Filter performance parameters for vectorial high-aperture wave fields

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Performance parameters have been presented that can be used to compare the focusing performance of different optical systems, including the effect of pupil filters. These were originally given for the paraxial case and recently extended to the high-aperture scalar regime. We generalize these parameters to the full vectorial case for an aplanatic optical system illuminated by a plane-polarized wave. The behavior of different optical systems is compared. © 2008 Optical Society of America

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Superresolving pupil filters have many potential applications in optical systems for microscopy, information storage, and material processing. Simple expressions for performance parameters for the scalar paraxial case were presented by Sheppard and Hegedus [1]. These are useful design tools as they allow the investigation of filter performance without the necessity for computing the focal intensity. These parameters hold for real filters, including the important case of phase filters with a phase change of 180°. The parameters include transverse and axial gains, the Strehl ratio, which is a measure of the filter efficiency, and the ratio of the intensity at focus to the total energy, which is a measure of how strong the central lobe is compared with the outer rings.

Usually filters are most useful when working with a high numerical aperture, and the parameters have recently been generalized to the high-aperture scalar case [2]. However, if the numerical aperture is large, then polarization effects become important, so parameters based on a full electromagnetic theory would be advantageous. A few papers have considered the effects of filters based on a vectorial theory [3–7]. The most important polarization case is that of a plane-polarized beam focused by a high-aperture lens.

We consider a high-aperture optical system in the Debye approximation, illuminated by a plane-polarized wave. For an axially symmetric pupil, the electric field in the focal region at the point ρ,ϕ,z in cylindrical coordinates can be written as [8]

$$E_x = -ikf(I_0 + I_2\cos 2\phi),$$

$$E_y = -ikfI_2\sin 2\phi,$$

$$E_z = -2kfI_1\cos\phi, \tag{1}$$

where

$$I_n = \int_{-\infty}^{\infty} Q(c)\left(\frac{1-c}{1+c}\right)^{n/2} J_n(k\rho\sqrt{1-c^2})\exp(ikzc)\mathrm{d}c. \tag{2}$$

Here $Q(c)$ is the pupil function expressed as a function of $c=\cos\theta$, where θ is the angle between the direction of the propagation of a plane wave component and the axis. $Q(c)$ includes the effect of a pupil filter, if any, an apodization factor that depends on the design of the optical system ($c^{1/2}$ for an aplanatic system) and an additional factor $(1+c)$ and is taken to be zero for $c>1$ and $c<\cos\alpha$, where $\alpha\leqslant\pi$. The limiting value $\alpha=\pi$ corresponds to a complete sphere of radiation and usually in practice $\alpha<\pi/2$. Note that Eq. (2) allows all three integrals to be written neatly in a compact form. Expanding in terms of power series to the second order in distance from the focus, we have

$$I_0 = \int_{-\infty}^{\infty} Q(c)\left[1 - \frac{(k\rho)^2}{4}(1-c^2) + ikzc - \frac{1}{2}(kz)^2c^2\right]\mathrm{d}c,$$

$$I_1 = \int_{-\infty}^{\infty} Q(c)(1-c)\frac{k\rho}{2}(1+ikzc)\mathrm{d}c,$$

$$I_2 = \int_{-\infty}^{\infty} Q(c)(1-c)^2\frac{(k\rho)^2}{8}\mathrm{d}c. \tag{3}$$

Introducing the moments of the pupil,

$$q_n = \int_{-\infty}^{\infty} Q(c)c^n\mathrm{d}c, \tag{4}$$

we then have

$$I_0 = q_0 + ikzq_1 - \frac{(kz)^2}{2}q_2 - \frac{(k\rho)^2}{4}(q_0 - q_2),$$

$$I_1 = \frac{k\rho}{2}(q_0 - q_1) + \frac{i(kz)(k\rho)}{2}(q_1 - q_2),$$

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$$I_2 = \frac{(k\rho)^2}{8}(q_0 - 2q_1 + q_2). \tag{5}$$

Often, we wish to produce the most localized wave field for a particular value of the numerical aperture, sin α. The simplest measure of localization is the normalized intensity at focus for a given focused power. It is well known that in the paraxial case this is maximized for a pupil of constant value (the Luneberg apodization problem [9], pp. 348–353), which we may take to be unity. For the high numerical aperture case, at the focal point only I_0 is nonzero, so the intensity (time-averaged electric energy density) at focus is q_0^2. We can use Schwarz's inequality to show that the ratio of the intensity at the focus to the focused power is maximized for a mixed-dipole field $Q(c)=(1+c)^2/2$, $\cos\alpha < c < 1$ [10].

The focused power is proportional to

$$E = \int_{-\infty}^{\infty} \frac{4|Q(c)|^2}{(1+c)^2}\mathrm{d}c. \tag{6}$$

We define a normalized time-averaged electric energy density F as

$$F = \frac{3q_0^2}{4E} \tag{7}$$

and find that for the mixed-dipole field this tends to one-half as $\alpha \to \pi$. The total normalized time-averaged energy density (electric and magnetic) at the focus for $\alpha \to \pi$ is then unity, which explains the factor 3/4 in Eq. (7).

It should be noted that, unlike the paraxial case, the integral of the intensity (energy density) over the focal plane is not proportional to the power crossing the focal plane [11]. The integral of the intensity is proportional to

$$I_{\text{total}} = \int_{-\infty}^{\infty} \frac{|Q(c)|^2}{c(1+c)^2}\mathrm{d}c, \tag{8}$$

where now $\alpha < \pi/2$ to include only forward propagating waves. The ratio of the intensity at the focus to the integrated intensity over the focal plane is maximized if

$$Q(c) = \frac{c(1+c)^2}{2}, \quad \cos\alpha < c < 1 \tag{9}$$

for any $\alpha < \pi/2$. This has been called a perfect wave by Stamnes [12]. We thus call it a perfect mixed-dipole wave. We define another normalized time-averaged energy density F_I as

$$F_I = \frac{3q_0^2}{16I_{\text{total}}}, \tag{10}$$

as then it behaves the same as F for small α. For the perfect mixed-dipole wave for $\alpha \to \pi/2$, $F_I \to 17/64 \approx 0.27$. In many practical cases F_I is a more useful criterion than F, as it describes how the intensity at the focus compares with the integral over the sidelobes. Figure 1 shows the variation of F and F_I with α for these different types of optical systems. Also included is the Herschel case, $Q(c)=(1+c)/2$, corresponding to a uniform angular distribution, the Helmholtz apodization, $Q(c)=c^{-3/2}$, corresponding to a planar diffractive lens, and the parabolic mirror apodization, $Q(c)=1$ [10,13].

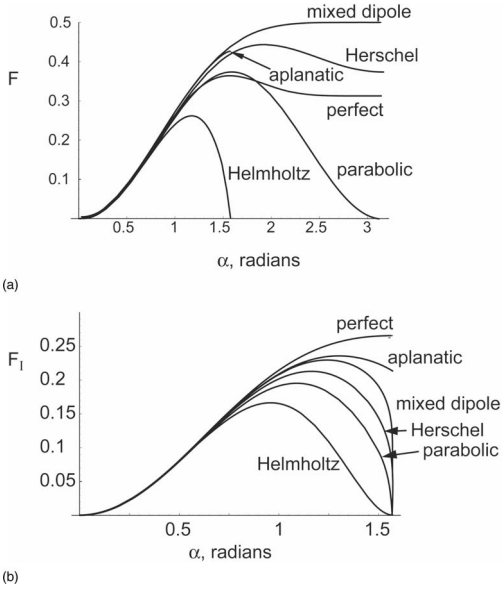

Fig. 1. (a) Variation in F, the ratio of the intensity at focus to the input power, for aberration-free focusing systems of different types. (b) Variation in F_I, the ratio of the intensity at focus to the integrated intensity in the focal plane.

Equations (1) and (5) show how the intensity falls off away from the focus and can be used as approximate measures of the FWHM. Compared with a complete scalar uniformly distributed spherical illumination, we can thus introduce gain parameters G_A and G_T for the linearly polarized case:

$$G_A = 3\left(\frac{q_0 q_2 - q_1^2}{q_0^2}\right), \tag{11}$$

$$G_T = \frac{3}{4}\left[\frac{(4q_0q_1 - 2q_0q_2 - 2q_1^2) - (3q_0^2 - 6q_0q_1 + q_0q_2 + 2q_1^2)\cos 2\phi}{q_0^2}\right]. \tag{12}$$

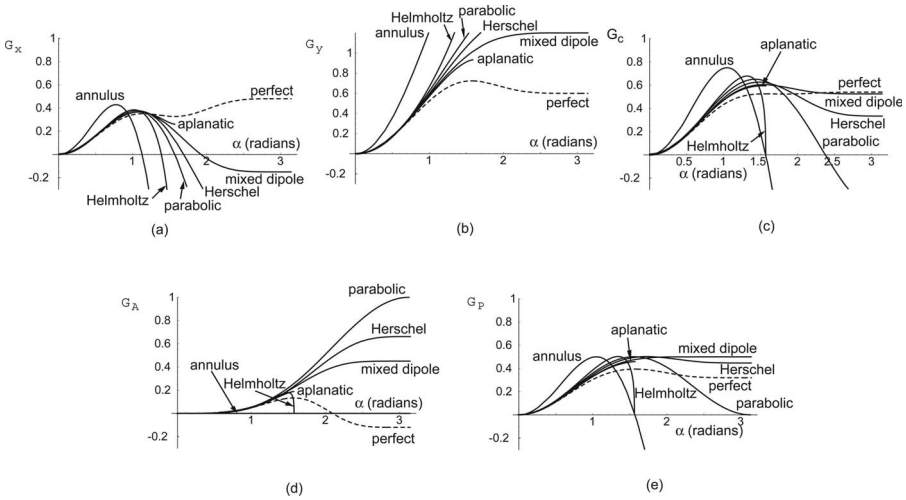

Fig. 2. Gains, compared with complete scalar spherical illumination, for aberration-free focusing systems of different types: (a) transverse x, (b) transverse y, (c) transverse circular-polarized, (d) axial gains, and (e) polar gains.

The central lobe of the focal spot is sharpened up by a factor $G_{A,T}^{1/2}$ compared with a scalar spherical wave. Along the x and y axes (ϕ=0 and $\pi/2$, respectively), we have

$$G_x = \frac{3}{4}\left(\frac{10q_0q_1 - 3q_0q_2 - 3q_0^2 - 4q_1^2}{q_0^2}\right), \tag{13}$$

$$G_y = \frac{3}{4}\left(\frac{3q_0^2 - 2q_0q_1 - q_0q_2}{q_0^2}\right), \tag{14}$$

and for circularly polarized illumination the average of these:

$$G_C = \frac{3}{4}\left(\frac{4q_0q_1 - 2q_0q_2 - 2q_1^2}{q_0^2}\right). \tag{15}$$

We can also define a polar (3D) gain $G_P=(G_x+G_y+G_A)/3$:

$$G_P = \frac{2q_1(q_0 - q_1)}{q_0^2}. \tag{16}$$

The variation of the gains with aperture is shown in Fig. 2. In the y direction, for the mixed-dipole case the gain increases monotonically with aperture becoming greater than unity for apertures $>\pi/2$. The aplanatic case and the perfect wave result in reduced gains. The Herschel, Helmholtz, and parabolic mirror apodizations give increased gains, as could be expected from the increased strength of the angular spectrum at high angles, but this is accompanied by a reduction in F and F_I as a consequence of the increased strength of the outer diffraction rings. For the aplanatic case the gain falls off slightly relative to the mixed-dipole case for apertures close to $\pi/2$. For circularly polarized illumination, the gain for the mixed-dipole case falls off a little for apertures approaching π. As an example of a pupil mask we also give results for a narrow annulus at α, for which the gain in the y direction is increased. In the x direction, the value of G_x for the mixed-dipole field and the annulus become negative (corresponding to a focal minimum in intensity) for large apertures as a result of the doughnut focal spot of the longitudinal electric field component. Thus a narrow annulus does not result in a sharper focal spot than a circular aperture for high numerical apertures, as has been pointed out previously [14].

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