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Short Communication

The stenopaeic slit: an analytical expression to quantify its optical effects in front of an astigmatic eye

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Summary

The stenopaeic slit is a trial case accessory used in subjective refraction, especially when high astigmatism is present. In spite of its simplicity, the effect of the slit when it is not oriented along one of the principal meridians of the examined eye is difficult to predict, even in terms of classical geometrical optics. In this paper, the optical principles of the slit are considered with full details in the theoretical framework of the dioptric power space. An analytical expression to obtain the residual refractive error when a stenopaeic slit is placed in front of an astigmatic eye at any orientation is deduced. In the light of these results, some aspects of the clinical procedure are discussed. © 2001 The College of Optometrists. Published by Elsevier Science Ltd. All rights reserved.

Introduction

Undertaking subjective refraction when the patient has low visual acuity caused by high astigmatism sometimes requires the use of unusual techniques such as refraction using a stenopaeic slit. With this simple trial case accessory the principal meridians of the refractive error are isolated and refraction performed along each one of them independently (Borish and Benjamin, 1998). Although the procedure is clinically quite easy to follow, its rationale is difficult to understand, since the effect of the slit in front of an astigmatic eye is not obvious, especially when the slit is not oriented along the principal meridians.

The stenopaeic slit reduces the effective pupil size in the meridian perpendicular to it (Bennett and Rabbetts, 1989), which produces an axial shift of the circle of least confusion (CLC), and consequently a change of the retinal blur size that depends on the orientation of the slit. As shown in *Figure 1*, when the slit orientation coincides with one of the principal meridians of the eye, it can be considered that it 'changes' the spherocylindrical refractive error into a purely spherical one. Of course, if the slit orientation does not coincide with one of them, the analysis is not so easy, requiring a more detailed study.

The purpose of this paper is two-fold. Firstly, a graphical and intuitive deduction of the optical principles of the stenopaeic slit is presented in the frame of a three-dimensional power space (Deal and Toop, 1993). Secondly, this previous interpretation leads us to obtain a general analytical expression of the residual refractive error when a stenopaeic slit is located in front of an astigmatic eye. Therefore, questions relative to the clinical procedure such as: what is the residual refractive state that results from the slit, where is the CLC located for its different orientations and which of these orientations improve or impair vision, will be answered.

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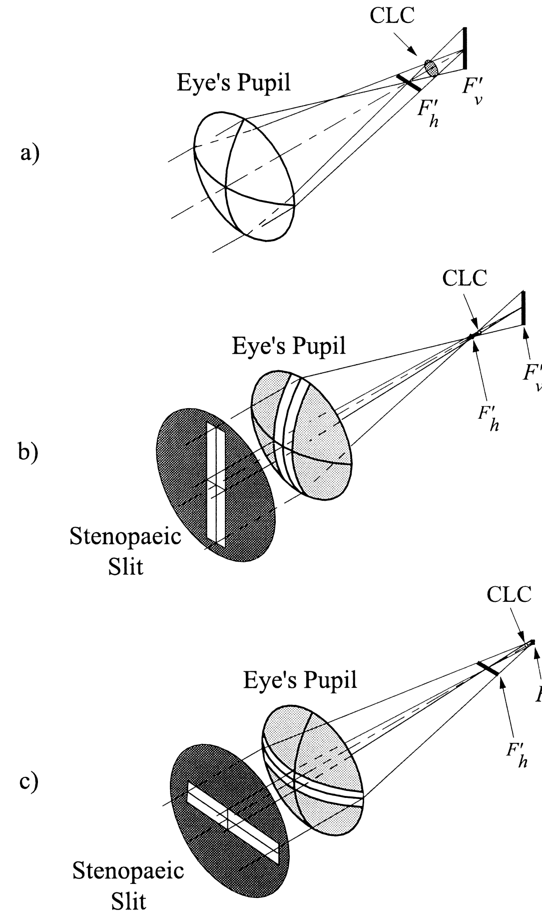

Figure 1. (a) Geometrical representation of the focal lines of an eye with a refractive error of $1/-2 \times 180$. (b) Same as (a) when a stenopaeic slit is placed vertically in front the eye. The CLC shifts axially towards the horizontal focal line and reduces to almost a focal point. (c) Same as (b) when the stenopaeic slit is placed horizontally.

Dioptric power space

Any spherocylindrical power expressed in the clinical form of sphere, cylinder and axis ($S/C \times \alpha$) can also be represented by a vector in a three-dimensional space, called the dioptric power space, for which several coordinate systems have been defined (Harris, 1991; Deal and Toop, 1993; Thibos *et al.,* 1994, 1997). One of the possible sets of coordinates in this space is given by (Deal and Toop, 1993):

$$X = -\frac{C}{2}\cos(2\alpha), \quad Y = -\frac{C}{2}\sin(2\alpha), \quad Z = S + \frac{C}{2}, \qquad (1)$$

where Z represents the spherical equivalent power and X and Y define the plane of the Jackson cross cylinders, as their Z coordinate is zero. In this space pure cylinders lie on the surface of a cone and each value of α (strictly speaking each meridian) defines a line that forms an angle of 45° with the XY plane, that is to say, in the dioptric power space all pure cylinders with the same axis α lie on the same line independently of their power (see *Figure 2*).

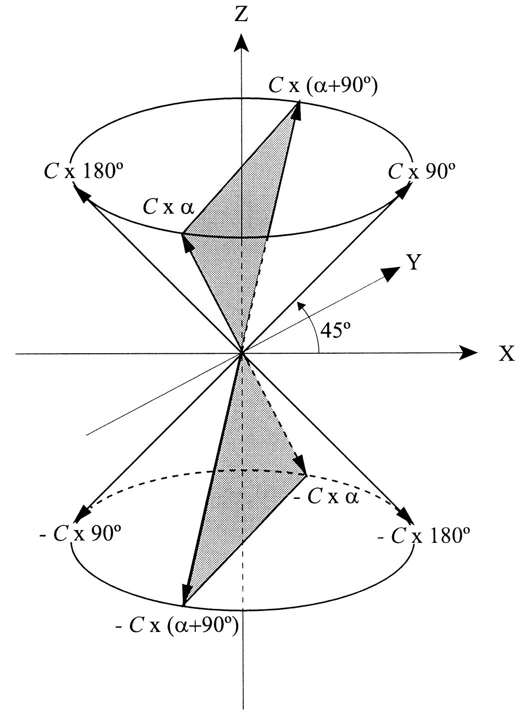

Figure 2. Representation of pure cylinders of power C in the three-dimensional space for all possible orientations of α. The shaded plane shows that the lines corresponding to two pure cylinders with axis α and $\alpha + 90°$, respectively, are orthogonal and define a plane containing the Z axis.

The stenopaeic slit in the dioptric power space

Let us describe the effect of the stenopaeic slit in front of an astigmatic eye within the dioptric power space. To simplify the analysis, we consider as a particular example an astigmatic eye with the refractive error $+1.00/-2.00 \times 180$, which according to Equation (1) has vector components $E = (X, Y, Z) = (1, 0, 0)$. By placing in front of it the stenopaeic slit at an angle $\beta = 90°$, the geometrical analysis of *Figure 1* predicts a residual spherical refractive error ($R_\beta = (0, 0, -1)$) coincident with the dioptric power of the horizontal focal line (-1 D, see *Figure 1b*). Therefore, the effect of a vertical stenopaeic slit in front of the eye in terms of vectors in three-dimensional space is to transform the error $E = (1, 0, 0)$ into the residual error $R_{90} = (0, 0, -1)$ (see *Figure 3a*). Similarly, when the slit is at 180°, the residual error R_{180} obtained is a sphere of $+1$ D with components (0,0,1), as predicted by the geometrical analysis (see *Figures 1c* and *3b*). Thus, the slit projects the vector E onto the plane that, contains the Z axis and is perpendicular to the one defined by the slit orientation. So, when a stenopaeic slit is aligned with one of the principal meridians of the eye, the projection of E for these two orientations of the slit always lies on the Z axis, and the resultant residual refractive error is a sphere. However, when the slit is 45° away from the

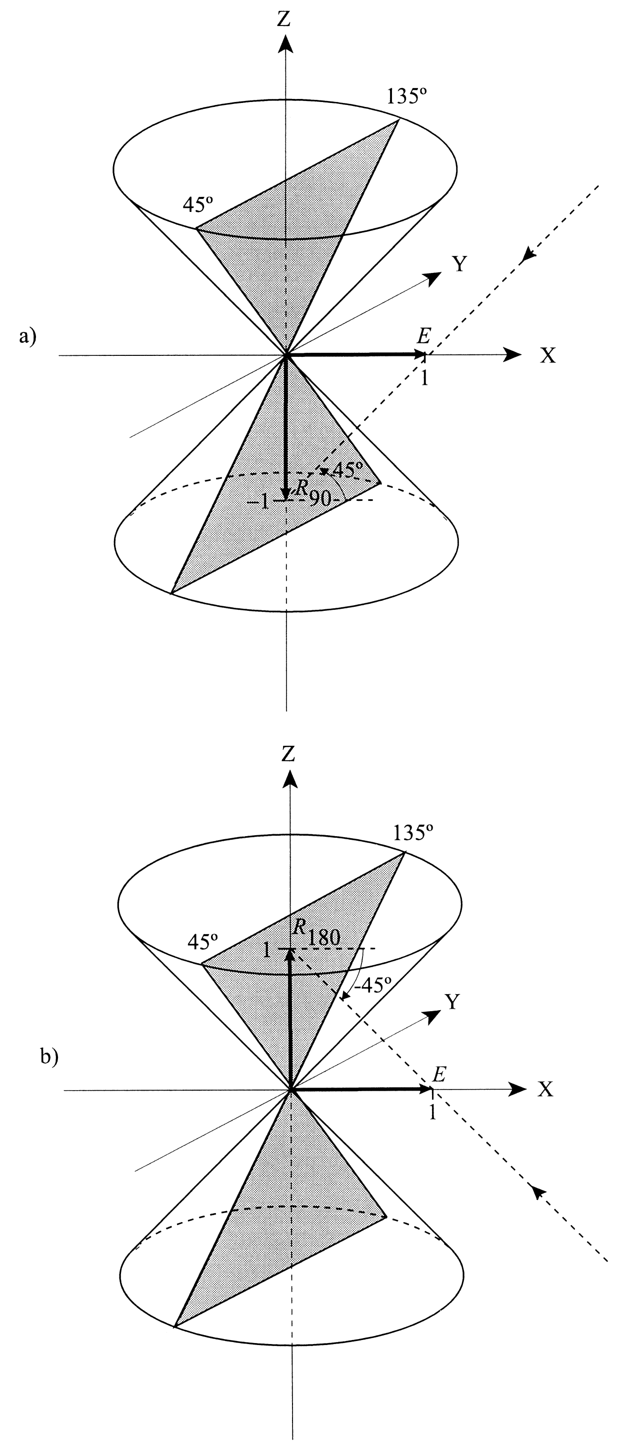

Figure 3. Representation in the three-dimensional space of the stenopaeic slit effect when it is placed in front of an eye with the refractive error $E = 1/-2 \times 180$: (a) at an angle of 90° and (b) at an angle of 180°.

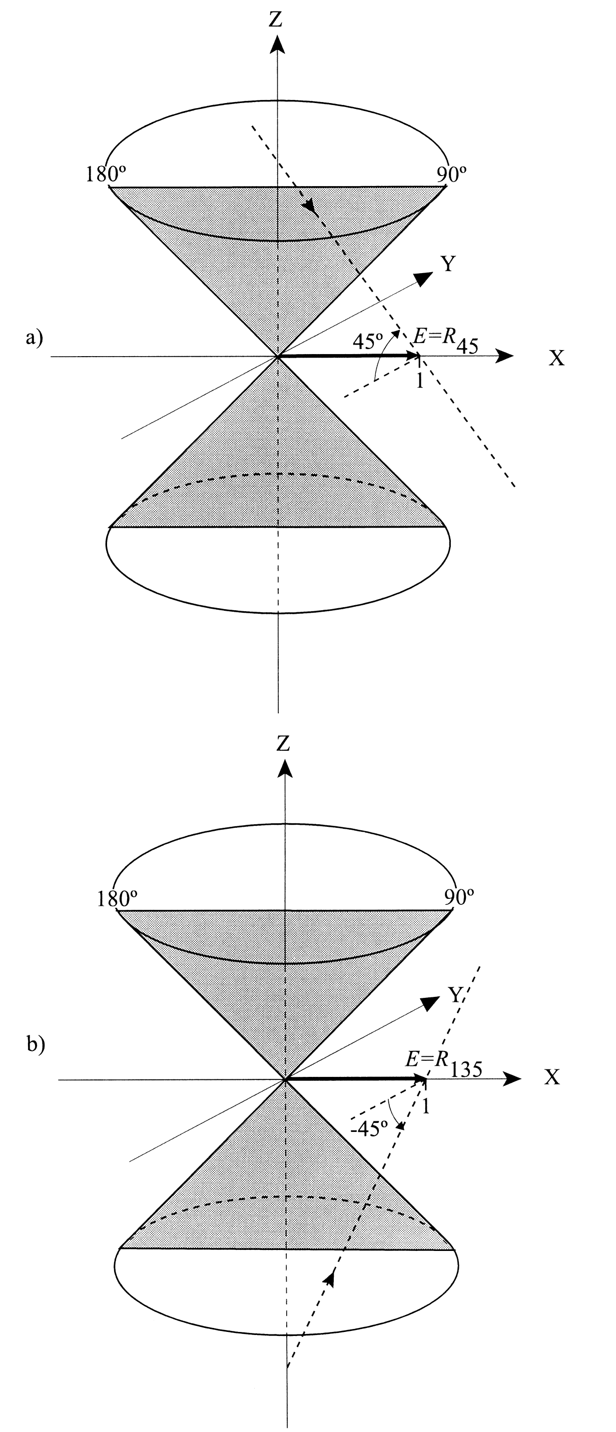

Figure 4. Same as in *Figure 3* when the stenopaeic slit is at: (a) 45° and (b) 135°.

principal meridians, the projection coincides with E for both $\beta = 45°$ and $\beta = 135°$ slit positions, as can be seen in *Figure 4*. Therefore, it is confirmed that a slit placed at 45° to the principal meridians of the ocular astigmatism produces no effect on the eye's refractive error.

Analytical expression

Having established the optical principles of a stenopaeic slit in the dioptric power space, the general equations to calculate the residual refractive error $R_\beta = (X_\beta, Y_\beta, Z_\beta)$ are deduced as follows.

We found that to describe the slit effect on a refractive error, its vector $E = (X, Y, Z)$ must be projected onto the plane perpendicular to the one defined by β and $\beta + 90°$ in the direction given by β. Therefore, it is convenient to change the reference system performing a rotation of the XY plane at an angle $\beta' = (180 - 2\beta)$ around the Z axis in order to match the plane defined by β and $\beta + 90°$ with the new $X'Z'$ plane. Then, the components of the vector E in the rotated system are (see for example Lenaghan and Levy, 1996):

$$X' = X\cos\beta' + Y\sin\beta', \qquad Y' = -X\sin\beta' + Y\cos\beta',$$

$$Z' = Z. \qquad (2)$$

After rotation, the vector E must be projected on the plane $Y'Z'$ in the direction given by β, which describes an angle of 45° with the $X'Y'$ plane (see *Figure 5*). The components of the projection produce (X'_P,Y'_P,Z'_P). As can be seen from the same figure, the X'_P component will be zero and the Y' component remains unchanged ($Y' = Y'_P$). The value of the Z'_P component can be deduced from *Figure 5* as

$$\tan 45° = (Z' - Z'_P)/X'; \qquad \text{i.e. } Z'_P = Z' - X'. \qquad (3)$$

Therefore the projected vector components are

$$E'_P = (X'_P, Y'_P, Z'_P) = (0, Y', Z' - X'). \qquad (4)$$

Finally, we must rotate the $X'Y'$ plane through an angle $-\beta'$ to obtain the residual refractive error R_β in the original coordinate system (X, Y, Z):

$$X_\beta = X'_P\cos\beta' - Y'_P\sin\beta'$$

$$Y_\beta = X'_P\sin\beta' + Y'_P\cos\beta' \qquad Z_\beta = Z'_P. \qquad (5)$$

Applying trigonometrical identities and substituting Equations (1–4) into Equation (5), we obtain:

$$X_\beta = -\frac{C}{2}\sin(2\beta)\sin(2\alpha + 2\beta)$$

$$Y_\beta = -\frac{C}{2}\cos(2\beta)\sin(2\alpha + 2\beta) \qquad (6)$$

$$Z_\beta = S + \frac{C}{2}(1 - \cos(2\alpha + 2\beta)).$$

Of course, from the values X_β, Y_β and Z_β one can revert to standard notation by (Thibos *et al.*, 1997):

$$C = -2\sqrt{X_\beta^2 + Y_\beta^2} \qquad E = Z_\beta - \frac{C}{2}$$

$$\alpha = \frac{1}{2}\arctan\left(\frac{Y_\beta}{X_\beta}\right). \qquad (7)$$

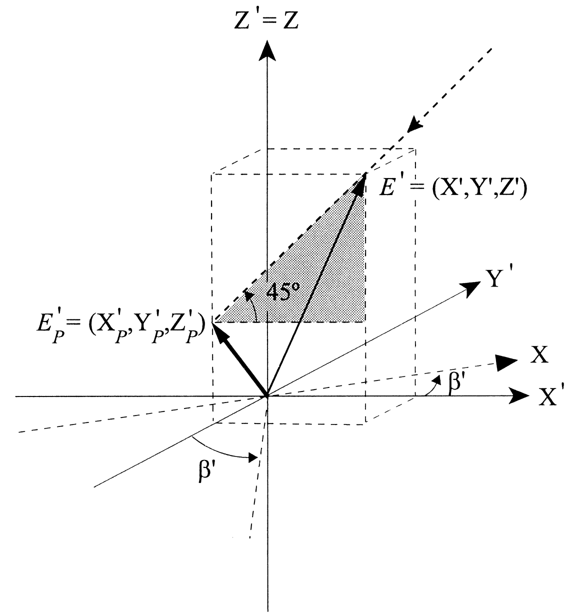

Figure 5. Projection of the vector E' onto the $Y'Z'$ plane.

Equations (6) and (7) are the main result of this paper. They permit us to obtain the refractive residual error that results from a stenopaeic slit located before an eye at any angle β. This result has certain clinical implications in subjective refraction as we discuss next.

Clinical implications

During stenopaeic slit refraction the slit is placed in front of the eye and rotated till it achieves the best visual acuity meridian. Three-dimensional space can be used to show how the tip of the vector representing the residual refractive error moves when the slit orientation is changed. In *Figure 6* the path of the tip of the vector R_β is obtained by means of Equation (6), for β values ranging from $\beta = 0°$ to $\beta = 180°$. Three different spherocylindrical errors have been considered: (a) a case of compound myopic astigmatism of $-1.00/-1.00 \times 180$; (b) a case of mixed astigmatism of $+1.00/-2.00 \times 180$; and (c) a case of compound hyperopic astigmatism of $+1.00/+1.00 \times 180$. In this figure the variation of the residual refractive error produced by the slit at different orientations can be clearly seen. However, these situations are not realistic since the effect of accommodation has not been considered. As blur is the stimulus to accommodation (Fincham, 1951; Morgan, 1968; Phillips and Stark, 1977), this mechanism is devoted to produce an in-focus retinal image. In cases of astigmatism it is generally admitted that accommodation produces minor changes in the cylindrical component of refraction, which are not of clinical importance (Millodot and Thibault, 1985; Bennett and Rabbetts, 1989). With this assumption, if a stenopaeic slit is placed before an astigmatic eye and the CLC is behind

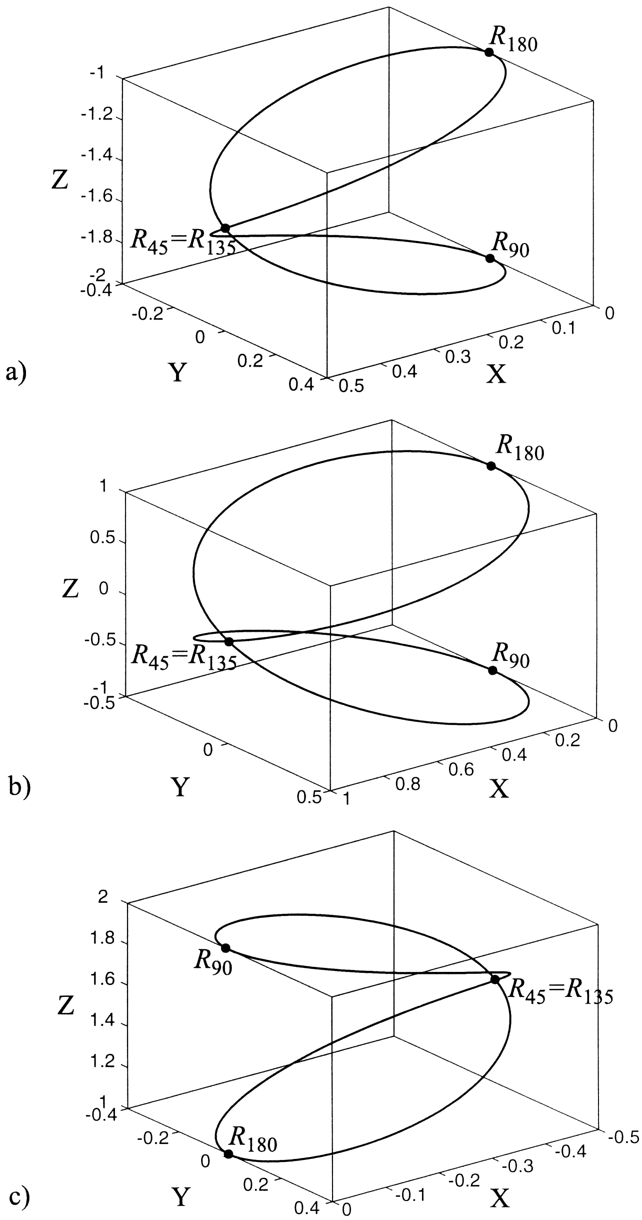

Figure 6. Residual refractive error in the three-dimensional space for slit orientations in the range [0–180°] for an eye with the refractive error: (a) $-1/-1 \times 180$ (b) $1/-2 \times 180$ and (c) $1/1 \times 180$.

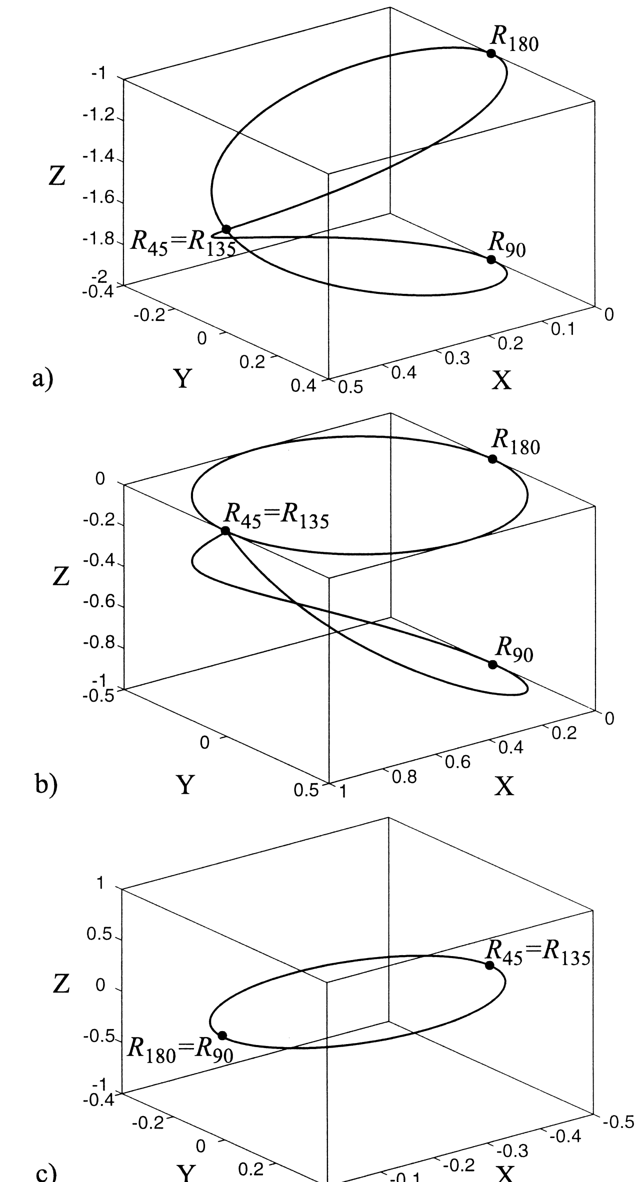

Figure 7. Same as in *Figure 6* when the accommodation is active.

the retina ($Z_\beta > 0$), whenever possible the eye accommodates to obtain the best focused image, making $Z_\beta = 0$. Thus, the profile that describes the R_β vector tip during ideal noncyclopegic refraction for the previous examples is represented in *Figure 7* (where it is assumed that accommodation is always sufficient to bring the CLC onto the retina). Of course, for an eye with compound myopic astigmatism the act of accommodation produces no change as the CLC is in front of the retina for any orientation of the slit ($Z_\beta < 0$ always, see *Figure 7a*). However, in the case of mixed astigmatism (*Figure 7b*), when the eye accommodates the curve changes with respect to the one represented in *Figure 6b*. One half is reduced to a circle in the plane $Z = 0$ and the other half remains unchanged as $Z_\beta < 0$. For the case of compound hyperopic astigmatism, as $Z_\beta > 0$ always, the eye accommodates and the tip of the vector describes twice a circle, so the resultant profile is contained in the plane *XY* (see *Figure 7c*).

The modulus of R_β for each slit position is related to the visual acuity reached by the patient, so that the bigger the modulus, the lower the visual acuity (Raasch, 1995). Besides, visual acuity is used during stenopaeic slit refraction to locate the principal meridians of the eye. Then, for each position of the slit, the modulus of R_β can be calcu-

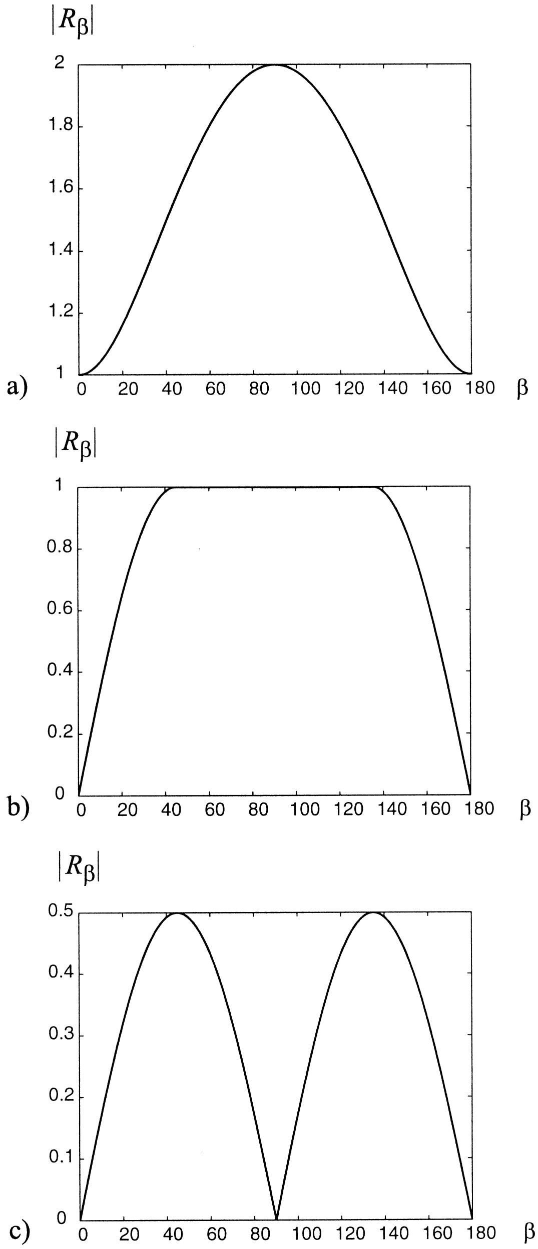

Figure 8. Modulus of R_β corresponding to each one of the curves of *Figure 7*, respectively.

lated by use of Equation (6) and the visual acuity changes induced by the slit can be analyzed. Let us compute the modulus of the residual error for the examples previously considered. For the eye with compound myopic astigmatism, the patient will report two extreme positions of better and worse visual acuity when the slit is at $\beta = 0°$ and $\beta = 90°$, respectively (see *Figure 8a*). However, for the eye with mixed astigmatism and the CLC at the retina, there is a unique orientation of the slit that provides the best visual acuity but not a single orientation of the slit for the worst visual acuity, since $|R_\beta|$ remains constant in the range [45–135°] (see *Figure 8b*). Then, clinicians cannot identify clinically both principal meridians. On the other hand, in cases of compound hyperopic astigmatism, accommodative activity would result in two slit orientations that provide better vision ($\beta = 0°$ and $\beta = 90°$, see *Figure 8c*) whereas two single orientations, $\beta = 45°$ and $\beta = 135°$, both give worse vision. In this case, the principal meridians can be clinically identified as the two orientations of the slit producing the best visual acuity. Consequently, if it is desired to identify clinically both principal meridians during stenopaeic slit refraction, it seems clear that any astigmatic refractive error should be converted into myopic or hyperopic astigmatism.

Conclusions

An analytical expression to obtain the residual refractive error when a stenopaeic slit is placed at any orientation in front of an astigmatic eye has been deduced in the framework of three-dimensional power space. The effect of accommodation on the residual refractive error and the changes in patient visual acuity have also been considered. Since it was assumed that astigmatism does not change due to accommodation, it would be of interest in a future work to verify these results experimentally in order to compare them with those obtained by other researchers (Garzia and Nicholson, 1988; Ukai and Ichihashi, 1991). Finally, it is hoped that the analytical expression for the residual refractive error generated by the stenopaeic slit (Equation (6)) may be used to allow clinicians to gain insights into other objective or subjective techniques that can be implemented with this trial case accessory.

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