly surprising is the slight attention paid to the aim of achieving three-dimensional (3D) superresolution [13,14] that is, of obtaining a narrowness of the central lobe of the irradiance point spread function (PSF) of the system simultaneously in both the axial and the transverse direction. This would be very important in 3D imaging.

In this paper we present a whole family of annular binary filters for increasing at will the 3D resolution capacity of imaging systems. The binary filters are composed of a central clear circle and a concentric annular aperture. The area of the annular aperture is bigger than that of the circle. It will be shown that by continuously increasing the area of the resulting

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annular mask, the width of the core of the irradiance PSF, both in the transverse and in the axial direction, gradually decreases. The transverse impulse response provided by these filters presents an additional advantage: the maximum strength of the outer rings remains reasonably low, as we will show below.

In a second step, we propose the use of the above filters for improving the performance of two important optical techniques: optical data storage and confocal scanning microscopy.

In Section 2, we discuss the influence of the filters transmittance on the transverse and axial resolving capacity of an imaging system. Then we apply this formalism to design a new set of binary filters for achieving 3D superresolution. In Section 3 we show that the above filters could be used to improve the bit packing density of optical disks. Finally, in Section 4 we define the confocal gains in resolution, and we show that the use of the proposed filters allows to obtain a significant improvement of 3D resolution capacity of confocal scanning setups.

2. Annular binary filters design

Let us consider the amplitude PSF of an imaging system apodized by a purely absorbing pupil filter, after we consider cylindrical symmetry, that is [15]

$$h(\upsilon,W_{20}) = 2\int_0^1 p(\rho)\exp\left(-i2\pi W_{20}\rho^2\right)\mathrm{J}_0(2\pi\upsilon\rho)\,\rho\,\mathrm{d}\rho\,. \quad (1)$$

In Eq. (1) $p(\rho)$ is the pupil function, ρ being the normalized radial coordinate. Besides, $\upsilon = r_o r/\lambda f$ corresponds to the transverse radial coordinate expressed in optical units, r_o being the maximum radial extent of the pupil and f the focal length of the system. The axial position is specified in terms of the well-known defocus coefficient, $W_{20} = r_o^2 z/2\lambda f^2$, which specifies the amount of defocus measured in units of wavelength λ [16], z being the axial coordinate as measured from the focus of the system

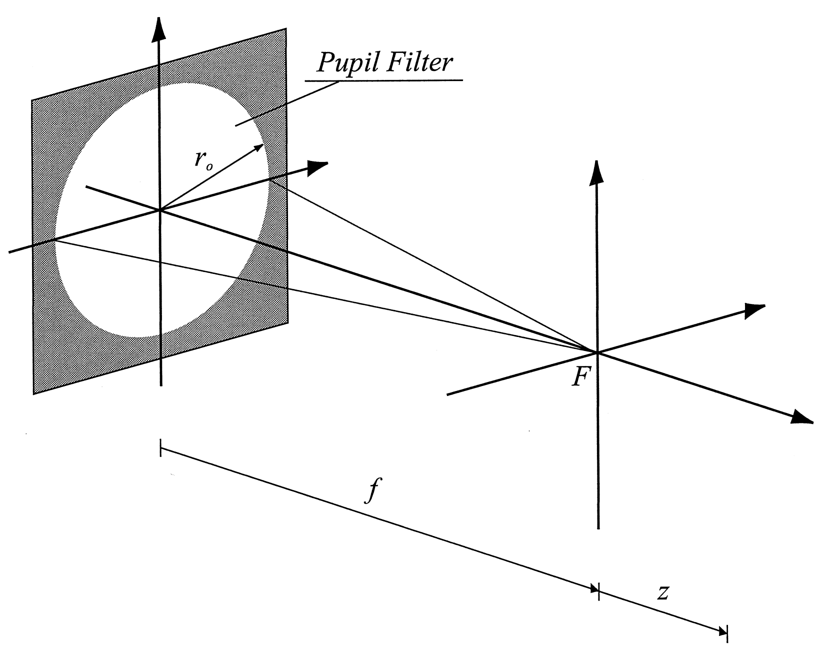

Fig. 1. Schematic layout of the focusing arrangement. The origin of the axial coordinate is taken at the focus F of the system.

(see Fig. 1). Finally, J_0 denotes the Bessel function of the first kind and zero order.

Next, it is convenient to perform the following geometrical mapping

$$\xi = \rho^2 - 0.5\,, \quad q(\xi) = p(\rho)\,. \qquad (2)$$

Then, except for an irrelevant phase factor, Eq. (1) can be rewritten as

$$h(v,W_{20}) = \int_{-0.5}^{0.5} q(\xi)\exp(-i2\pi W_{20}\xi) \times J_0\left(2\pi v\sqrt{\xi+0.5}\right)\,\mathrm{d}\xi\,. \qquad (3)$$

We are interested in the design of annular binary filters to increase the 3D resolving capacity of imaging systems, i.e. to reduce the volume of the central lobe of the 3D PSF. To this end, the width of this lobe both in the transverse and in the axial direction should be reduced. Then, to analyze the axial and transverse sections of the 3D PSF, it is convenient to particularize Eq. (3) for the two cases of our interest.

For the optical axis we set $v = 0$ in Eq. (3) to give

$$h(0,W_{20}) = \int_{-0.5}^{0.5} q(\xi)\exp(-i2\pi W_{20}\xi)\,\mathrm{d}\xi\,. \qquad (4)$$

It is apparent from Eq. (4) that the axial behavior of the system is governed by the one-dimensional (1D) Fourier transform of the mapped transmittance $q(\xi)$.

On the other hand, for the image plane case, $W_{20} = 0$, we have that

$$h(v,0) = \int_{-0.5}^{0.5} q(\xi)\,J_0\left(2\pi v\sqrt{\xi+0.5}\right)\,\mathrm{d}\xi\,. \qquad (5)$$

Therefore, the transverse behavior is also governed by the function $q(\xi)$, but through a different type of mathematical transformation.

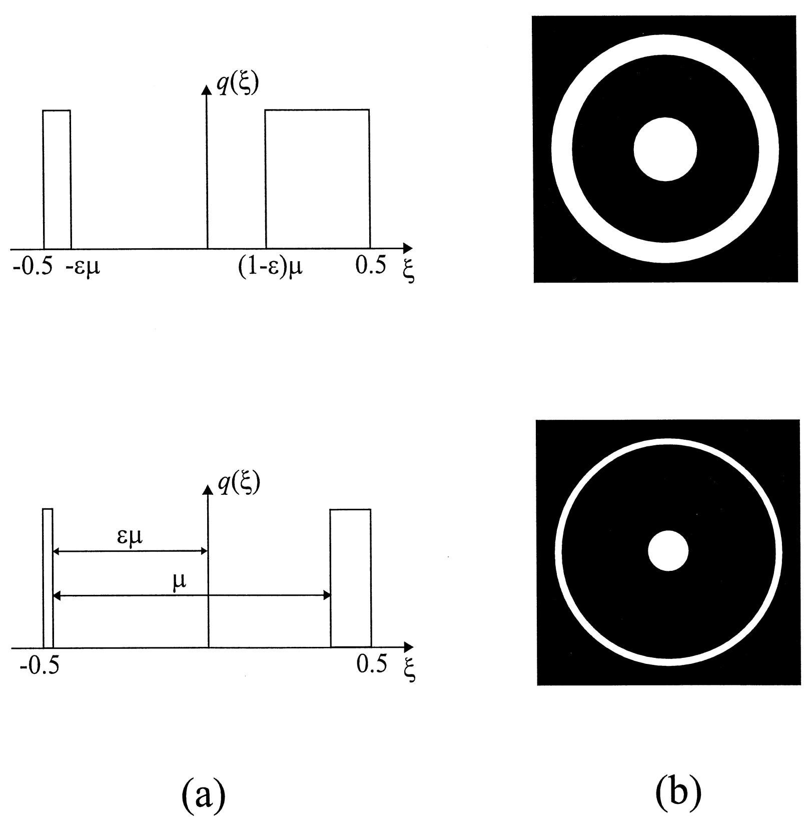

Fig. 2. Two members of the family of 3D superresolving pupil filters of Eq. (8): (a) mapped function $q(\xi)$ for $\mu = 0.6$, $\varepsilon = 0.7$ and for $\mu = 0.85$, $\varepsilon = 0.55$; (b) actual 2D representation.

The two-point resolution of an imaging system is usually evaluated in terms of the full width at the half maximum (FWHM) of the central lobe of the irradiance PSF [17]. However, since the parabolic term in the power-series expression for the irradiance distribution in the focal region dominates within the central lobe, an alternative method for evaluating the resolution of optical systems are the transverse and axial resolution gains defined by Sheppard and Hegedus [7]. The gains are defined as the ratio of the

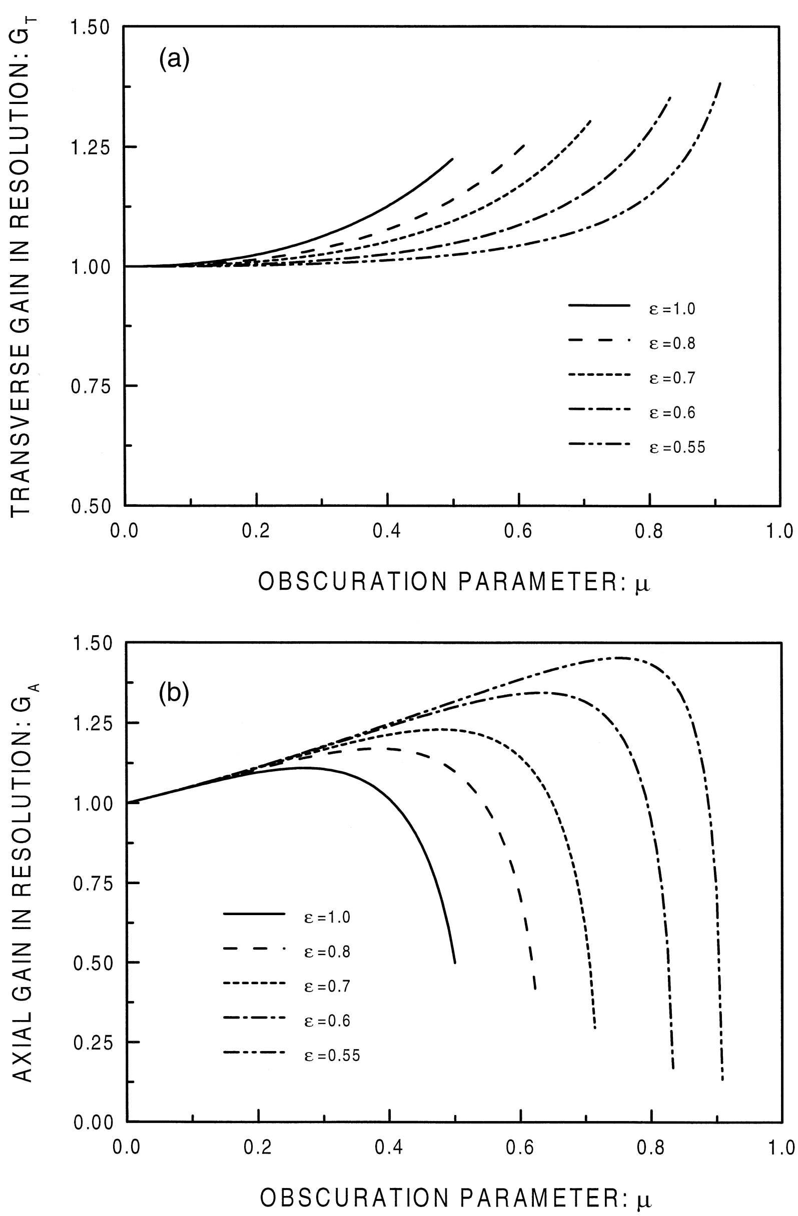

Fig. 3. Variation of: (a) transverse; (b) axial; and (c) 3D resolution gains versus the obscuration parameter μ, and for some typical values of the asymmetry parameter ε. Note that for a fixed value of ε one can tune the 3D gain by simply continuously varying the amount of obscuration.

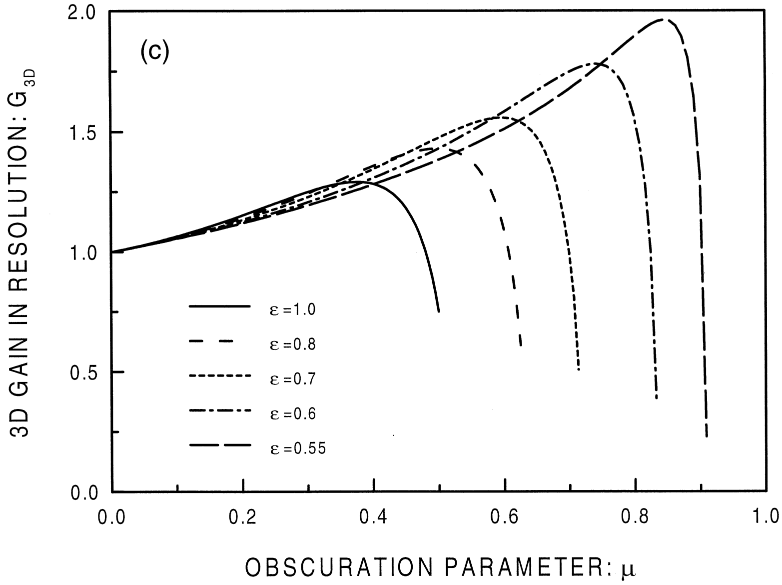

Fig. 3 (continued).

parabolic approximation of the irradiance response provided by the nonapodized imaging system to that corresponding to the apodized one.

The transverse, G_T, and the axial, G_A, resolution gains are [11]

$$G_T = \sqrt{\frac{1+2\bar{\xi}_a}{1+2\bar{\xi}_c}} \quad \text{and} \quad G_A = \frac{\sigma_a}{\sigma_c}, \tag{6}$$

respectively, where subscript 'c' corresponds to the nonapodized circular pupil, and subscript 'a' to the apodized pupil. In Eq. (6) $\bar{\xi}$ is the mean abscissa of the function $q(\xi)$ and σ the standard deviation. A superresolving effect in one of both directions, transverse or axial, is obtained when the value of the corresponding gain is bigger than unity.

For the clear circular pupil, $q(\xi) = \mathrm{rect}(\xi)$, the mean abscissa and the standard deviation are $\bar{\xi}_c = 0$ and $\sigma_c = 1/\sqrt{12}$, respectively. Thus, the gains can be definitively written as

$$G_T = \sqrt{1+2\bar{\xi}_a} \quad \text{and} \quad G_A = \sqrt{12}\,\sigma_a . \tag{7}$$

Finally, since we are analyzing the 3D resolving power of optical systems, we can define the 3D gain in resolution as the product $G_{3D} = G_A G_T^2$, where we have considered that the quadratic 3D irradiance PSF has an ellipsoidal form.

As it is made clear above, for obtaining axial superresolution it is necessary that $\sigma_a > \sigma_c$. Thus, the value of $q(\xi)$ in the vicinity of $\xi = -0.5$ and $\xi = +0.5$ should be much bigger than at $\xi = 0$. In other words, we need a purely absorbing pupil filter whose amplitude transmittance, $p(\rho)$, is maximum at $\rho = 0$ and at $\rho = 1$, and minimum at $\rho = 1/\sqrt{2}$. Note that this kind of filter enhances the low and the high frequencies over the intermediate frequencies.

On the other hand, to produce an effect of transverse superresolution we need that the mean abscissa of $q(\xi)$ is bigger than zero. Then the light throughput in the vicinity of $\xi = +0.5$ should be much bigger than in the vicinity of $\xi = -0.5$. In other words, we need a pupil filter, $p(\rho)$, that enhances the high frequencies over the low frequencies.

Since our aim here is designing pupil filters to achieve 3D superresolution, the above two conditions should be simultaneously fulfilled. This requirement could be satisfied by continuously-varying purely-absorbing filters. However, although digital-halftoning techniques have been recently adapted to produce radially-symmetric pupil filters [18], such filters are still difficult to fabricate for practical use. In order to be useful for mass production, the apodizers should offer the following: simple structure, low production cost and high tolerance. Thus, according to these constrains we propose the set of annular binary filters characterized by the mapped function

$$q(\xi) = \mathrm{rect}(\xi) - \mathrm{rect}\left[\frac{\xi + (\varepsilon - 0.5)\mu}{\mu}\right], \qquad (8)$$

with $0.5 < \varepsilon \leq 1$, and $0 < \varepsilon\mu < 0.5$. From now on, parameter μ will be called as the obscuration parameter and ε as the asymmetry parameter.

Note that any member of the above family of filters consists of a circular pupil obstructed by an annular mask, in such a way that the area of the resulting transparent annulus is higher than that of the central circular aperture, as it is illustrated in Fig. 2.

To investigate the superresolving properties of the proposed set of annular binary filters, we first calculate the moments of the function $q(\xi)$. We obtain

$$m_0 = 1 - \mu, \; m_1 = (\varepsilon - 0.5)\mu^2,$$

and

$$m_2 = \tfrac{1}{12}\left[1 - 4\mu^3(3\varepsilon^2 - 3\varepsilon + 1)\right]. \qquad (9)$$

Therefore, the resolution gains provided by the set of binary filters are

$$G_T = \sqrt{1 + \frac{(2\varepsilon - 1)\mu^2}{1 - \mu}}, \qquad (10)$$

and

$$G_A = \frac{1}{1 - \mu}\sqrt{\mu^4 - 4\mu^3[1 + 3\varepsilon(\varepsilon - 1)] - \mu + 1}. \qquad (11)$$

In Fig. 3 we have plotted the variation of the transverse, axial, and 3D resolution gains as a function of the obscuration parameter μ, and for some typical values of the asymmetry parameter ε. Note from Fig. 3a that, for a fixed value of ε the function $G_T(\mu)$ has a slowly-varying behavior, with positive slope, so that the maximum transverse resolution is attained when μ reaches its maximum value. On the contrary, the function $G_A(\mu)$ shows a slowly increasing variation only for low and intermediate values of μ, where the gain reaches the maximum value. For high values of μ the axial gain dramatically decreases (see Fig. 3b). This fact limits the amount of obscuration that can be used. Finally, in Fig. 3c we have depicted the variation of the 3D resolution gain. From this figure it is clear that for a fixed value of ε one can gradually increase, between certain limits, the 3D resolution of the system simply by continuous variation of the parameter μ. Moreover, as the value of ε decreases, the maximum achievable value of the 3D resolution gain increases. However, a price is paid: the irradiance peak, S, drops as the obscuration parameter increases according to $S = |h(0,0)|^2 = m_0^2 = (1 - \mu)^2$.

3. Application to optical data storage systems

The design of superresolving filters has constituted the aim of many research efforts. The irradiance impulse response provided by such filters has a narrow central lobe, but, unfortunately, large sidelobes. As a result, the usable field of view is dramatically reduced. Therefore, the application of such filtering technique is not very useful in conventional imaging systems. Nevertheless, over the last few years two novel important optical techniques have been developed in which the above drawback is overcome.

On the one hand, in optical data storage setups the use of superresolving filters for decreasing the laser writing spot size and increasing the read optics resolution, with the aim of improving the bit packing density of optical disk systems, has been extensively proposed [19–26]. The requirements that pupil filters should fulfill are: (i) narrow central lobe of the irradiance transverse impulse response, (ii) maximum transverse sidelobe irradiance under a threshold level, and (iii) not too low irradiance transmission efficiency. The threshold level is dependent on the recording medium, but in general it can be assumed that to avoid recording errors the allowable sidelobe irradiance must be $\leq 30\%$ of the main peak irradiance [22].

Another important feature, which indeed was taken into account only by Lee et al. [26], is an improvement of sensitivity on the focus error signal, i.e. an improvement in the axial resolution.

The use of pupil filters that fulfill some of the above constrains has been already proposed [19–26]. However, up to now no pupil filters that satisfy all the constraints have been proposed. At this point, it is remarkable that many members of the here presented family of annular binary filters simultaneously fulfill all the above requirements. To exemplify this statement, we select the filter whose asymmetry parameter is $\varepsilon = 0.7$ and whose obscuration parameter is $\mu = 0.6$. For this filter the values for the gains and the transmission efficiency are $G_A = 1.15$, $G_T = 1.17$, and $S = 0.16$, respectively. In Fig. 4 we show the axial and the transverse behavior of this filter, calculated according to Eqs. (4) and (5). Note from this figure that an important reduction in the width of the central lobe is obtained both in the transverse and in the axial direction. Moreover, the largest transverse sidelobe is low (about 11%), which is quite important in the writing process. This is because the proposed radially-symmetric filter attenuates the intermediate frequencies [4].

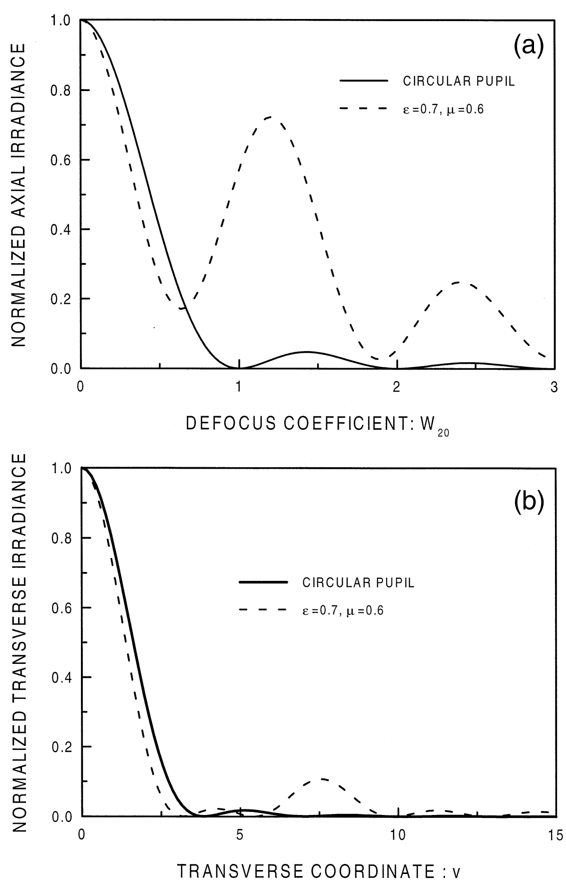

Fig. 4. (a) Normalized axial irradiance PSF corresponding to a system apodized by the annular filters whose parameters are $\mu = 0.6$, $\varepsilon = 0.7$ (dashed line), and a nonapodized system (solid line); (b) normalized transverse irradiance PSF for the same cases. Note that an important reduction in the FWHM is obtained both in the axial and in the transverse direction.

As for the readout process, it is remarkable that by the use of a confocal readout system [27], which involves the use of a narrow aperture in the detector plane, both the transverse and the axial sidelobes are extinguished.

Finally, we would like to remark that whereas the use of a shading band as a pupils filter, as proposed by some researchers [22–26], permits a transverse resolution improvement only in the direction of scan, the annular binary filter presented here is appropriate for improvement in two dimensions.

4. Application to confocal scanning systems

The second imaging technique where superresolving filters are useful is confocal scanning microscopy [28–37]. In the case of bright-field confocal scanning systems, the irradiance PSF is given by

$$I(v,W_{20}) = \left| h_{il}(v,W_{20})\, h_{col}(v,W_{20}) \right|^2, \quad (12)$$

where

$$h_{il}(v,W_{20}) = \int_{-0.5}^{0.5} q_{il}(\xi) \exp(-i2\pi W_{20}\xi) \times J_0\left(2\pi v\sqrt{\xi + 0.5}\right) d\xi, \quad (13)$$

represents the amplitude PSF of the illumination set, whereas

$$h_{col}(v,W_{20}) = \int_{-0.5}^{0.5} q_{col}(\xi) \exp(\mp i2\pi W_{20}\xi) \times J_0\left(2\pi v\sqrt{\xi + 0.5}\right) d\xi, \quad (14)$$

is the amplitude PSF of the collecting arm. The upper and lower signs correspond to the reflection and the transmission architectures, respectively. Note that in Eq. (12) it is assumed that the pupil functions of the two arms of the confocal setup have the same radial extent.

What we propose now is the use of our annular binary filters in the illuminating arm of the confocal system to improve its 3D resolution power. Thanks to the multiplicative character of the 3D irradiance PSF, the large axial and transverse sidelobes inherent to the use of the annular filters are drastically reduced by placing a clear circular aperture as the collecting-set pupil.

In order to select, among the set of annular binary filters, the most suitable for obtaining 3D superresolution we should take into account that the resolution gains defined in Section 2 are not valid for describing the focal behavior of confocal systems. This is because now the irradiance PSF is given by the squared modulus of the product of two individual PSFs. Therefore, it is convenient to define the confocal gains in resolution.

It is straightforward to find (see Appendix A) that, within the second order approximation, the

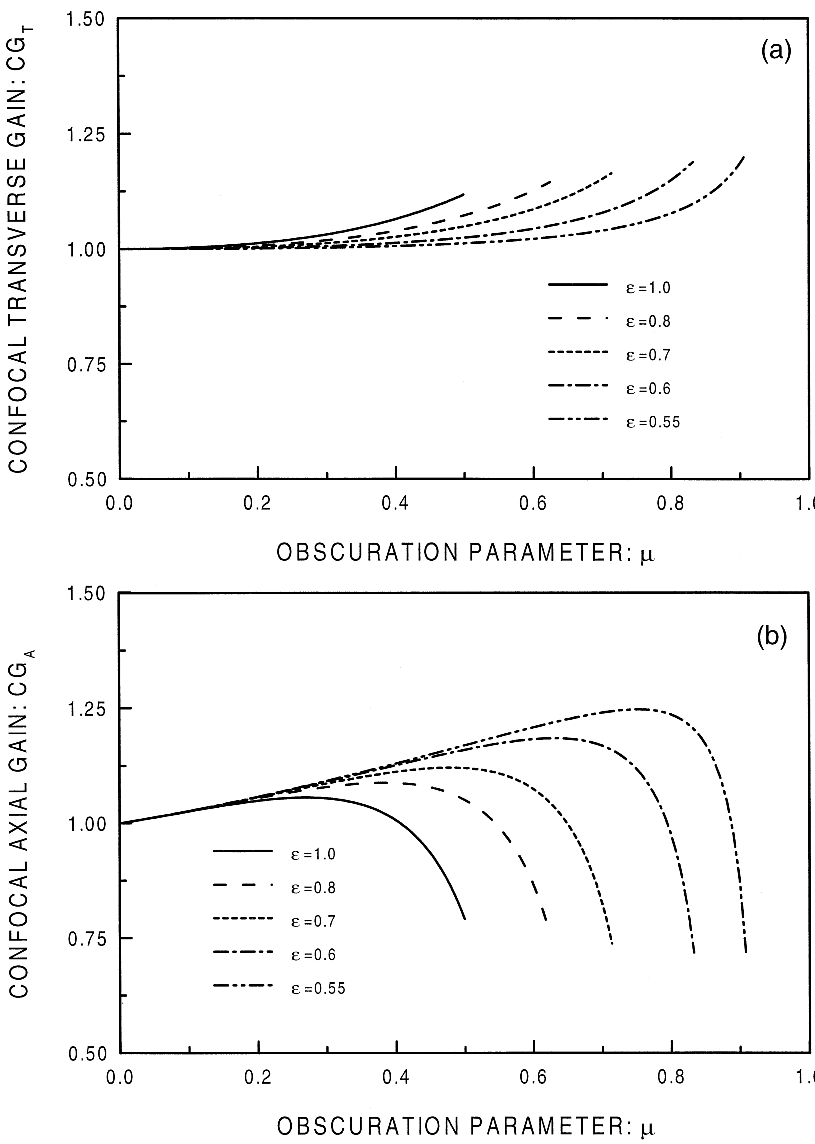

Fig. 5. Variation of: (a) transverse; (b) axial; and (c) 3D resolution confocal gains versus the obscuration parameter μ, and for some typical values of the asymmetry parameter ε.

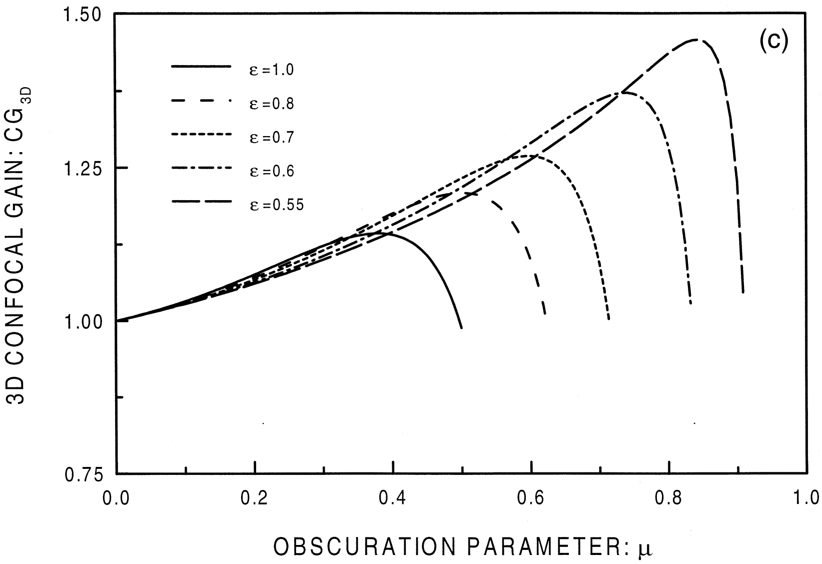

Fig. 5 (continued).

variation of the normalized irradiance in the focal region of an apodized confocal system is given by:

(a) the focal plane (see Eq. (A.8)),

$$I_N(v,0) = 1 - 2\pi^2\left(1 + \bar{\xi}_{il} + \bar{\xi}_{col}\right)v^2 . \qquad (15)$$

Following a reasoning similar to that of Section 2, i.e. by comparing the quadratic fall-off for the irradiance responses provided by the non-apodized and the apodized confocal set-ups, the confocal gain in transverse resolution is then given by

$$CG_T = \sqrt{1 + \bar{\xi}_{il} + \bar{\xi}_{col}} . \qquad (16)$$

Since we have assumed that a clear circular aperture is used as the collecting-set pupil (and then $\bar{\xi}_{col} = 0$), the gain can be rewritten as

$$CG_T = \sqrt{1 + \bar{\xi}_{il}} = \frac{1}{\sqrt{2}}\sqrt{1 + G_T^2} , \qquad (17)$$

where we have expressed the value of the confocal gain in terms of the conventional gain.

(b) the optical axis (see Eq. (A.4)),

$$I_N(0,W_{20}) = 1 - 4\pi^2\left(\sigma_{il}^2 + \sigma_{col}^2\right)W_{20}^2 . \qquad (18)$$

Therefore, the confocal gain in axial resolution is

$$CG_A = \sqrt{6\left(\sigma_{il}^2 + \sigma_{col}^2\right)} . \qquad (19)$$

If we assume again that $q_{col}(\xi) = \mathrm{rect}(\xi)$, then $\sigma_{col}^2 = 1/12$. Consequently, the axial gain is rewritten as

$$CG_A = \sqrt{1/2 + 6\sigma_{il}^2} = \frac{1}{\sqrt{2}}\sqrt{1 + G_A^2} , \qquad (20)$$

where G_A represents the conventional gain in axial resolution.

Finally, the confocal gain in 3D resolution is defined as the product $CG_{3D} = CG_T^2\, CG_A$.

By using the values of the conventional gains (see Eqs. (10) and (11)) corresponding to the proposed set of annular binary filters, we can evaluate the dependence of the confocal gains with the parameters μ and ε. In this context, in Fig. 5 we have plotted the variation of the transverse, axial and 3D confocal resolution gains as a function of μ, and for some typical values of ε.

Note from this figure that, for a given value of the asymmetry parameter, the 3D resolution power of the confocal system can be tuned by simply gradually increasing the obscuration parameter μ.

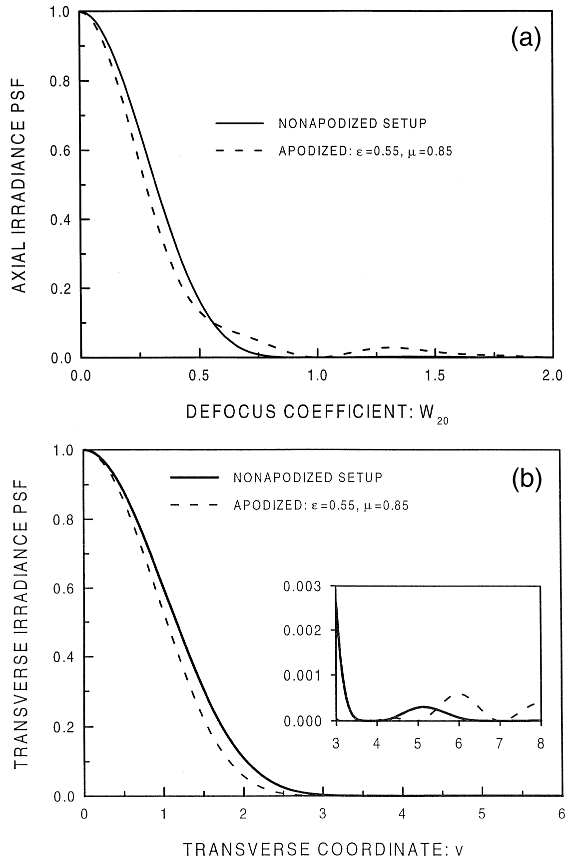

Fig. 6. (a) Normalized axial irradiance PSF of a confocal microscope whose illuminating set incorporates the annular filter whose parameters are $\mu = 0.85$, $\varepsilon = 0.55$. The solid curve corresponds to the nonapodized set-up; (b) bormalized transverse irradiance PSF for the same confocal cases.

As an example of the improvement in 3D resolution that can be achieved by the use of this method we consider the annular filter whose parameters are $\varepsilon = 0.55$ and $\mu = 0.85$. The value for the 3D confocal gain provided by this filter is $CG_{3D} = 1.45$. Therefore, the filter produces a quite important reduction in the width of the 3D irradiance PSF, and then a very important improvement in the 3D resolution power of the confocal system. To illustrate the effect produced by this filter in the PSF of the confocal set-up, in Fig. 6 we show the transverse and the axial irradiance PSF of a confocal system whose illuminating-set pupil is apodized by the proposed annular filter [1]. The solid curve correspond to the nonapodized confocal system.

Note from this figure that by simply placing an annular binary filter in the illuminating path of the confocal system, an important reduction in the central lobe of the quadratic irradiance PSF in the axial and in the transverse direction (and therefore in any other direction passing through the focus) is achieved.

As for the influence of the above annular filter in the transfer function of the confocal system, it can be easily found that the cut-off spatial frequency remains unaffected in both that transverse and the axial direction. Moreover, it is achieved, also in both directions, a slight diminution for the low frequencies and a significant enhancement for the high frequencies. Providing then an equalization in the transfer functions. Therefore, it could be stated that by using the above annular filter it is obtained an improvement in the quality of images of extended objects.

We would like to emphasize that although we have focused our study on confocal systems in which only one of the pupils is apodized, our formalism could be also applied to analyze the performance of systems in which both arms are apodized. In such a case, the confocal gains are those defined in Eqs. (16) and (19). It is remarkable that these formulae constitute a very useful tool for the design of the optimal pair of filters when dealing with twofold apodized confocal set-ups, as those reported in references [36] and [38].

Note, finally, that although the results obtained in this paper have been deduced for the bright-field mode, they also hold for the fluorescence mode.

5. Conclusions

We have designed a new set of annular binary filters which have the ability of tuning the 3D resolu-

[1] The term apodization etymologically comes from the Greek (to remove foot), and involves the suppression, or at least a considerable decrease, of the sidelobes of the diffraction pattern. However, during the last decades the use of this term has been extended, and then now it is generally accepted the use of the word apodization to denote any modification of the uniform amplitude distribution of the pupil (see reference [1]).

tion power of imaging systems. The filters consist of a central circular aperture and a concentric annulus such that its area is bigger than that of the inner circle. The corresponding 3D resolution gain depends on the value of the design parameters ε and μ. It was shown that for a given value of ε, the 3D resolving power of the system can be tuned by simply gradually change the obscuration ratio μ.

We have recognized that the members of the proposed family of annular filters can be used to improve the performance of two important, recently developed, optical techniques. In the case of optical data storage set-ups, the use of the proposed filters permits to increase the bit packing density of optical disks. This is because the filters provide a narrow central lobe of the irradiance impulse response, both in the axial and the transverse direction, a low value for the maximum transverse sidelobe irradiance, and a reasonable irradiance transmission efficiency. Furthermore, we estimate that the use of these filters can be also very useful for the readout system of volumetric optical disks, since they can reduce the interlayer cross talk [39].

To analyze the ability of the filters to improve the resolution of confocal scanning set-ups, we have defined the confocal gains in resolution. Although in our study we have centered our attention in the case in which a clear circular aperture is used as the collecting-set pupil, a more general study, in which pupil filters are placed in the two arms of the confocal set-up, could also be performed. The study of the dependence of the confocal gains with the asymmetry and the obscuration parameters, allows us to recognize that an important improvement in 3D resolution of confocal systems can be obtained by using our binary filters.

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Appendix A

The axial amplitude PSF of the illuminating and the collecting arms of a confocal system are given by the 1D Fourier transform of the corresponding pupil filter (as it can be inferred from Eqs. (13) and (14)). The kernel of these Fourier transforms can be expressed as a power series as

$$\exp(\mp i2\pi W_{20}\xi) = \sum_{n=0}^{\infty} \frac{(\mp i2\pi W_{20}\xi)^n}{n!}. \quad (A.1)$$

Then, the individual axial amplitude PSFs can be written in the form

$$h_{\mathrm{il}}(0,W_{20}) = \sum_{n=0}^{\infty} \frac{(-i2\pi W_{20})^n}{n!} m_n^{\mathrm{il}},$$

$$\text{and } h_{\mathrm{col}}(0,W_{20}) = \sum_{n=0}^{\infty} \frac{(\mp i2\pi W_{20})^n}{n!} m_n^{\mathrm{col}}, \quad (A.2)$$

where m_n^{il} and m_n^{col} represents the nth moment of the functions $q_{\mathrm{il}}(\xi)$ and $q_{\mathrm{col}}(\xi)$, respectively.

Now, the amplitude PSF of the confocal set-up can be approximated by

$$h_{\mathrm{il}}(0,W_{20})\,h_{\mathrm{col}}(0,W_{20}) \simeq m_0^{\mathrm{il}} m_0^{\mathrm{col}} \left[1 - i2\pi\left(\frac{m_1^{\mathrm{il}}}{m_0^{\mathrm{il}}} \pm \frac{m_1^{\mathrm{col}}}{m_0^{\mathrm{col}}}\right) W_{20} - 2\pi^2\left(\frac{m_2^{\mathrm{il}}}{m_0^{\mathrm{il}}} + \frac{m_2^{\mathrm{col}}}{m_0^{\mathrm{col}}} \mp 2\frac{m_1^{\mathrm{il}}}{m_0^{\mathrm{il}}}\frac{m_1^{\mathrm{col}}}{m_0^{\mathrm{col}}}\right) W_{20}^2\right]. \quad (A.3)$$

Therefore, taking into account that we deal with purely absorbing pupil filters, the normalized axial irradiance PSF of the confocal system is given, in parabolic approximation, by

$$I_{\mathrm{N}}(0,W_{20}) = \frac{I(0,W_{20})}{I(0,0)} \simeq 1 - 4\pi^2\left[\frac{m_2^{\mathrm{il}}}{m_0^{\mathrm{il}}} - \left(\frac{m_1^{\mathrm{il}}}{m_0^{\mathrm{il}}}\right)^2 + \frac{m_2^{\mathrm{col}}}{m_0^{\mathrm{col}}} - \left(\frac{m_1^{\mathrm{col}}}{m_0^{\mathrm{col}}}\right)^2\right] W_{20}^2 = 1 - 4\pi^2(\sigma_{\mathrm{il}}^2 + \sigma_{\mathrm{col}}^2) W_{20}^2. \quad (A.4)$$

For the case of the transverse PSF, we use the formula of the power expansion of J_0 [40], that is

$$J_0\left(2\pi\sqrt{\xi+0.5}\right) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(n!)^2}\left(\pi v\sqrt{\xi+0.5}\right)^{2n}. \tag{A.5}$$

Then, the individual amplitude transverse PSFs can be written, for both the illuminating and the collecting sets, in terms of the moments of $q(\xi)$ as

$$h(v,0) = m_0 - \pi^2\left(\frac{m_0}{2} + m_1\right)v^2 + \dots . \tag{A.6}$$

So, the amplitude transverse PSF of the confocal system can be approximated by

$$h_{\mathrm{il}}(v,0)\,h_{\mathrm{col}}(v,0) \simeq m_0^{\mathrm{il}} m_0^{\mathrm{col}}\left[1 - \pi^2\left(1 + \frac{m_1^{\mathrm{il}}}{m_0^{\mathrm{il}}} + \frac{m_1^{\mathrm{col}}}{m_0^{\mathrm{col}}}\right)v^2\right]. \tag{A.7}$$

Finally, the normalized transverse quadratic irradiance PSF is

$$I_{\mathrm{N}}(v,0) = \frac{I(v,0)}{I(0,0)} \simeq 1 - 2\pi^2\left(1 + \bar{\xi}_{\mathrm{il}} + \bar{\xi}_{\mathrm{col}}\right)v^2 . \tag{A.8}$$

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