# 3D resolution in computationally-reconstructed integral photography

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## ABSTRACT

In this research we have proposed a new definition for three-dimensional (3-D) integral imaging resolution. The general concept of two-dimensional (2-D) resolution used also for 3-D is failed to describe the 3-D resolvability completely. Thus, the researches focused on resolution improvement in 3-D integral imaging systems, didn't investigate thoroughly the effect of their method on the 3-D quality. The effect has only been shown on the 2-D resolution of each lateral reconstructed image. The newly introduced 3-D resolution concept has been demonstrated based on ray patterns, the cross-section between them and the sampling points. Consequently the effect of resulting sampling points in 3-D resolvability has been discussed in different lateral planes. Simulations has been performed which confirm the theoretical statements.

**Key words:** Integral imaging, 3D imaging and display, 3D resolution

## Introduction

Integral imaging (InI), based on the Integral Photography concept proposed in 1908 by Gabriel Lippmann [1], recently re-attracted researchers interest, due to its capacity of capturing many views in real-time. Integral imaging offers many advantages as opposed to the other existing 3D sensing techniques as it uses natural light, can offer full parallax in real-time and does not need special glasses [2]. Therefore, it constitutes a promising technology for the production of real-time 3D image capturing and displaying systems. The main reason for the low quality of InI is the limited resolution of the reconstructed image. Many researchers have focused their work on improving the resolution of the output 3D images by improving the number of pixels in each elemental image (EI) or decreasing the lens pitch [3-9]. Therefore in all of these works the resolution improvement was done in the EI side while the real effect on the 3D image side was not investigated thoroughly. Motivated for this lack, in this work we analyze the concept of resolvability in 3D InI systems and demonstrate its difference with its conventional counterpart. To this end, we first analyze the distribution of optical rays in the 3D reconstruction space and the patterns of intersection with different planes parallel to the microlens array. Based on this analysis, a new definition for 3D resolvability in InI is proposed. The theoretical analysis is confirmed through experiments.

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## Three-dimensional resolvability in InI

In a conventional single-lens imaging system, the final resolution is determined by the minimum value of the sensor pixel size and the lens resolution. Therefore, a the most attempt in improving the resolution is to improve the lens resolution or to decrease the sensor pixel size. This simple imaging system can be used also for the capture of images of 3D objects. This can be made by obtaining a stack of 2D images after scanning the 3D object along the two coordinates of the integral image plane. The 3D image is built computationally as the 3D matrix synthesized from the captured 2D images. In such case, the lateral resolution is still determined by the pixel size, whereas the scanning pitch determines the axial resolution.

In a 3D InI system the concept of resolution is not so simple. Here, an array of lenses and a 2D matrix sensor are used to capture the lateral and the directional information. Papers have been devoted to the analysis of resolution of InI systems. A fraction of them are focused on improving the number of pixels in each elemental image and thus output image [3-6], while a second group proposed methods to improve angular resolution by decreasing the lens pitch and increasing the number of lenses in the array [7-9]. The most comprehensive work in this area is done by Hoshino et al [10], Okoshi [11] and Burckhardt [12]. Specifically, Okoshi proposed the concept of the resolvability of a point in 3D space i.e. from neighboring lateral points and depth points. In other words 3D resolution is a mixture of spatial lateral resolution and depth resolution.

Here we use a more comprehensive definition for 3D resolvability based on the concept of ray-crossing. The crossing of the rays at a sampling point causes it to become more distinguished from its axial neighbors. More crossing rays means better depth discrimination. Therefore, although in the image obtained through a microlens array (MLA) the spatial resolution is worse than in the one obtained through a single lens, the depth resolvability is better. Regarding this fact, we can propose a new and comprehensive definition for 3D resolvability. The 3D resolvability of an integral imaging system is defined with not only the distance between cross-points but also the number of rays crossing in each point. Both factors for a point, determine the amount of its differentiation from neighboring points in 3D space and thus its 3D resolution.

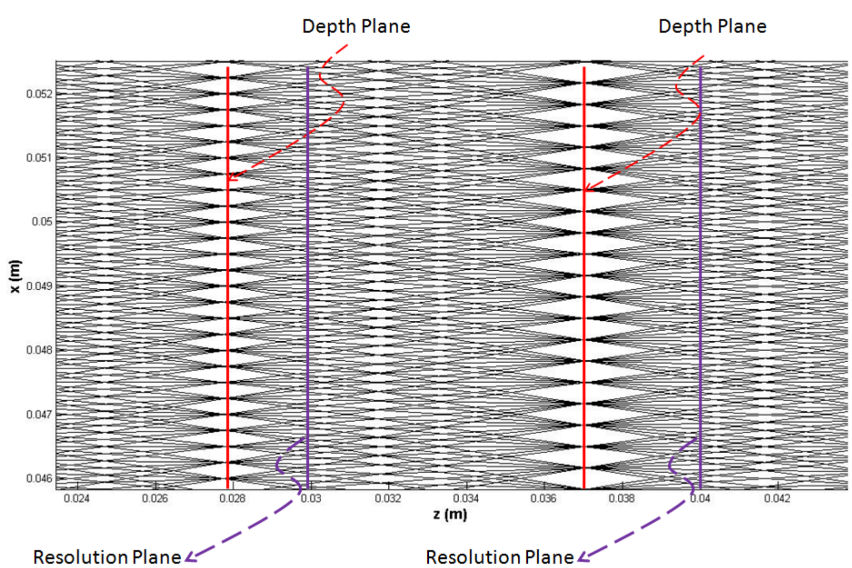



Fig. 1. Ray-crossing pattern in a sample InI system. The pattern was calculated assuming a microlens array with pith p=1 mm, focal length of 5 mm, and elemental images composed by 25 x 25 pixels.

With respect to the proposed definition of the 3D resolvability, we should sketch the ray patterns of an InI system to derive the 3D resolution in each image plane. As is clear from Fig. 1, both the depth and lateral resolution change as the reconstruction plane changes. This change follows a quasi-periodic pattern and does not necessarily increase with axial distance despite what Okashi's formula states. It is noticeable in Fig. 1 that in some reconstruction planes there are more intersection points giving rise to better lateral resolution. Nevertheless, in some other planes the number of sampling points is smaller but each point contains more crossing rays and thus has better depth discrimination. Based on this fact, we define two special kinds of planes. The "resolution planes" have the highest number of sampling points and thus good lateral resolution and poor depth discrimination. The "depth planes" have the least number of sampling points but the most ray crossing occurs in each point improving depth differentiation with respect to points of neighbor planes. These two kinds of planes are shown in Fig. 1. Naturally, the 3D resolution in other planes is between these two extremes depending on the number of samples and the number of crossing in each sampling point.

Of course, the most important parameters affecting the distance between depth and resolution planes are the lens pitch, and the number of pixels of each elemental image. The smaller the pitch the higher the number of rays crossing in each point and therefore the shorter the distance between different depth planes. On the other hand, the more the number of pixels in each elemental image the more the number of resolution planes. An important conclusion is that the 3D resolvability has a periodic dependence with distance from the lens array, contrarily to what Okoshi's definition states.

## Analytical Justification

The principle of InI is the simultaneous capturing of multiple views of the 3D scene using an array of microlenses (Fig. 2). To simplify the following analysis we will just consider one lateral direction (x-direction), though the analysis is easily expandable to the two-dimensional lateral plane. In the matrix sensor behind each microlens an elemental image is recorded, obtained from a slightly different point of view. This image has very low resolution due to the limited size of each elemental image.

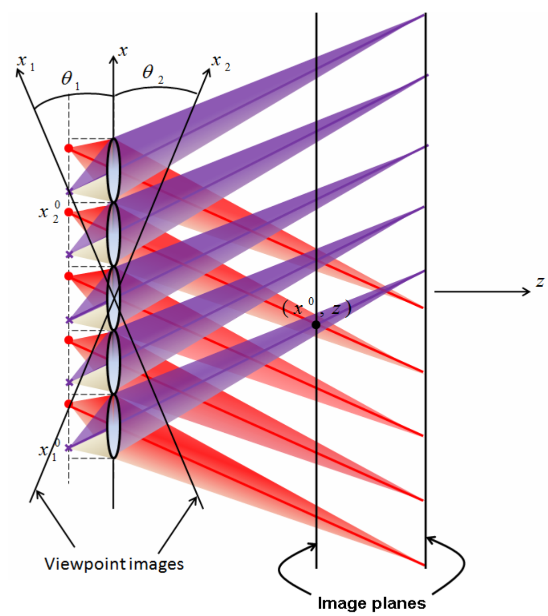



Figure 2. Lens set-up for integral image generation using a one-dimensional microlens array and the local coordinate systems associated with viewpoint images.

A re-arrangement of the pixels of the elemental images leads to the formation of the viewpoint images (Fig. 3).

The first viewpoint image is formed by the first pixel of each elemental image, the second by the second and so on. As parallel rays are recorded at the same position under each microlens, each viewpoint image contains information recorded from one particular direction. The resolution of the viewpoint images is equal to the pitch period of the microlens array.

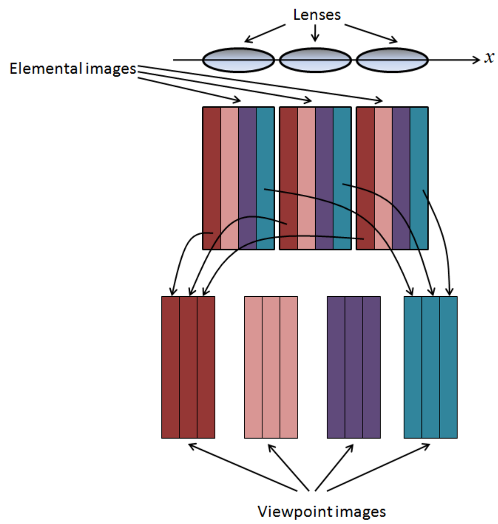



Figure 3. Viewpoint image generation from a unidirectional integral image.

Consider an integral image produced by an array of M microlenses. Thus, there will be M elemental images, each having N pixels. Decomposing the integral image to its viewpoint images, results to N viewpoint images, each of M pixels. It should be noted that viewpoint images are different from common images. They are parallel projections, recording the 3D scene from a different angle rather than perspective projection in the traditional 2D recording. Orthographic projection is a type of parallel projection with rays perpendicular to the projection plane. Similarly, viewpoint images are orthographic projections of the 3D scene in planes rotated, with respect to the image plane (Fig. 2), by angle $\theta$ specified by the lens parameters. This angle depends on the pitch, P, of each microlens and f (the focal length), and is ranging from $-Pitch/2f$ to $Pitch/2f$ (Fig. 2).

Analyzing the recording process of integral imaging, the connection between a 3D point and its projections is established. A local coordinate system for each viewpoint image is used (Fig. 2) that is rotated by angle $\theta$ around the local coordinate system of the central viewpoint, which is the one that has zero angle with the image plane. The projection of a 3D point on the kth viewpoint image plane includes a rotation around the y axis (normal to the $x$-$z$ plane):

$$x_k = x\cos\theta_k + z\sin\theta_k \tag{1}$$

Obviously, when two projections $x_1^0$, $x_2^0$ of the same 3-D point, $(x^0, z)$, on the planes with angles $\theta_1$ and $\theta_2$ respectively, are known, then the coordinates of this point can be easily calculated by:

$$\begin{cases} x_1^0 = x^0\cos\theta_1 + z\sin\theta_1 \\ x_2^0 = x^0\cos\theta_2 + z\sin\theta_2 \end{cases} \tag{2}$$

In general, if the projections of a 3-D point on more than two viewpoint image planes, $h$ viewpoint image planes, are available, then its coordinates can be computed by solving the following system of equations:

$$\begin{cases} x_1^0 = x^0 \cos\theta_1 + z\sin\theta_1 \\ x_2^0 = x^0 \cos\theta_2 + z\sin\theta_2 \\ \qquad \vdots \\ x_h^0 = x^0 \cos\theta_2 + z\sin\theta_2 \end{cases} \tag{3}$$

By increasing the number of equations the dimension of the error matrix of other 3D points in satisfying these equations will increase. Therefore, the norm of this error matrix will enhance causing other 3D points to become more differentiated from the corresponding point. This fact is the main key in describing the properties of different points in 3D image space. The number of rays, h, crossing in each 3D point is equal to the number of its corresponding viewpoint images. On the other side, the map of the point on all h viewpoint images leads to h equations, which consequently assign an h dimensional error matrix to other points in satisfying these equations. Therefore, the number of rays crossing in each 3D point determines the amount of error in mapping other points on this 3D point in 3D integral image reconstruction. The value of error in term determines the capability of this point to resolve itself from other 3D points in reconstruction stage. This error is especially more important for the points very close to the corresponding 3D point while the value of error in each equation is small. In this case, the more dimension of the error gives raise to its norm. The closest 3D points with the error norm more than a specific value, determine its 3D resolution.

In intensity point of view, when we have just one $x_k$ corresponding to a 3D point then in Eq. (1), many 3D $(x,z)$ coordinates will be imaged on $x_k$ with nearly the same intensity and so depth discrimination will not be perfect at all. This is the case occurs in a single lens imaging system. But, when there is more number of images of a 3D point, $(x,z)$, we will have more number of equations in Eq. (3), then many points will capture the intensity of this 3D point. Then in reconstruction all these points will be mapped to the same 3D point and thus this point will be created with more intensity, differentiating it from its neighboring points more robustly, compared to the other points along the same ray corresponding to each $x_k$.

Therefore when a 3D point is imaged through multiple rays, each mapping it to a different viewpoint image plane, in the reconstruction stage, it will be differentiated from other 3D points more strongly. Thus, there are in general two main factors determining the amount of resolvability of a point in the 3D space and consequently the 3D resolution. The first is distance between points in the image plane, which is determined by the number of rays intersecting the lateral image plane. Second is the number of rays crossing each other at the same 3D point. The higher the number of these rays, the better this point will be differentiated from other points along the axial (depth) direction. Therefore to measure the resolution of 3D integral imaging system we should both consider the number of samples in each lateral plane and the number of rays crossing each other in one sample point.

To better demonstrate the capability of the proposed definition of 3D resolution in expressing the image quality, we have performed some experiments. To this end, we have first simulated the pattern of all rays emanating from all elemental images and passing through the lens array deriving the intersection of these rays and lateral planes. Then we have selected a "resolution plane" and a "depth plane". As explained in the above section, in depth planes each point is imaged through a number of rays and thus in a number of viewpoint images. Therefore, the images in these planes will be discriminated from other depths more robustly. On the other side, in resolution planes, we have more number of sampling points and so the resolution of image in these planes is better but will not be discriminated fully from other depth planes because of the less number of constraining equations in Eq. (3). Then we will place a group of objects one time in depth planes and the other time in resolution planes and will record the integral images. In reconstruction, we expect more depth discrimination in depth planes and higher lateral resolution and thus more frequency content in resolution planes.

## Experiments

In this section, to better demonstrate the proposed concepts we have performed the following experiment, in which we selected a real object to show the analyzed 3D resolution property of integral imaging systems. The selected object is an optometrist with a resolution board in his hand. In this experiment, we have used digital camera which was displaced in a rail following an array of 11x11 positions separated by 10 mm. The gap of the camera was fixed to $g = 35\,mm$. Each elemental image was composed of $201 \times 201$ px. Then, we sketched the ray patterns, as in the Fig.1, and selected a pair of depth plane and resolution plane very close to each other. The resolution plane is in 35.0 cm distance and the depth plane is in 35.2 cm distance from the lens array plane. Then, we recorded two sets of EIs. For the first set, we placed the optometrist so that the board was set at the depth plane. In the second, the board was set at the resolution plane. The experimental set up and the recorded elemental images for depth plane imaging are shown in Fig. 4 and Fig. 5 respectively.

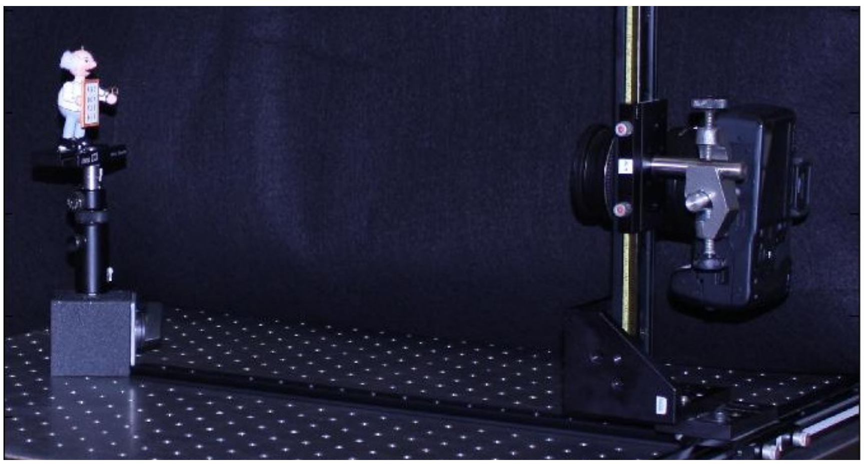


Figure 4. The experimental setup.

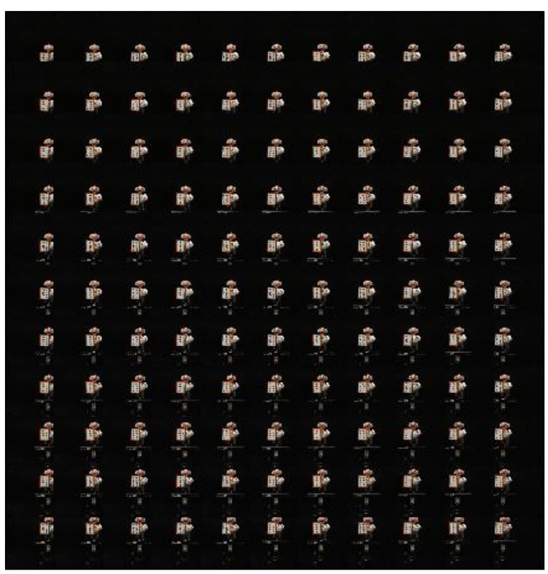
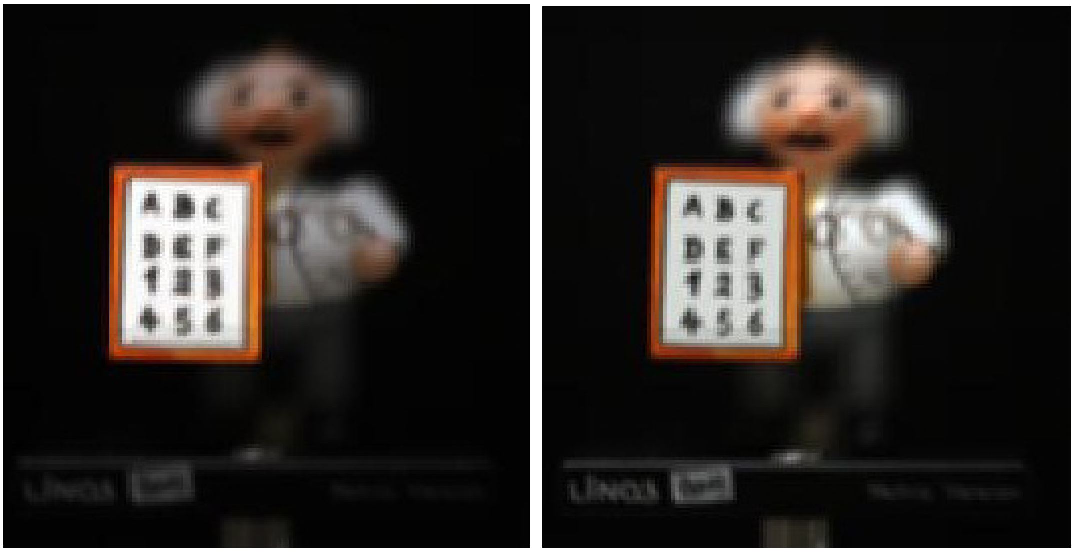


Figure 5. The elemental images

We performed computational reconstruction for the two imaging planes using the two sets of captured EIs. The resulted images in depth plane and resolution plane are shown in Fig. 6.



(a) (b)

Figure 6. Reconstructed image of optometrist while the board is in (a) depth plane and (b) resolution plane.

The expected properties are clearly noticeable. In the depth plane image the board is more distinctly reconstructed than the other out-of-focus parts of the image while in resolution plane, the other parts of the image that are not in the reconstruction plane are also as distinguishable as the written board. On the other hand, the resolution of the board is better preserved in resolution plane imaging as the letters on the board are reconstructed more visibly.

Therefore, in a 3D InI system, the amount of desired 3D resolution in each direction depends on the applications. In applications where the amount of desired spatial resolution in each 2D image plane (lateral plane) is more advantageous, the resolution planes are best candidates to do imaging in. Therefore, the lens array and imaging system structure should be designed in a way to have more number of resolution planes. But in the cases where depth discrimination and 3D information is more important or the general images doesn't have a high resolution content, depth planes will better satisfy the imaging goals. In this case the parameters of the integral imaging system should be chosen in a way to lead more number of depth planes and more ray-crossing in each sampling points.

## Conclusions

A new definition for three-dimensional resolution in an integral imaging system was proposed. Despite previous definitions, which were just based on visualization, here the concept is explained more thoroughly using ray optics. The concept of lateral and axial resolvability was explained based on geometrical optics and sampling properties of rays and ray crossing. Then two different imaging planes were introduced while one of them had appropriate depth resolution and the other one resolved lateral points more accurately. The selection between these imaging planes depends on the amount of desired 3D resolution in lateral and axial (depth) directions. The proposed concept was confirmed through mathematical analysis. To better demonstrate the analyzed properties a experiment was. The result of experiment verifies the claimed concepts and features. In our future works, we are going to also consider the effect of eye and observer's distance along the ray crossing on the 3D resolution. The other related work will be designing the integral imaging system for the purpose of better axial and/or lateral resolution.

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