Extension of depth of field using amplitude modulation of the pupil function for bio-imaging

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ABSTRACT

In this paper we present a novel approach to generate images of extended depth of field (DOF) without compromising the lateral resolution to support realization of three-dimensional imaging systems such as integral imaging. In our approach in extending DOF, we take advantage of the spatial frequency spectrum of the object specific to the task in hand. The pupil function is thus engineered in such a fashion that the modulation transfer function (MTF) is maximized only in these selected spatial frequencies. We extract these high energy spatial frequencies using PCA method. The advantage of our approach is illustrated using an amplitude modulation and a phase modulation example. In these examples, we split the pupil filter and choose the optimum transmission/phase value of each section in the filter in a way that the response of the system in all the DOF range as well as spatial frequencies of interest is optimized. Consequently, we have optimized the DOF extension process with blocking the minimum possible area in the pupil plane. This maximizes the output image quality (e.g. 10% DOF improvement) compared to the existing methods where non-optimal blocking of the lens area may cause more degradation in output image quality. Experimental results are presented to illustrate our proposed approach.

Keywords: Integral Imaging, Depth of field, Modulation Transfer Function, PCA

1. INTRODUCTION

Three-dimensional (3D) incoherent imaging systems have many advantages and applications in different areas such as entertainment, data visualization, industrial design, microscopy, and cartography, among other. Various methods have thus far been proposed and studied for 3D image pickup and display.^{1–3} One of these methods is integral imaging (II) where 3D images are reconstructed by integration of different rays coming from two dimensional elemental images formed by a lens array. This method^{4–10} is proven to be a promising 3D imaging technique as it does not require any special glasses. Moreover, it forms auto-stereoscopic images with both horizontal and vertical parallaxes and thus provides continuously varying viewpoints within the field of view. This is unlike conventional stereoscopic systems such as lenticular lens method. However, II faces two major challenges before it can become a widely used technology: low viewing resolution, and limited depth of displayed 3D images.^{11–13} In this manuscript we focus on the latter challenge and present a method of extending image depth of field (DOF) in II. The results can benefit many 3D imaging systems, including 3D integral imaging of microscope objects.^{14–16}

The DOF is often defined as the range of object distances within which the imaging system is satisfying its goal. The goal to be satisfied is defined as having the spatial frequency spectral signal-to-noise ratio (SNR)

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above a specific threshold level. This SNR is based on the system's modulation transfers function (MTF) and is to be maximized along a specific range (i.e. the DOF range). The majority of the literature on the DOF extension topic employ an optical power absorbing apodizer and/or aspheric phase filters, as a means to increase the DOF.^{17–21} However, all these methods suffer from a major drawback. In previous methods that are based on either amplitude modulation or phase modulation, often, the modulating pattern is selected without reference to the frequency spectrum of the object to be imaged.^{22,23} Thus, these methods attempt to maximize the MTF function, the frequency response of the optical system, in all spatial frequencies; even at the spatial frequencies where the signal has no considerable energy. This approach leads to a decrease of optical power at the image plane and deteriorates the image quality. This is particularly important in task-specific imaging systems, where the imaging system is designed to image a special class of objects. It should be however noticed that only one particular object/image is considered in this paper. This object is thus meant to act as a typical example representing a class of similar objects to be imaged.

In our previous work,²⁴ this major drawback was partly overcome by introducing a novel cost function for extending the DOF. This new approach was not only significantly faster compared to the previously used methods in maximizing the DOF range, but also optimized to take advantage of the spatial frequency content of the objects relevant to the task-specific imaging systems. The problem was solved by taking into account the spatial frequency spectrum of the input object and maximizing the MTF at these spatial frequencies to arrive at the task-specific pupil function. It should be noted that in this and previous research we are interested in a particular class of objects (for instance micro-organism). Given the shape, structure and zoom-level of the imaging system (here, a microscope), the images typically have the significant part of their energy concentrated at some spatial frequencies and a negligible amount of energy in other spatial frequencies. So, in designing a task-specific imaging system for a class of objects, the performance of the imaging system at low spatial frequency regions would be irrelevant. Therefore, maximizing MTF at these spatial frequencies would be non-optimal of energy (or in another words, a misuse of limited spectral SNR^{25,26}) and degrades the efficiency of the final pupil filter. Thus, in that work, we attempted to limit the MTF maximization to the relevant spatial frequencies of the object. To extract the high energy spatial frequencies of the image spectrum, we use the principal component analysis (PCA) algorithm. PCA is a simple and efficient algorithm which uses simple mathematical concepts to identify the principal parts of an image.

In this work, we are going to more deeply explore the proposed concept in our previous work.²⁴ In that paper, we simply analyzed the proposed idea in designing a sample amplitude filter. But, this idea could be used to improve the performance of different DOF extension method including amplitude and/or phase modulation. Thus, here we are going to investigate the performance of the proposed task-specific DOF extension method for different pupil function engineering approaches. The resultant images are compared with the same counterparts obtained using the conventional amplitude/phase modulation algorithms.

In Section 2 the previously proposed pupil function engineering method, where object's frequency spectrum is taken into account for improving the DOF extension methods in a task specific imaging system, will be explained briefly. The performance of the proposed method have been tested with a biological object in Section 3 for different pupil function engineering approaches. The obtained results show that the quality of the images constructed by the proposed pupil function is better than that formed by conventionally engineered pupil function. In Section 4 The obtained results have been quantitatively analyzed with some metrics. Section 5 concludes this manuscript.

2. TASK-SPECIFIC PUPIL FUNCTION ENGINEERING

Assume an optical imaging system whose lens has a focal length of f and an aperture's outer diameter of D. The distance between the object plane and the pupil plane is d_o and the distance between the image plane and the pupil plane is d_i . The DOF denotes the range of object distances within which the spatial frequency spectral signal-to-noise ratio (SNR) is above a threshold level, $SNR^{*:22}$

$$DOF = \{d_0 : SNR(\mathbf{u}) \ge SNR^*, \forall \mathbf{u} \in \mathbf{U}^*\},$$
 (1)

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where $\mathbf{u} = [u, v] = [\lambda d_i f_x / D, \lambda d_i f_y / D]$ is the normalized spatial frequency, f_x and f_y (with unit of line-pairs/mm) is the image spatial frequency, and \mathbf{U}^* is the working range of interest for normalized spatial frequencies. In other words, Eq. 1 defines the DOF of a task-specific imaging system as the range of defocus beyond which the spectral SNR drops below a minimum value within a band of spatial frequencies of interest, \mathbf{U}^* .

The spectral SNR in Eq. 1 is defined as follows: 23,25

$$SNR(u,v) = KR^{2}(u,v)MTF^{2}(u,v),$$
⁽²⁾

where K is a constant coefficient which depends on the parameters of the optical system and R(u, v) is the spectral density of object reflectivity at spatial frequencies u and v in the image plane. Based on this relation, maximizing SNR in DOF extension process can be performed via not only through MTF but also the spatial spectrum density of the object, R(u, v). Therefore, in the proposed method we used the object's spectrum in the new proposed cost function.

Generally, in the DOF extension method we try to compensate the blurring effect of defocus on the resulted image by introducing a pupil filter, h(x, y), as follows:

$$P(x,y) = h(x,y) \times \operatorname{circ}(1) \times \exp\left\{2\pi \mathbf{i} \left[W_{20}(x^2 + y^2)\right]\right\},\tag{3}$$

where in general h(x, y) is the pupil filter which could be an amplitude and/or phase modulation pattern. The DOF extension methods are based on engineering this filter in a way that the resulted SNR is maximized in the desired DOF range. For example, reducing the size of the aperture of the lens (amplitude) is a common way of extending the DOF. This is done using different methods such as annular apodization.^{11,22,27} In such methods, the system is made less susceptible to defocus error by blocking some angle range of marginal rays. This approach may adversely affect the system in two ways: (i) the overall performance of the optical system may be reduced, since the smaller aperture blocks some range of spatial frequencies $[u_{max} = D/(1.22\lambda d_i)]$; and (ii) the total number of photons reaching the photodetector array within an exposure frame is reduced proportionally to D^2 , thus leading to a lower mean *SNR*. Consequently, it is desired to block as little area as possible of the pupil plane, i.e. the pupil filter needs to be made more efficient. In our method which will be explained in the following section, we discuss an improved form of h(x, y) (pupil filter) engineered based on the spectrum of the input image. In this section as well as the remainder of this manuscript, we use terms such as *spectrum, spectral* and *frequency* with reference to the spatial spectrum and spatial frequency exclusively. Also, the terms *pupil filter* and *pupil function* refer to the same concept and may be used interchangeably.

In most previous DOF extension methods, SNR maximization is conventionally done by controlling the MTF function through a modulation of the pupil function, h(x, y). The reason lies in the relation between MTF and spectral SNR. Let us investigate this relation in more depth to see how one can improve this objective in DOF extension algorithms used in task-specific imaging system. A deeper analysis of the SNR relation (Eq. 2) clarifies that besides MTF, the frequency spectrum of the input image is also an important factor. Thus in task-specific pupil function engineering, given that R(u, v), the representative of the intended class of objects, has most of its energy only at a specific frequency region, we propose to maximize R(u, v)MTF(u, v) instead of only the MTF(u, v) function. In this way we have escaped from dedicating energy to MTF where there is no signal energy. Although this new objective function help to improve the system response in task-relevant spatial frequencies, the search and optimization is again done in all frequencies which increases the number of required computations. Hence, for the algorithm to be more efficient, we introduce the effect of object spectral content in the new objective function by extraction of the high energy spatial frequencies of the object. Then, we can maximize MTF just in these spatial frequencies which is not only more energy efficient but also is computationally better. Thus, here the new introduced objective function is to maximize MTF(u,v)R(u,v)S(u,v) where S(u,v) is a weighting function added to discard the unimportant frequencies related to low energy part of the object's spatial frequency spectrum. It is worth noting that even though S(u, v) is extracted by using the spatial spectrum of the representative of the intended class of objects, it is not necessarily appropriate for all possible object orientations and/or scaling. It is thus necessary to have a bank of S(u, v)s to account for different object orientations and imaging magnifications.

This effect can also be viewed in the spatial domain. Indeed, maximizing MTF just in the dominant frequencies, prevents the misuse of energy by not distributing it in task-irrelevant spatial frequencies. This translates to minimum dissipation of energy in the spatial domain, based on Parseval's principal which in term minimizes the blocked region in methods such as annular apodization and thus minimizes the two shortcomings of DOF extension algorithms (resolution and mean SNR) as mentioned before.

There is another point that worth mentioning here. Numerical calculation of the double integral in the MTF relation for all frequencies and all defocus values [Eq. (4)] is a tedious and time consuming task. By limiting the MTF optimization to the limited frequency region where objects have most of their energy besides optimizing the used algorithms, we are gaining another benefit. This advantage is the less time we need to calculate the MTF integral which in term lessens the above mentioned computational complexity and time consumption.

3. IMPLEMENTATION OF THE OPTIMIZATION OF THE TASK-SPECIFIC PUPIL FUNCTION ENGINEERING

In this section we illustrate the implementation of the proposed fast optimization method for two main categories of pupil function engineering: 1. amplitude modulation and 2. phase modulation. For the purpose of amplitude/phase modulation, we divide the pupil filter to n sections. Each section may have any of the m allowable transmission values between 0 and 1 for amplitude modulation and 0 and π for phase modulation(in general, the transmission values may be any value within the unit complex disk for the general form of amplitude-phase modulation), where the larger the value of m, the more exact is the amplitude/phase quantization of the obtained results. The exact choice of m and n is based on the calculation capacity, among other things. We then find the optimum transmission/phase values for each n sections of the pupil filter (between 0 and 1/0 and π) to maximize the MTF in the frequency region extracted with the PCA method, for the DOF range of interest. In the following sections the proposed method for the two main DOF extension approaches will be analyzed. Please see Ref. 24 for mathematical details of our method.

3.1 EXAMPLE OF USING THE PROPOSED METHOD FOR AMPLITUDE OR PHASE MODULATION

Regarding the computational capacity and the required precision, We start with n = 16 and m = 8. In this case, the pupil filter is divided into four sections along the x direction and into four sections along the y direction. We construct the pupil function for different possible transmission values for these regions (i.e. different amplitude modulation) in different defocus values, W_{20} . Then, we can calculate the MTF from autocorrelation of the pupil function for each of these scenarios²⁴.

By using the PCA method, we can extract the dominant frequency region which indicates exactly what region in the function $MTF(u, v, W_{20})$ is of importance to us. Then, all is left is to optimize the amplitude modulation to maximize the MTF or SNR in those regions:²²

$$\max_{h_{ij} \in \mathcal{T}} \left\{ \min_{\mathbf{u} \in \mathbf{U}^*, d_o \in DOF} \left[\text{SNR}(\mathbf{u}, d_o) \right] \right\},\tag{4}$$

where $i \in \{1, \ldots, i_{max}\}$, $j \in \{1, \ldots, j_{max}\}$ are the indexes of the pupil sections while $i_{max} \times j_{max} = n$ and $\mathcal{T} = \{t_1, \ldots, t_m\}$ denotes the range of possible transmission values. In brief, Eq. 4 states that we find the worst SNR for all the range of U^* and DOF and then maximize it with optimum selection of the pupil filter transmission values. The maximization process could be done using an exhaustive search optimization algorithm. Yet, one can use a local optimization to find the final optimal pupil filter.

To analyze and implement the proposed method in extending DOF for amplitude modulation, we use a biological sample (the cross section of rat's brain). Brain is a 3D object and in many studies its 3D image is needed. Thus, in these specific tasks we have many of these images, captured by each lens in the II method, to



Figure 1. Comparison between obtained images with amplitude modulation(the number of used grids is 4). (a) Filtered image at $W_{20} = 0.5$ with PCA. (b) Filtered image at $W_{20} = 0.5$ with no PCA, and comparison between obtained images with phase modulation(the number of used grids is 4) (c) Filtered image at $W_{20} = 0.5$ with PCA. (b) Filtered image at $W_{20} = 0.5$ with PCA. (b) Filtered image at $W_{20} = 0.5$ with PCA. (b) Filtered image at $W_{20} = 0.5$ with PCA. (c) Filtered image at $W_{20} = 0.5$ with PCA. (c) Filtered image at $W_{20} = 0.5$ with PCA. (c) Filtered image at $W_{20} = 0.5$ with PCA. (c) Filtered image at $W_{20} = 0.5$ with PCA. (c) Filtered image at $W_{20} = 0.5$ with PCA. (c) Filtered image at $W_{20} = 0.5$ with PCA. (c) Filtered image at $W_{20} = 0.5$ with PCA. (c) Filtered image at $W_{20} = 0.5$ with PCA. (c) Filtered image at $W_{20} = 0.5$ with PCA. (c) Filtered image at $W_{20} = 0.5$ with PCA. (c) Filtered image at $W_{20} = 0.5$ with PCA. (c) Filtered image at $W_{20} = 0.5$ with PCA. (c) Filtered image at $W_{20} = 0.5$ with PCA. (c) Filtered image at $W_{20} = 0.5$ with PCA.

be processed for forming the 3D image. The microscope can be designed to work better for this task-specific imaging system using here proposed approaches. The test image has its most significant edges at a specific direction which leads to a particular high energy region in its spatial frequency spectrum. Thus, this image is an appropriate candidate for testing the efficiency of our proposed approach.

The next step is to divide the pupil filter into n regions while the transmission value (i.e. amplitude and/or phase) in these regions are independently optimized. For the sake of simplicity, we are going to optimize the transmission amplitude and phase of these sections, separately (i.e. amplitude modulation or phase modulation). The resulted MTF was applied on the image in two defocus extremes, 0 and 0.5. The obtained images were compared with their traditional counterpart (in the same defocus values) obtained without using PCA (previous DOF extension methods). The high frequency parts of the image taken with the PCA-based optimized amplitude filter [Fig. 1(a)] is clearly superior in 0.5 defocus value compared to the image taken with the traditional optimum pupil filter [Fig. 1(b)]. It is apparent from these figures that the obtained images using the proposed method contains more detail and information compared to images of the previous methods (i.e. pupil function engineering without PCA).

In the next experiment we implemented the proposed method for deriving the appropriate phase modulations. The followed steps are just like the previous section. The only difference is that here each pupil section could have a transmission phase between 0 and π while the resulting phase filter is applied to the pupil function. Figures, 1(c) and 1(d) show the image resulted from applying this filter in $W_{20} = 0.5$ (the worst case) compared to the conventional phase modulation.

As it is clear from these figures, the result of applying phase modulation filter using the proposed method is only a bit better than the image resulted from conventional phase filter which does not use the object's spectrum. this result is also acceptable, as in phase modulation the blocking of pupil does not occur and therefore optimizing the modulation pattern will not help to improve the performance as much as the amplitude modulation case.

4. DISCUSSION ON RESULTS WITH QUANTITATIVE METRICS

In the previous section we tested the performance of the proposed method for different pupil function engineering approaches. The resulted images from pupil filter obtained using PCA (proposed method) and conventional method (no PCA) was shown where the superiority of the obtained images using PCA was perceptible. This better performance was more noticeable in cases including amplitude modulation where the blocking effect were minimized. To show this superiority more accurately, we have used two quantitative merits as follows:

Difference between high frequency contents: One merit for showing the improved performance of the resulted images is to measure the high frequency content of the images which shows the amount of details they contain. The more the value of this merit, the more detail in the image and thus the higher quality of the image.



Figure 2. Difference between high frequency contents for different approaches;(a) Amplitude modulation (b) Phase modulation

Figure 2 shows the amount of this parameter for the conventional imaging system (no pupil function engineering), the obtained images from the proposed method (using the exact MTF formula plus PCA) and traditional pupil function engineering (with no PCA). As expected the high frequency content which shows details of image is more in the filtered images using PCA in all defocus values.

Gradient between consequent axial images: Another figure of merit that can be used to verify the superiority of the resulted images is the defocus independence or the difference between consequent images in the axial direction (i.e. along the optical axis or in other words through the DOF as defocus change). In calculating this merit we subtract the two consequent axial image and then average the obtained difference values. By using the proposed method for DOF extension we expect the image to be more consistent in the expected defocus range compared to the previous method's images. Thus, the smaller the gradient between consequent images in the axial direction, the higher the quality of the resulted images. Figure 3 contains the results of this parameter for the conventional imaging system (no pupil function engineering), the proposed method (using the exact MTF formula plus PCA) and traditional pupil function engineering (with no PCA). Again, as expected the gradient between consequent axial images in the new method (using PCA filter) is smaller; showing less variation in different defocus values compare to the focal point. The resultant data of these two merits confirm the superiority of the method. In particular we can see a 14% increase in DOF compared to traditional pupil function engineering.

In each of the two tested scenarios the two introduced metrics where computed for different defocus values. The resulted curves for the first metric, difference high frequency contents, are shown in Figs. 2(a) and 2(b) for amplitude and phase modulation, respectively. In each figure we have two curves for comparing the performance of the proposed method with PCA and the conventional method.

It is clear from the figures that in cases where we have amplitude modulation the difference between two curves is more, suggesting that the amount increase in high frequency contents (resolution) is larger as as was discussed in the previous section.

The next metric which shows the amount of change of images along DOF range was also measured for two tested approaches. The results are shown in Figs. 3(a) and 3(b) for two modulation methods.

Again, we see that the second metric is better (i.e. smaller), for the filtered images obtained using PCA. Thus, shown curves in Figs. 2 and 3 clearly demonstrate the better performance of the method especially when using amplitude modulation.

5. CONCLUSION

In this paper, we proposed and analyzed a novel approach to generate images of extended DOF to support realization of three-dimensional task-specific imaging systems such as integral imaging. We illustrated that in task-specific imaging systems maximizing MTF (and thus SNR) at all spatial frequencies will be sub-optimal and



Figure 3. Gradient between consequent axial images for different approaches; (a) Amplitude modulation (b) Phase modulation.

a misuse of the available spectral SNR. Therefore, we proposed to maximize MTF just at the spatial frequencies where the object has significant energy. To extract these high energy spatial frequency regions, we used a PCA-based algorithm.

Finally, we analyzed and implemented the proposed method in extending the DOF on a biological sample object for two main approaches in pupil function engineering. The resulted images from the proposed pupil filter with PCA with amplitude or phase modulation were obtained and compared to the images resulted from traditional pupil filters (without PCA). This comparison was made rigorous by introducing two figure of merits which showed the superiority of the proposed approaches. In particular, we achieved more than 10% increase in DOF using our method. This method may be extended to a variety of dimensionality reduction schemes other than spatial frequency, in the context of task-specific imaging such as infra-red wavelength imaging.

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REFERENCES

- [1] Javidi, B. and Okana, F., "Three dimensional television, video, and display technologies.," Springer (2002).
- [2] Benton, S. A., "Selected papers on three-dimensional displays.," SPIE (2001).
- [3] Okashi, T., [Three-Dimensional Imaging Techniques], Academic (1976).
- [4] Lippmann, G., "La photographic integrale," Comtes-Rendus 146, 446–451 (1908).
- [5] Ives, H. E., "Optical properties of a lippman lenticulated sheet," J. Opt. Soc. Am. 21, 171–176 (1931).
- Burckhardt, C. B., "Optimum parameters and resolution limitation photography," J. Opt. Soc. Am. 58, 71-76 (1968).
- [7] Stern, A. and Javidi, B., "3d image sensing, visualization, and processing using integral imaging," Proceedings of the IEEE Journal 94, 591–608 (2006).
- [8] Yang, L., McCornick, M., and Davies, N., "Discussion of the optics of a new 3d imaging system," Appl. Opt. 27, 4529–4534 (1988).
- [9] Okano, F., Arai, J., Mitani, K., and Okui, M., "Real-time integral imaging based on extremely high resolution video system," *Proceedings of the IEEE Journal* **94**, 490–501 (2006).
- [10] Choi, H., Min, S. W., Yung, S., Park, J. H., and Lee, B., "Multiple viewing zone integral imaging using dynamic barrier array for three-dimensional displays," *Opt. Exp.* 11, 927–932 (2003).
- [11] Martnez-Cuenca, R., Saavedra, G., Martnez-Corral, M., and Javidi, B., "Extended depth-of-field 3-d display and visualization by combination of amplitude-modulated microlenses and deconvolution tools," *IEEE Journal of Display Technology* 1, 321–327 (2005).

- [12] Castro, A., Frauel, Y., and Javidi, B., "Integral imaging with large depth of field using an asymmetric phase mask," Opt. Exp. 15, 10266–10273 (2007).
- [13] Bagheri, S. and Javidi, B., "Extension of the depth of field in integral imaging: an overview," Three-Dimensional Imaging, Visualization, and Display, SPIE Proc. of 7329, 732906 (2009).
- [14] Javidi, B., Moon, I., and Yeom, S., "Three-dimensional identification of biological microorganism using integral imaging," Opt. Exp. 14, 12095–12107 (2006).
- [15] Jang, J. S. and Javidi, B., "Three-dimensional integral imaging of micro-objects," Opt. Lett. 29, 1230–1232 (2004).
- [16] Levoy, M., Zhang, Z., and McDowall, I., "Recording and controlling the 4d light field in a microscope objects," J. of Microscopy 235, 144–162 (2009).
- [17] Bagheri, S., Silveira, P. E. X., and de Farias, D. P., "Analytical optimal solution of the extension of the depth of field using cubic-phase wavefront coding. part i. reduced-complexity approximate representation of the modulation transfer function," J. Opt. Soc. Am. A 25, 1051–1063 (2008).
- [18] Bagheri, S., Silveira, P. E. X., Narayanswamy, R., and de Farias, D. P., "Analytical optimal solution of the extension of the depth of field using cubic-phase wavefront coding. part ii. design and optimization of the cubic phase," J. Opt. Soc. Am. A 25, 1064–1074 (2008).
- [19] Ojeda-Castaeda, J., Ramos, R., and Noyola-Isgleas, A., "High focal depth by apodization and digital restoration," Appl. Opt. 27, 2583–2586 (1988).
- [20] Ojeda-Castaeda, J., Tepichin, E., and Diaz, A., "Arbitrary high focal depth with a quasi-optimum real and positive transmittance apodizer," *Appl. Opt.* 28, 2666–2670 (1989).
- [21] Ojeda-Castaeda, J. and Berriel-Valdos, L. R., "Zone plate for arbitrarily high focal depth," Appl. Opt. 29, 994–997 (1990).
- [22] Bagheri, S. and Javidi, B., "Extension of depth of field using amplitude and phase modulation of the pupil function," Opt. Lett. 33, 757–759 (2008).
- [23] Silveira, P. E. X. and Narayanswamy, R., "Signal-to-noise analysis of task-based imaging systems with defocus," Appl. Opt. 45, 2924–2934 (2006).
- [24] Bagheri, S., Kavehvash, Z., Mehrany, K., and Javidi, B., "A fast optimization method for extension of depth-of-field in three-dimensional task-specific imaging systems," *To Appear in J. of Display Technology* (2010).
- [25] Bagheri, S., Silveria, P. E. X., and Barbastathis, G., "Signal-to-noise-ratio limit to the depth-of-field extension for imaging systems with an arbitrary pupil function," J. Opt. Soc. Am. A 26 (2009).
- [26] Bagheri, S., "Signal-to-noise-ratio limit to the depth-of-field extension for task-specific imaging systems with an arbitrary pupil function," *Computational Optical Sensing and Imaging (COSI)* (2009).
- [27] Welford, W. T., "Use of annular aperture to increase focal depth," Opt. Soc. Am. A. 50, 749–753 (1960).