Authors' reply

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In the analysis of the stenopaeic slit performance given in our previous work (Muñoz-Escrivá and Furlan, 2001) we used the framework of the tri-dimensional dioptric power space. This kind of representation assumes the existence of a dioptric power in meridians other than the principal ones, as predicted by the sine-squared law (Thibos *et al.*, 1997). As Dr Naeser points out, the discussion about the validity of this assumption is not new, but without doubt, this approximation became very helpful in numerous applications in optometry and ophthalmic optics (see for example Harris, 1991; Deal and Toop, 1993; Raasch, 1995 and references therein). We also consider our approach as one of these innovative applications. In fact, we have shown the effect of a stenopaeic slit in the dioptric power space for the first time in the literature. We have proved that, depending on the refractive state of the eye ($\boldsymbol{E}$), the net effect of the slit is to project the vector defined by $\boldsymbol{E}$ onto a particular plane of that space. In this sense, we called this projection 'the residual refractive error': $\boldsymbol{R}_\beta$. Ideally, the effect of the slit is to isolate a single meridian of the eye. This means that within the above-mentioned approximation the power of this eye can be tuned continuously between the powers of its two principal meridians as predicted by our Eqs. (6) and (7) (Muñoz-Escrivá and Furlan, 2001).[1] In fact, these equations say that the net result of the slit is equivalent to modifying, or even to partially compensating, the ocular refraction. Moreover, by use of our analysis in the dioptric power space, the power of the lens, whose effect is equivalent to the one produced by a slit at a given orientation can be easily deduced. In other words, the same effect got with a slit at an arbitrary orientation in front of an astigmatic eye can be obtained with a single lens. The theory and a few simple examples that support this affirmation are presented next.

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[1]Erratum: The sphere in the standard notation represented by **E** in Eq. (7) must be represented by **S** according to Eq. (1).

Referring to our previous work (Muñoz-Escrivá and Furlan, 2001), the stenopaeic-slit equivalent lens (SSEL) can be obtained by the difference between the refractive state and the residual refractive error when the slit is present, i.e. $\boldsymbol{R}_S = \boldsymbol{E} - \boldsymbol{R}_\beta$. The components of this vector ($\boldsymbol{X}_S$, $\boldsymbol{Y}_S$, $\boldsymbol{Z}_S$) can be obtained from Eqs. (1) and (5) of that reference as:

$$\begin{aligned} \boldsymbol{X}_S &= \boldsymbol{X} - \boldsymbol{X}_\beta = -\frac{C}{2}\cos(2\beta)\cos(2\alpha + 2\beta) \\ \boldsymbol{Y}_S &= \boldsymbol{Y} - \boldsymbol{Y}_\beta = \frac{C}{2}\sin(2\beta)\cos(2\alpha + 2\beta) \\ \boldsymbol{Z}_S &= \boldsymbol{Z} - \boldsymbol{Z}_\beta = \frac{C}{2}\cos(2\alpha + 2\beta) \end{aligned} \qquad (1)$$

From these values, it can be easily shown that the SSEL is a pure cylinder of power

$$C_S = C\cos(2\alpha + 2\beta)$$

orientated at an angle

$$\alpha_S = -\beta \qquad (2)$$

Taking up the example on p. 328 of that reference for an eye with the refractive error +1.00/–2.00 × 180, whose components in the dioptric space are $\boldsymbol{E} = (1, 0, 0)$ [see also *Figures 6(b), 7(b) and 8(b)*], we have calculated $\boldsymbol{R}_\beta$ for four different orientations of the slit. From each one of these values, the SSEL was obtained. *Table 1* summarises our results. It can be seen that the SSEL is a pure cylinder that can be positive or negative depending on the slit orientation. In order to 'examine the situation in practise' we also performed the simple experiment suggested by Dr Naeser. A sphero-cylinder lens of power +1.00 sph, −2.00 cyl was placed in an optical bench simulating a with-the-rule mixed astigmatism. An iris diaphragm was used as an artificial pupil fixed to 4-mm diameter. A slide with letters of different sizes [see *Figure 1(a)*] was used as a test chart, and a CCD camera connected to a PC computer was used as an artificial retina. The experimental results are sketched in *Figure 1(b)–(f)*. For comparison, in *Figure 2* we have repeated *Figure 8(b)* of our previous paper showing the modulus of $\boldsymbol{R}_\beta$ for this particular

Table 1. Residual refractive error and the slit equivalent lens corresponding to an astigmatic eye +1.00/–2.00 × 180 for four values of the slit orientation

β	Residual refractive error	Slit equivalent lens
0°	$\boldsymbol{R}_\beta = (0, 0, +1) = +1.00$ DS	$\boldsymbol{R}_S = (+1, 0, -1) =$ plano/–2.00 × 180°
20°	$\boldsymbol{R}_\beta = (0.41, 0.49, 0.77) = +1.41/-1.28 \times 25°$	$\boldsymbol{R}_S = (0.59, -0.49, -0.77) =$ plano/–1.53 × 160°
45°	$\boldsymbol{R}_\beta = (+1, 0, 0) = +1.00/-2.00 \times 180°$	$\boldsymbol{R}_S = (0, 0, 0) =$ nil
90°	$\boldsymbol{R}_\beta = (0, 0, -1) = -1.00$ DS	$\boldsymbol{R}_S = (+1, 0, +1) =$ plano/+2.00 × 90°

case. When the stenopaeic slit is placed at 180°, our theory predicts $\boldsymbol{R}_{180} = (0, 0, 1)$, i.e. the obtained residual error is a sphere of $+1.00$ DS (see *Table 1*, first row). Thus, assuming that the eye can accommodate, it can compensate the resulting hypermetropia bringing the best focused image to the retina; thus, the modulus of $\boldsymbol{R}_\beta$ is reduced to zero by accommodation [see point (1) in *Figure 2*]. The observed image is shown in *Figure 1(b)*, it can be seen that although the three rows of letters are legible, the best image is not perfect because of the finite extent of the slit, which is implicitly assumed to be infinitesimal in the theory.

When the slit is rotated by 20° (see *Table 1*, second row), the circle of least confusion (CLC) also lies behind the retina, and the eye can also accommodate to obtain the best image. However, in this case the residual refractive error has a cylindrical component of 1.28 DC. Thus, when the CLC is at the retina $|\boldsymbol{R}_{20}| = 0.64$ [see point (2) in *Figure 2*]. Consequently, a worse visual acuity than the previous case is attained. In fact, as shown in *Figure 1(c)* some letters of the third row are now not recognisable. At $\beta = 45°$ the modulus of $\boldsymbol{R}_\beta$ reaches its maximum value [see point (3) in *Figure 2*], so the visual acuity is the worst, as seen in *Figure 1(d)*. As the values of $|\boldsymbol{R}_\beta|$ remain constant in the range 45°–135°, no improvement in the visual acuity is obtained in this range of slit orientations. In these cases, in spite of the different appearance of the retinal images as a result of the orientation of the slit, the legibility of the letters in *Figure 1(d)–(f)* (corresponding to the points 3, 4 and 5, respectively, in *Figure 2*) is almost the same.

In conclusion, although strictly speaking the stenopaeic slit behaviour is not refractive, its optical effect

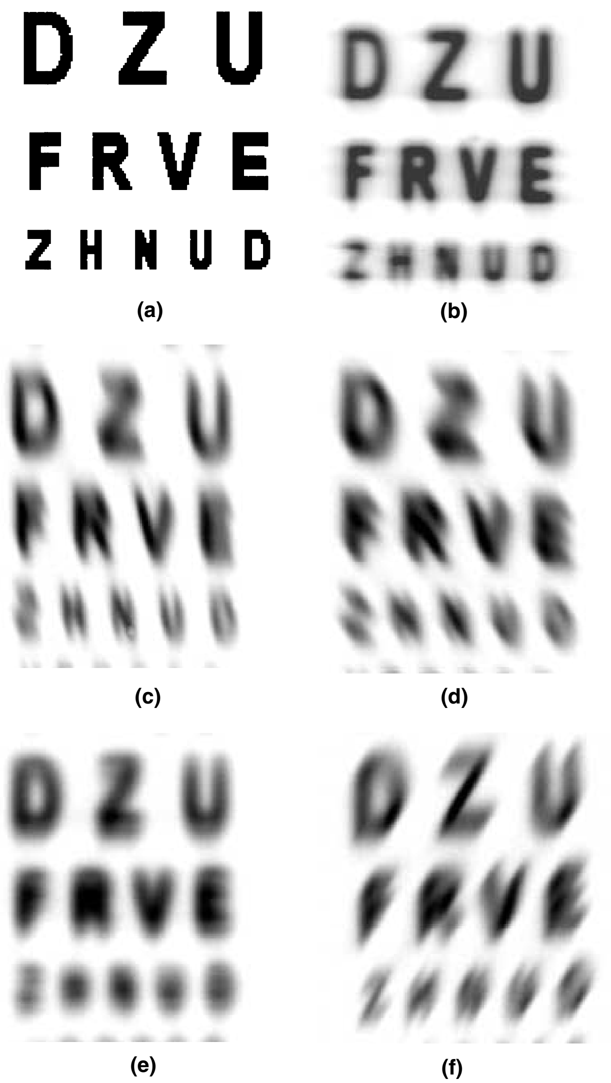

Figure 1. (a) Test chart. (b) Image perceived by an astigmatic eye +1.00/–2.00 × 180 when a stenopaeic slit is at 180°. (c, d, e and f) Same as in (b) when the slit is at 20°, 45°, 90° and 135°, respectively.

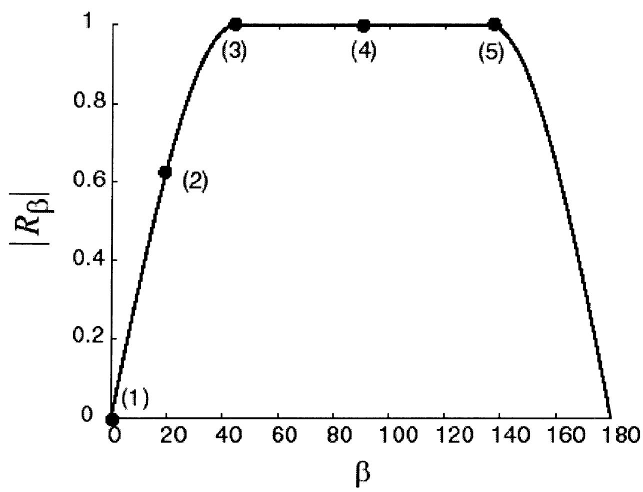

Figure 2. Modulus of $\boldsymbol{R}_\beta$ corresponding to the example in *Figure 1*. The points labelled with numbers (1)–(5) are the values corresponding to images (b)–(f) in *Figure 1*, respectively.

(in front of an a stigmatic eye) can be matched with the refractive one produced by a cylindrical lens. Moreover, our experimental results confirm that the visual acuity declines proportionally with the increment of the modulus of the vector that defines the refraction of the eye in the dioptric power space (Raasch, 1995). We are currently investigating other clinical implications of these results.

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