1 October 1999

OPTICS COMMUNICATIONS

ELSEVIER

Optics Communications 169 (1999) 1–8

www.elsevier.com/locate/optcom

Analytical formulation of the axial behavior of apodized general Bessel beams

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Received 7 May 1999; received in revised form 1 July 1999; accepted 9 July 1999

Abstract

We present an analytical formula for the evaluation of the axial-irradiance distribution of general Bessel beams apodized by a radially-nonsymmetric window. Our approach is based on the similarity between the axial behavior of such beams and the propagation properties of a properly modified version of the window transmittance. To illustrate our formalism, we analyze the axial behavior of some complex beams.

1. Introduction

The concept of nondiffracting beams (also called Bessel beams) was introduced by Durnin and co-workers [1,2]. The main feature of these beams is that they undergo absolutely nondiffracting spreading, but their transverse intensity is highly localized. Such waves are exact solutions of the Helmholtz equation. However, since these exact solutions of the wave equation are not square integrable, the nondiffracting beams carry an infinite power, therefore they are not physically realizable.

In the last few years many research efforts have been addressed to the experimental realization of beams whose behavior approximates that of ideal Bessel beams in an extended propagation range. In this sense, it is remarkable that Bessel beams have been produced by the use of a narrow annular slit [2], by holographic optical elements [3,4], by the use of Fabry–Perot resonators [5], or even by refracting systems designed for beam transformation [6].

Since the introduction of nondiffracting beams, a number of publications have been devoted to the analysis of their propagation properties [7–10]. However, since a realistic realization includes the presence of a windowing profile, an analytical description of their propagation features is not trivial, and therefore the wavefield generally has to be numerically evaluated [11,12]. It is noticeable that the above studies have dealt with radially-symmetric window profiles applied to the simplest member of the family of nondiffracting beams, i.e., to that whose transverse amplitude is described by the zero-order Bessel

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0030-4018/99/$ - see front matter
PII: S0030-4018(99)00387-9

function of the first kind. Therefore, a thorough analysis of the propagation of a nondiffracting beam in its general form (which will be referred to in this paper as a general Bessel beam) under radially-nonsymmetric windowing has not been performed to date.

The goal of this paper is to report on an analytical formulation to evaluate the axial-irradiance distribution corresponding to a general Bessel beam apodized [1] by a rotationally-nonsymmetric window. The formula, which is a generalization of that reported by Borghi et al. [14], is based on the similarity between the axial-irradiance distribution of the apertured Bessel beam and the irradiance distribution, along a certain straight line, of a modified version of the window transmittance. Our formulation will allow us not only to revisit the analysis of well-known Bessel beams, but also to tackle, in a quite simple way, the study of the propagation features of more complex beams, which have not been previously reported.

Describing our approach, in Section 2 we formulate the basic theory for evaluating the axial-irradiance distribution corresponding to radially-nonsymmetric apodized general Bessel beams. In Section 3 we apply our formulation to revisit the study of the propagation properties of the well-known class of Bessel–Gauss (BG) beams. To exploit the power of our formalism, in Section 4 we perform the analysis of a more general situation in which we deal with a general Bessel beam windowed by a radially-nonsymmetric function.

2. Basic theory

To analyze the axial behavior of an apodized general Bessel beam, we start by considering that in a reference plane, transverse to the propagation direction of the beam, the amplitude distribution can be expressed as the product

$$v(\rho,\theta) = t(\rho,\theta)\, B(\rho,\theta)\,, \qquad (1)$$

ρ and θ being polar coordinates over the reference plane, taking its axial point as origin. In this equation the function

$$B(\rho,\theta) = \sum_{m=-\infty}^{+\infty} \mathrm{i}^m B_m J_m(\beta\rho)\exp(\mathrm{i}m\theta) \qquad (2)$$

represents the amplitude distribution associated with a Bessel beam in its general form [3]. This beam consists of a proper superposition of elementary Bessel beams, $J_m(\beta\rho)\exp(\mathrm{i}m\theta)$, all with the same scale factor $1/\beta$. The phase factor $\exp(\mathrm{i}m\theta)$ gives rise to a spiral wave front, rotating upon propagation. Let us recall that the coefficients B_m in Eq. (2) correspond to the coefficients of the circular harmonic expansion for the spatial-frequency spectrum of the general Bessel beam. Since this spectrum is confined into a thin annulus of radius $\eta = \beta/2\pi$ [15], the values B_m simply correspond to the Fourier expansion coefficients of the angular variation of the spatial-frequency spectrum, $A(\phi)$, within this annulus. In this way, a nondiffracting beam can be also understood as the result of the superposition of plane waves whose wave-vectors describe a cone of semiaperture α such that $\sin\alpha = \beta/k$, and whose relative amplitude is given by the function $A(\phi)$.

Note that Eq. (1) may describe two different diffracting geometries: (a) the case of a diffracting screen, of amplitude transmittance $t(\rho,\theta)$, normally illuminated by a general Bessel beam; and (b) the situation corresponding to an optical beam, whose amplitude distribution at the reference plane is $t(\rho,\theta)$, that illuminates an holographic plate designed to produce a Bessel beam [3,4].

To evaluate the axial-amplitude distribution of the apodized general Bessel beam we assume that the propagation is in the paraxial regime, and then we particularize the Fresnel–Kirchhoff diffraction formula for the axial points [16], i.e.

$$h(z) \approx \frac{\exp(\mathrm{i}kz)}{\mathrm{i}\lambda z}\int_0^{\infty}\int_0^{2\pi} t(\rho,\theta)\, B(\rho,\theta) \times\exp\left(\mathrm{i}\frac{k}{2z}\rho^2\right)\rho\,\mathrm{d}\rho\,\mathrm{d}\theta\,, \qquad (3)$$

[1] The term apodization etymologically comes from the Greek (to remove foot), and involves the suppression, or at least a considerable decrease, of the sidelobes of the diffraction pattern. However, during the last decades the use of this term has been extended, and now the use of the word apodization is generally accepted to denote any modification of the uniform amplitude distribution of the pupil (see Ref. [13]).

where z is the axial distance from the reference plane. By substituting now Eq. (2) into Eq. (3), we obtain

$$h(z) = \frac{k\exp(\mathrm{i}kz)}{\mathrm{i}\,z}\sum_{m=-\infty}^{+\infty}\mathrm{i}^m\int_0^\infty B_m\,t_{-m}(\rho)\,J_m(\beta\rho)\times\exp\left(\mathrm{i}\frac{k}{2z}\rho^2\right)\rho\,\mathrm{d}\rho\,, \quad (4)$$

where the function $t_m(\rho)$ represents the radial part of the mth-order circular harmonic of the function $t(\rho,\theta)$, that is

$$t_m(\rho) = \frac{1}{2\pi}\int_0^{2\pi}t(\rho,\theta)\exp(-\mathrm{i}m\theta)\,\mathrm{d}\theta\,. \quad (5)$$

Finally, if we take into account that $J_{-m}(x) = (-1)^m J_m(x)$, Eq. (4) can be rewritten as

$$h(z) = \frac{k\exp(\mathrm{i}kz)}{\mathrm{i}\,z}\sum_{m=-\infty}^{+\infty}\mathrm{i}^m\int_0^\infty[B_{-m}\,t_m(\rho)]\times J_m(\beta\rho)\exp\left(\mathrm{i}\frac{k}{2z}\rho^2\right)\rho\,\mathrm{d}\rho\,. \quad (6)$$

We have, then, obtained an analytical formula that describes the axial behavior of nondiffracting beams, in their general form, apodized by a diffracting window, with or without radial symmetry. From this formula it is apparent that the axial amplitude distribution is governed by the products $B_{-m}t_m(\rho)$, i.e., by the coefficients of the expansions into circular harmonics of both the Bessel-beam spectrum and the apodizing function $t(\rho,\theta)$. Moreover, Eq. (6) also permits us to state that zero-axial irradiance is achieved if the apodization is such that all the products $B_{-m}t_m(\rho)$ are equal to zero. Note that this is a generalization of the condition for obtaining zero-axial irradiance when a pupil filter is uniformly illuminated [17], and it may be particularly suitable for precision alignment applications.

Let us now consider a, in principle, different situation in which the amplitude distribution at the reference plane is $p(\rho,\theta)$. This situation may correspond to: (a) a monochromatic plane wave that normally illuminates a diffracting screen whose amplitude transmittance is $p(\rho,\theta)$; and (b) an optical beam whose amplitude distribution at $z=0$ is $p(\rho,\theta)$.

If we apply again the Fresnel–Kirchhoff diffraction formula, we find that the transverse field amplitude distribution at a distance z from the reference plane can be expressed as

$$u(r,\varphi,z) = \frac{\exp(\mathrm{i}kz)}{\mathrm{i}\,\lambda\,z}\exp\left(\mathrm{i}\frac{k}{2z}r^2\right)\int_0^\infty P(\rho,\varphi;r,z)\times\exp\left(\mathrm{i}\frac{k}{2z}\rho^2\right)\rho\,\mathrm{d}\rho\,, \quad (7)$$

where r and φ are polar coordinates over the plane of interest, taking its axial point as origin, and the function $P(\rho,\varphi;r,z)$ is given by

$$P(\rho,\varphi;r,z) = \int_0^{2\pi}p(\rho,\theta)\exp\left[-\mathrm{i}\frac{2\pi}{\lambda z}r\rho\cos(\theta-\varphi)\right]\mathrm{d}\theta\,. \quad (8)$$

This function can be expressed in terms of a series of Bessel functions as [2]

$$P(\rho,\varphi;r,z) = 2\pi\sum_{m=-\infty}^{+\infty}(-\mathrm{i})^m J_m\left(\frac{2\pi r\rho}{\lambda z}\right)\times\exp(\mathrm{i}m\varphi)\,p_m(\rho)\,, \quad (9)$$

where $p_m(\rho)$ represents the radial part of the mth-order circular harmonic of $p(\rho,\theta)$.

By substituting Eq. (9) into Eq. (7) we find that

$$u(r,\varphi,z) = \frac{k\exp[\mathrm{i}k(z+r^2/2z)]}{\mathrm{i}\,z}\times\sum_{m=-\infty}^{+\infty}(-\mathrm{i})^m\exp(\mathrm{i}m\varphi)\int_0^\infty p_m(\rho)\times J_m\left(\frac{2\pi r\rho}{\lambda z}\right)\exp\left(\mathrm{i}\frac{k}{2z}\rho^2\right)\rho\,\mathrm{d}\rho\,. \quad (10)$$

Comparing now Eqs. (6) and (10) we find that the following relation holds

$$|h(z)|^2 = \left|u\left(r=\frac{\beta}{k}z,\,\varphi=\pi,\,z\right)\right|^2, \quad (11)$$

[2] Hint: $\exp(-\mathrm{i}A\cos B) = \sum_{m=-\infty}^{\infty}(-\mathrm{i})^m J_m(A)\exp(\mathrm{i}mB)$; see for example Ref. [18].

provided that the circular-harmonic expansion of the pupil function $p(\rho,\theta)$ is in the form

$$p(\rho,\theta) = \sum_{m=-\infty}^{\infty} p_m(\rho)\exp(\mathrm{i}m\theta) = \sum_{m=-\infty}^{\infty} B_{-m}\, t_m(\rho)\exp(\mathrm{i}m\theta). \tag{12}$$

The above equations indicate that the axial-irradiance distribution of a general Bessel beam apertured by a window with or without radial symmetry is the same as that produced, under plane-wave illumination, by the window function of Eq. (12) along the straight line defined by $r=\beta z/k = z\sin\alpha$, $\varphi=\pi$.

On the basis of this general result, many particular results can be obtained. In this sense, it is straightforward to find from our approach that the axial irradiance associated with an apertured $J_0(\beta\rho)$ beam, is the same as that produced by the zero-order circular harmonic of the aperture transmittance (under plane-wave illumination) along the line $r = z\sin\alpha$, as reported by Borghi et al. [14].

We center now our attention into the case in which at $z=0$ the amplitude distribution is in the form

$$v(\rho,\theta) = t(\rho,\theta)\, J_m(\beta\rho)\exp(\mathrm{i}m\theta), \tag{13}$$

that is, into the case of an apertured elementary Bessel beam of order *m*. Since in this case the coefficients $B_n = \delta_{n,m}$ we find that the corresponding axial-irradiance distribution is the same as that produced, along the above mentioned straight line, by a radially-nonsymmetric pupil filter $p(\rho,\theta)$ whose circular-harmonic decomposition only contains the term of order $-m$ of the apodizing pupil $t(\rho,\theta)$.

From that result it can be stated, as a general proposition, that the circular-harmonic decomposition of the function $p(\rho,\theta)$ is governed by the form of the general Bessel beam. In other words, the general Bessel beam selects the circular harmonics that are present in the function $p(\rho,\theta)$, and their relative weight.

Our approach allows us also to perform a thorough analysis of other quite interesting situations. This is the case of the axial behavior of the well-know BG beams [19], or the more general case in which a radially-nonsymmetric laser mode illuminates a plate designed to produce a Bessel beam.

3. The Bessel–Gauss beam

Let us consider the case of the BG beams. In this case, the amplitude distribution at $z=0$ is given by

$$v(\rho,\theta) = J_0(\beta\rho)\exp\left(-\frac{\rho^2}{w_0^2}\right), \tag{14}$$

where w_0 represents the waist of the Gaussian beam.

As stated in the previous section, since we deal with the elementary zero-order Bessel beam, to calculate the axial-irradiance distribution we simply need to evaluate the propagation properties of a, radially symmetric, Gaussian beam whose amplitude distribution at $z=0$ is in the form

$$p(\rho) = \exp\left(-\frac{\rho^2}{w_0^2}\right). \tag{15}$$

Recalling the propagation characteristics of a Gaussian beam [20], it is easy to give an analytical expression for the irradiance distribution at any plane transverse to the propagation direction. The expression is

$$I(r,z) = \frac{w_0^2}{w^2(z)}\exp\left(-2\frac{r^2}{w^2(z)}\right), \tag{16}$$

where the parameter

$$w(z) = w_0\sqrt{1+\left(\frac{z}{z_\mathrm{R}}\right)^2} \tag{17}$$

represents the variation of the width of the beam when propagation, $z_\mathrm{R} = \pi w_0^2/\lambda$ being the so-called Rayleigh range for the Gaussian beam.

If we now evaluate the irradiance distribution along the straight line $r = z\sin\alpha$, we find that

$$I_\alpha(z) \equiv I(r=\alpha z, z) = \frac{1}{1+\left(\frac{z}{z_\mathrm{R}}\right)^2}\exp\left[-\frac{2\left(\frac{z}{z_\mathrm{R}}\right)^2\left(\frac{\alpha}{\vartheta}\right)^2}{1+\left(\frac{z}{z_\mathrm{R}}\right)^2}\right], \tag{18}$$

where the parameter $\vartheta = \lambda/\pi w_0$ represents the angular divergence of the Gaussian beam. In Eq. (18) we assume that α is small, since the propagation is in the paraxial regime, and therefore $z\sin\alpha \simeq z\alpha$.

Taking into account now the similarity between the axial irradiance of the BG beam and the irradiance distribution of the sole Gaussian beam along the straight line $r = z\alpha$, it is clear that Eq. (18) also represents the on-axis irradiance distribution for the BG beam represented by Eq. (14). Let us emphasize that Eq. (18) reproduces the result reported in Eq. (5) of Ref. [9], where the propagation characteristics of BG beams are analyzed.

In order to perform a rigorous analysis of the axial behavior of BG beams, we make now the following geometrical mapping

$$\zeta = \frac{z}{z_R}, \ \mathscr{I}_\alpha(\zeta) = I_\alpha(z). \tag{19}$$

Then, Eq. (18) can be rewritten as

$$\mathscr{I}_\alpha(\zeta) = \frac{1}{[1+\zeta^2]} \exp\left[-\frac{2\zeta^2 C(\alpha)}{1+\zeta^2} \right], \tag{20}$$

where the ratio

$$C(\alpha) = \left(\frac{\alpha}{\vartheta} \right)^2 \tag{21}$$

is a real and positive parameter that relates the scale of the Bessel beam with the Gaussian-beam angular divergence.

Note that, as it is shown in Fig. 1, $\mathscr{I}_\alpha(\zeta)$ is a monotonically-decreasing function for values $\zeta > 0$, with maximum value at the origin. This value corresponds to the irradiance at the axial point of the waist plane of the Gaussian beam, which due to the normalization taken in our study is $\mathscr{I}_\alpha(0) = 1$. The slope of $\mathscr{I}_\alpha(\zeta)$ depends on the value of the parameter $C(\alpha)$ in such a way that the higher the value of $C(\alpha)$ is the narrower the function $\mathscr{I}_\alpha(\zeta)$ is.

Since we are interested in finding out which value of the ratio α/ϑ provides the more slowly-varying axial-irradiance distribution for the BG beam, next we investigate the relation between the value of ζ, say ζ_H, corresponding to the half-maximum value of $\mathscr{I}_\alpha(\zeta)$, and the ratio parameter $C(\alpha)$. From Eq. (20) it is straightforward to find that such a relation is given by the transcendental equation

$$C(\alpha) = -\frac{1+\zeta_H^2}{2\zeta_H^2} \ln\left(\frac{1+\zeta_H^2}{2} \right). \tag{22}$$

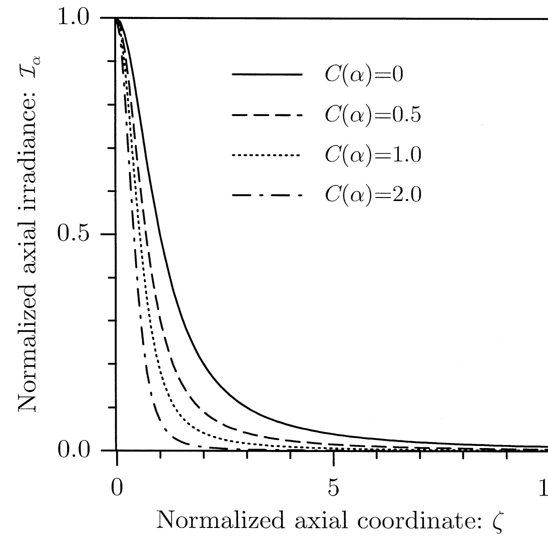

Fig. 1. Normalized axial-irradiance distribution for Bessel–Gauss beams with different values of the ratio parameter $C(\alpha)$, as a function of the normalized axial coordinate $\zeta = z/z_R$.

For a given value of the ratio parameter $C(\alpha)$, the root of this equation is the abscissa of the point of intersection of the curve

$$y = -\frac{1+\zeta^2}{2\zeta^2} \ln\left(\frac{1+\zeta^2}{2} \right) \tag{23}$$

and the straight line $y = C(\alpha)$. A plot of these functions (see solid lines in Fig. 2) shows that the highest value for ζ_H ($\zeta_H = 1$) is obtained when $C(\alpha) = 0$, that is, when $\alpha = 0$. Note that this is just the case of a Gaussian beam illuminated by an on-axis plane wave. In this case, of course, the half-maximum irradiance is attained at $z = z_R$. On the other hand, as the value of the parameter $C(\alpha)$ increases the value of ζ_H continuously decreases. To analyze this effect, let us consider that the value of w_0 (and then the angular divergence of the Gaussian beam) remains constant. Then, a continuous increase in the value of the parameter α (which implies a proportional narrowness of the core of the Bessel beam) gives rise to a gradual diminution of the value of the half-maximum coordinate ζ_H. Thus, if we are interested in obtaining a slow variation for the axial-irradiance pattern, it is necessary that the scale of the Bessel beam core be as high as possible, compared with the Gauss-beam waist.

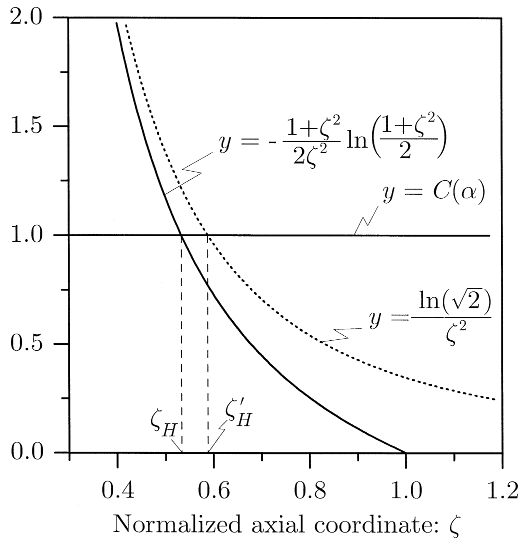

Fig. 2. Plot of the functions used to graphically solve the transcendent equations given in Eqs. (22) and (24).

It should be noted that a similar study, for the propagation features of BG beams, was already performed in reference [19]. In that paper, it was established, on the basis of parageometric reasoning, that the half-maximum value for the axial-irradiance distribution is attained at a distance $z_{\rm H} = \sqrt{\ln(\sqrt{2}\,)}\, w_0/\alpha$ from the reference plane. Taking into account the notation employed in our paper, this relation can be rewritten as

$$C(\alpha) = \frac{\ln(\sqrt{2}\,)}{\zeta_{\rm H}'^{2}}, \tag{24}$$

where $\zeta'_{\rm H} = z_{\rm H}/z_{\rm R}$. As it is shown in Fig. 2, where the function $y = \ln(\sqrt{2}\,)/\zeta^2$ is plotted with a dashed line, this result overestimates, for any value of $C(\alpha)$, the width of the axial-irradiance pattern. This is because, as the authors pointed out, the parageometric study did not consider the influence of two factors: the angular spread of the Gaussian beam, and the attenuation suffered by its amplitude when propagating, which is proportional to the factor $1/w(z)$.

4. Application to radially-nonsymmetric beams

In Section 3 we have used our approach to revisit the quite simple case in which the simplest elementary Bessel beam is windowed by the fundamental, or lowest-order, mode of a stable laser resonator. Since both beams are radially symmetric, it is clear that in this study we have not exploited all the power of our approach. Therefore, in this section we will tackle the analysis of a more general situation in which we deal with a higher-order, radially-nonsymmetric laser mode and with a general Bessel beam.

Let us consider that, for example, the window is given by the Hermite–Gauss function of (1,1) order, that is

$$t(x,y) = \mathscr{H}_{1,1}\left(\frac{x}{w_0},\frac{y}{w_0}\right) = \exp\left(-\frac{x^2+y^2}{w_0^2}\right) H_1\left(\frac{\sqrt{2}\,x}{w_0}\right) H_1\left(\frac{\sqrt{2}\,y}{w_0}\right), \tag{25}$$

where $H_1(\bullet)$ represents the first-order Hermite polynomial, and w_0 stands for the beam waist. A gray-scale representation of $|t(x,y)|$ is given in Fig. 3. To perform our analysis it is convenient to express this function, in polar coordinates, in terms of a

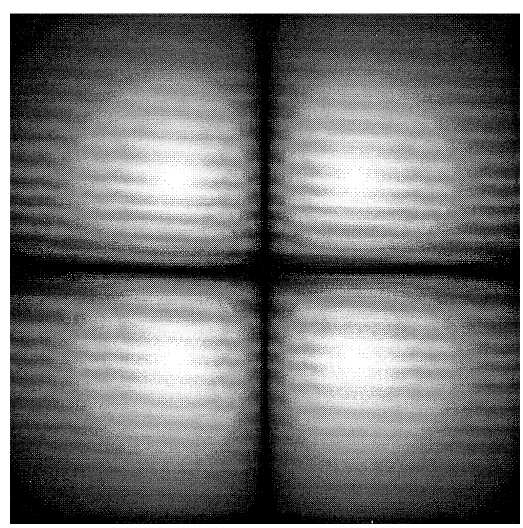

Fig. 3. Gray-scale representation of the modulus of the transmittance indicated in Eq. (25), for values $(x,y) \in (-2w_0, 2w_0) \times (-2w_0, 2w_0)$.

linear combination of Laguerre–Gauss functions, defined as

$$\mathscr{L}_{p,l}\left(\frac{\rho}{w_0},\theta\right) = \exp\left(-\frac{\rho^2}{w_0^2}\right)\left(\frac{\sqrt{2}\,\rho}{w_0}\right)^{|l|} L_p^{|l|}\left(\frac{2\rho^2}{w_0^2}\right) \mathrm{e}^{\mathrm{i}l\theta}, \qquad (26)$$

where $L_p^{|l|}(\bullet)$ are the associated Laguerre polynomial of orders p and $|l|$. In this way, we obtain

$$t(\rho,\theta) = -\mathrm{i}\left[\mathscr{L}_{0,2}\left(\frac{\rho}{w_0},\theta\right) - \mathscr{L}_{0,-2}\left(\frac{\rho}{w_0},\theta\right)\right]. \qquad (27)$$

Since Eq. (27) provides, indeed, the circular-harmonic decomposition of $t(\rho,\theta)$, to obtain the axial behavior of the Bessel Hermite–Gauss (BHG) beam under study we simply need to analyze the propagation properties of the next sole beam (see Eq. (12))

$$p(\rho,\theta) = -\mathrm{i}\left[B_{-2}\,\mathscr{L}_{0,2}\left(\frac{\rho}{w_0},\theta\right) - B_2\,\mathscr{L}_{0,-2}\left(\frac{\rho}{w_0},\theta\right)\right], \qquad (28)$$

where $B_{\pm 2}$ are coefficients of the general Bessel-beam expression.

Now, using the well-known propagation properties of the Laguerre–Gauss functions [20], it is easy to find that the three-dimensional irradiance distribution of this beam is

$$|u(r,\varphi,z)|^2 = \frac{w_0^2}{w^2(z)}\left| B_{-2}\,\mathscr{L}_{0,2}\left(\frac{r}{w(z)},\varphi\right) - B_2\,\mathscr{L}_{0,-2}\left(\frac{r}{w(z)},\varphi\right)\right|^2, \qquad (29)$$

where the function $w(z)$ is the same as that defined in Eq. (17).

Finally, by particularizing Eq. (29) to the points belonging to the straight line $r = \alpha z$, $\varphi = \pi$, and by performing the geometrical mapping described in Eq.

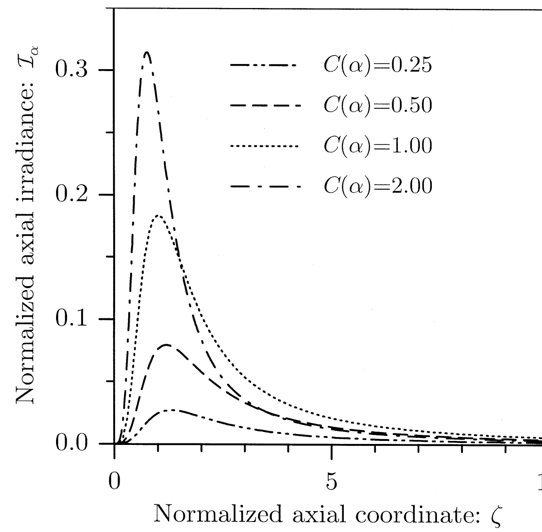

Fig. 4. Normalized axial-irradiance distribution for Bessel Hermite–Gauss beams of (1,1) order with different values of the ratio parameter $C(\alpha)$, as a function of the normalized axial coordinate $\zeta = z/z_{\mathrm{R}}$.

(19), we find that the axial-irradiance distribution provided by the BHG beam is given by

$$\mathscr{I}_\alpha(\zeta) = 4|B_{-2} - B_2|^2 \zeta^4 \frac{C^2(\alpha)}{(1+\zeta^2)^3} \times \exp\left(-2\frac{\zeta^2 C(\alpha)}{1+\zeta^2}\right). \qquad (30)$$

From the analysis of Eq. (30) we can obtain some conclusions on the propagation properties of that beam. First, we note that among all the components of the general Bessel beam, only those of order ± 2 contribute to the axial-irradiance distribution of the BHG beam. It is remarkable that the values of such coefficients do not govern the axial-irradiance pattern profile, but they only provide a weight factor. Second, it is now apparent that zero-irradiance is obtained along the optical axis if, and only if, the Bessel beam is such that $B_{-2} = B_2$, independently from the value of the rest of the coefficients. Therefore, it can be established that to obtain zero-axial irradiance, it is sufficient for $A(\phi)$ to be an even function.

To add more remarks on the propagation features of the beam we deal with, we also perform a graphical representation of $\mathscr{I}_{\alpha}(\zeta)/|B_{-2}-B_2|^2$ in Fig. 4. From this figure it is clear that the lower the value of the parameter $C(\alpha)$ (i.e., the higher the width of the beam core, compared with the Hermite–Gauss beam divergence, is), the smoother the variation of the axial irradiance. Therefore, the best performance of the beam is obtained, in this sense, when $\alpha \ll \vartheta$. Note, however, that in this case the BHG beam behaves, basically, as an ordinary Hermite–Gauss beam.

The analysis of curves in Fig. 4 reveals the presence of a quite interesting, highly surprising, and, to the best of our knowledge, novel focusing effect. Note that as the value of the ratio parameter, $C(\alpha)$, increases (which implies a departure from the, in principle, ideal axial behavior of the BHG beam), there appears an increasingly sharp axial-irradiance peak. The normalized axial coordinate for this maximum is given by

$$\zeta_{\max} = \sqrt{(1/2 - C(\alpha)) + \sqrt{(1/2 - C(\alpha))^2 + 2}}\ . \tag{31}$$

From this equation, it is straightforward to find that the higher the value of $C(\alpha)$ is, the closer from the window the peak is. Note that we have found a new focusing effect, that is surprising because the BHG beam under study is composed of two beams that, separately, provide a zero-axial irradiance pattern.

5. Summary

We have derived a novel formulation for calculating the axial behavior of general Bessel beams apertured by a radially-nonsymmetric window. We have shown that the axial-irradiance pattern generated by such beams is the same as that produced (under plane-wave illumination) along a certain straight line by certain window. The window is such that its circular-harmonic decomposition is governed by the Bessel-beam form.

Our result allowed us to formulate the necessary condition to obtain zero-axial irradiance, and to revisit the study of propagation properties of BG beams. Specifically, we have obtained, in a quite simple way, an analytical formula for the on-axis irradiance distribution. Moreover, we have found a transcendental equation that provides the value of the axial range where the axial-irradiance distribution remains nearly constant; in other words, the axial range where the BG beam exhibits a nearly nondiffracting behavior.

Additionally, with the aim of exploiting all the power of our approach, we have analyzed the axial behavior of more complex, radially-nonsymmetric, apertured Bessel beams. In particular, we have obtained an analytical expression for the on-axis irradiance distribution corresponding to a HGB beam. From this formula we have found that, depending on the value of the ratio parameter $C(\alpha)$, two different kinds of axial behavior can appear. For low values of this parameter, smooth variation of the axial irradiance is observed. However, for large values of $C(\alpha)$, a novel, unexpected, focusing effect is observed.

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