One-dimensional iterative algorithm for three-dimensional point-spread function engineering

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We present a new method with which to binarize pupil filters designed to control the three-dimensional irradiance distribution in the focal volume of an optical system. The method is based on a one-dimensional iterative algorithm, which results in efficient use of computation time and in simple, easy to fabricate binary filters. An acceptable degree of resemblance between the point-spread function of the annular binary filter and that of its gray-tone counterpart is obtained.

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Control of the beam structure in the three-dimensional (3D) region that surrounds the focal point of an optical system is an important task in various applications, such as in conventional imaging systems,[1,2] optical data storage readout heads,[3] and confocal scanning microscopy.[4] Specifically, in confocal microscopes the 3D point-spread function (PSF) of the system is a matter of critical interest, so several efforts to control its shape by the use of radially symmetric pupil filters have been reported.[5–8]

Iterative halftoning algorithms[9] constitute an important class of digital halftoning methods.[10] They can be used not only for binarization of images but also for binarization of diffractive optical elements, in particular, pupil filters that shape the PSFs of imaging systems. Taking into account the required computational effort, we believe that the iterative Fourier-transform algorithm (IFTA) is one of the most effective iterative halftoning procedures.[11] In a previous paper a digital halftoning technique derived from a classic IFTA was developed that binarizes radially symmetric gray-tone pupil filters such that their symmetry is preserved.[12] The filters calculated with this technique[12] were aimed only at shaping the axial profile of the three-dimensional (3D) PSF. From the point of view of 3D imaging, e.g., confocal scanning microscopy, it is of importance to shape the whole 3D distribution of the PSF. Here we demonstrate that this can be done by means of annular binary filters that are computed by a one-dimensional (1D) IFTA. This means that one can shape the 3D light-field distribution about the focal point by using an iterative procedure in which both pupil and PSF constraints are imposed on one-column matrices. Our approach is based on an axial form of the sampling theorem.[13,14] To demonstrate the utility of our method we use it to compute the binary versions of axially superresolving and Gaussian filters, and we analyze the corresponding 3D PSFs, which closely approximate those of the original gray-tone filters. The extremely high computing-time efficiency of our method could be useful, for example, for implementing in real-time radially symmetric pupil filters by means of programmable liquid-crystal spatial light modulators.[15]

Let us start by considering the normalized amplitude PSF of an aberration-free focusing system that is apodized by a purely absorbing radially symmetric pupil filter, namely,

$$h(u,v) = 2\pi \int_0^1 t(\rho)\exp(-i2\pi u\rho^2)J_0(v\rho)\rho\,\mathrm{d}\rho\,. \quad (1)$$

In Eq. (1) the function $t(\rho)$ stands for the properly scaled amplitude transmittance of the pupil filter. Variables u and v represent, respectively, the axial and the transverse focal coordinates as expressed in optical units.[16]

It is known that the function $h(u.v)$ can be written as

$$h(u,v) = \frac{1}{2\pi}\sum_{m=-\infty}^{+\infty} h(m,0)h_C(u-m,v)\,, \quad (2)$$

where $h_C(u,v)$ represents the normalized 3D amplitude PSF that corresponds to the circular aperture and has to be expressed in terms of the Lommel functions. This important formula, which represents the axial form of the sampling theorem,[13] indicates that the 3D amplitude PSF of an apodized system results from the coherent superposition of an infinite number of properly axially shifted PSFs that correspond to the circular aperture. The shifts are equal to integer numbers. We construct the weighting-factor set of this superpositon by sampling the axial PSF of the purely absorbing pupil filter in the axial nulls of a circular-aperture PSF.

Because $h(u,v)$ is completely determined by the values of $h(u,0)$ at $u = 0, \pm1, \pm2, \ldots$, assuming that the circular-aperture PSF is known, it is convenient to express $h(u,0)$ in a simpler form. To this end we perform the nonlinear mapping $q(\mu) = t[\rho(\mu)]$ such that $\rho(\mu) = +\sqrt{\mu + 0.5}$. Then, from Eq. (1) with $v = 0$, the axial PSF can be written, aside from an irrelevant phase factor, in the following way:

$$h(u,0) = \pi \int_{-0.5}^{0.5} q(\mu)\exp(-i2\pi u\mu)\mathrm{d}\mu\,. \qquad (3)$$

Functions $q(\mu)$ and $h(u,0)$ constitute a Fourier-transform pair in which the function $h(u,0)$ is band limited. Thus the function can be completely recovered, according to the sampling theorem, from a set of its samples properly distributed in the whole axis u.

Let us suppose now that we are interested in calculating a binary filter, $t_B(\rho)$, whose PSF closely approximates that of a gray-tone filter $t(\rho)$, which suits a particular need for 3D imaging (e.g., for superresolution). From the above reasoning [Eqs. (2) and (3)] it is apparent that $q(\mu)$ is precisely the function that one should binarize to reach this end. As we need an algorithm specially designed to produce a strong resemblance between the low-frequency spectra at the appropriate sampled points of the function $q(\mu)$ and its binary counterpart $q_B(\mu)$, the 1D version of the IFTA was selected as the binarization technique. The IFTA has been described elsewhere (see, for example, Ref. 12, in which a block diagram and a thorough study of the constraints that are necessary for implementing the IFTA are provided).

To execute the IFTA efficiently in this particular problem, one must take into account that in most cases of interest the terms that significantly contribute to the series of Eq. (2) are those with small m. Then the geometrical parameters of the IFTA should be chosen such that the axial points $u = 0$, ± 1, ± 2 coincide with those for which $h(u,0)$ is sampled first on execution of the IFTA. Those parameters are M, the number of equidistant samples of $q(\mu)$, and N, the number of elements in the one-column matrix $[h_i]$ whose ith element is $h_i \equiv h(i\Delta u, 0)$, where Δu is the axial sampling distance in the PSF domain ($i = 0, \pm 1, \pm 2, \ldots, \pm N/2$). It is evident that the coincidence of $u = 0, \pm 1, \pm 2, \ldots$ with at least some $u = i\Delta u$ takes place when $\Delta u = 1/k$, where k is a positive integer number. From Eq. (3) it results that the simplest case of $k = 1$ implies that $M = N$. For $k > 1$ the above coincidence takes place for $N/M = k$. This case is extensively used in computation by means of the fast Fourier transform, in which, to obtain sufficient sampling in the spectral domain, functions are sampled at M points and surrounded by zeros to form a vector of $N > M$ pixels. This is usually done to produce sufficient sampling in the spectral domain.

The utility of the proposed method was established in two numerical experiments. First, we considered the axially superresolving parabolic filter $q(\mu) = 4\mu^2$, whose real amplitude transmittance is $t(\rho) = (2\rho^2 - 1)^2$. This filter provides a significant narrowing of the central lobe of the axial PSF. Second, we considered the Gaussian filter $t(\rho) = \exp(-\pi^2\rho^2)$ or $q(\mu) = \exp[-\pi^2(0.5 + \mu)]$. We used these gray-tone filters to check the accuracy of our method in two quite different situations. The parabolic filter compresses the focal volume, whereas the Gaussian filter expands it.

In our numerical simulation we started the algorithm by sampling the function $q(\mu)$, for both filters, in a small number of equally spaced points, $M = 33$. As a result of a comprehensive empirical study, we established that no improvement is achieved by setting $N > M$. In other words, an increase in the size of the one-column matrices used in the algorithm does not improve the results and merely increases the computational effort. Therefore we formed a vector composed of $N = 33$ pixels.

The amplitude transmittances of the binary filters obtained by the 1D iterative technique are shown in Fig. 1. In Fig. 2 we have plotted the contours of constant irradiance in the meridian plane for the parabolic filter and its binary version. From comparison of the two figures it is apparent that the differences between the PSFs are negligible and that the binary pupil provides the desired result. Specifically, we calculated the differences, at any point of the focal volume, between the irradiance PSF of the gray-tone filter and that of its binary version. The maximum value for this difference was found to be less than 1% of the peak normalized irradiance.

In Fig. 3 we present the normalized 3D irradiance PSFs for a Gaussian filter and its binary counterpart. Note that in this case the figures do not seem to match so well as in Fig. 2. This result is a visual effect that is due to the smoothness of the shape of the 3D PSF. In fact, in this case the maximum

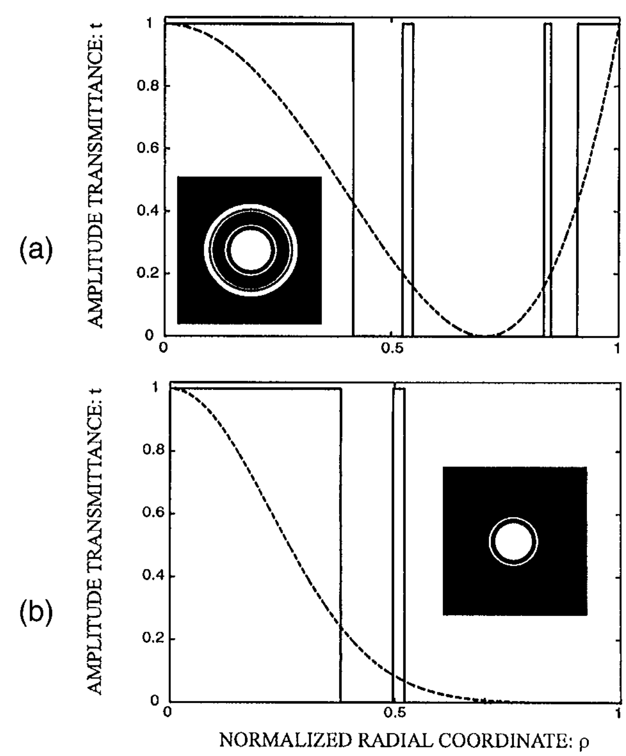

Fig. 1. Binary filters obtained by use of a 1D IFTA for $M = 33$: (a) parabolic filter, (b) Gaussian filter. Dashed curves, amplitude transmittance of the gray-tone filters.

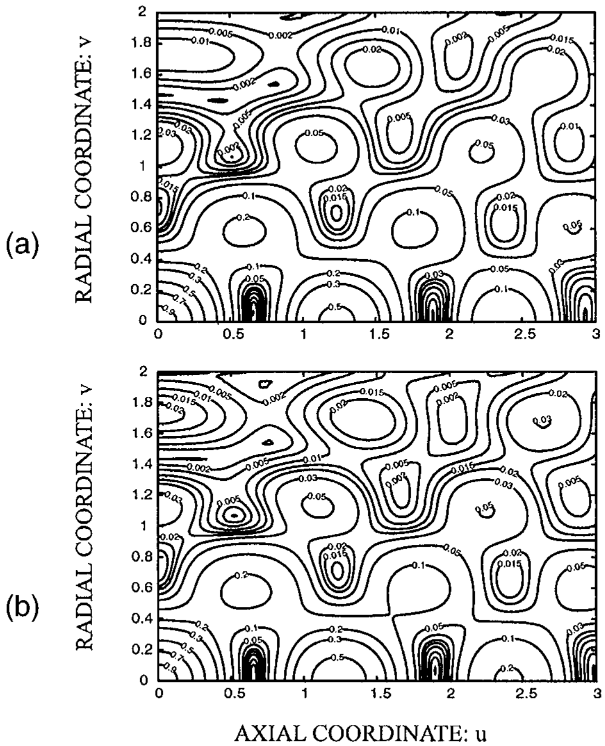

Fig. 2. Contours of constant normalized irradiance in the meridian plane corresponding to (a) a parabolic filter, (b) the binary version.

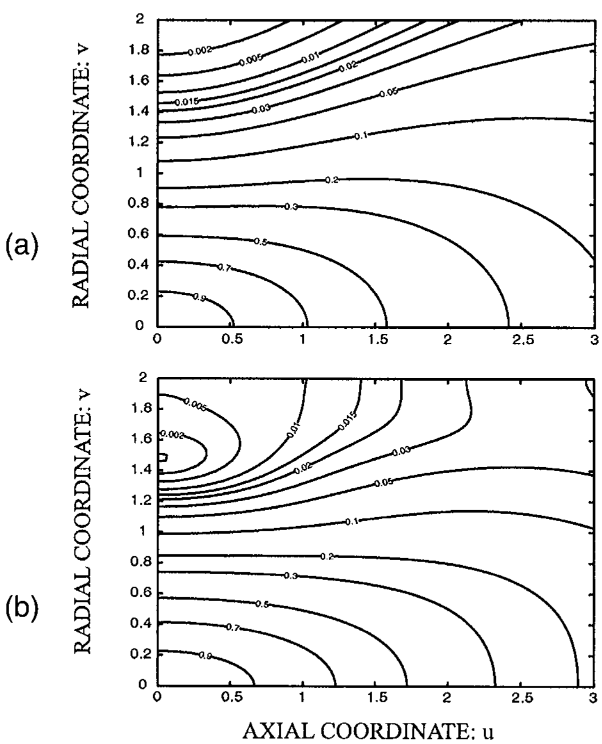

Fig. 3. Contour of constant normalized irradiance in the meridian plane corresponding to (a) a Gaussian filter, (b) the binary version.

value for the irradiance differences was less than 2%. We emphasize how, by simply illuminating, with a monochromatic plane wave, a pupil filter consisting of a circular aperture obstructed by an annular mask [see Fig. 1(b)], one can accurately reproduce the 3D structure of a Gaussian beam.

To summarize, we have presented a method for binarizing radially symmetric, purely absorbing pupil filters designed to control the 3D structure of the focal volume in an apodized system. Our method is based on the use of a properly adapted version of the IFTA. We apply the 1D IFTA to the mapped transmittance of the filter. Importantly, although we deal with 3D fields, we need to apply the Fourier tools only to one-column matrices. As follows from the axial sampling theorem, the 3D PSF of an apodized system can be recovered with a set of regularly spaced axial samples. Hence our algorithm uses matrices with a very low number of pixels (1×33 in our numerical experiment). If we compare our approach with other common realizations of Fourier matching in which, for any transverse plane, 2D matrices with at least 256×256 pixels are used, it exhibits a highly remarkable saving in computing time.

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