Nonlinear adjustment in the real euro-dollar exchange rate: the role of the productivity differential as a fundamental

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Abstract

In this paper we analyze the influence of productivity differentials in the dynamics of the real dollar-euro exchange rate. Using nonlinear procedures for the estimation and testing of ESTAR models during the period 1970-2006 we find that the dollar-euro real exchange rate shows nonlinear mean reversion towards the productivity differential fundamental. This fundamental does not only matter for the real exchange rate within each regime, but also drives changes between regimes. In addition, we provide evidence about the ability of the productivity differential to capture the overvaluation and undervaluation of the dollar against the euro.

JEL classification: C22, F31.

Keywords: nonlinearities, real exchange rate, productivity differential fundamental.

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1 Introduction

The purchasing power parity (PPP) theory postulates that national price levels should be equal when expressed in a common currency. Since the real exchange rate is the nominal exchange rate adjusted for relative national price levels, variations in the real exchange rate represent deviations from PPP. It has become something of a stylized fact that the PPP does not hold continuously, reflecting that deviations of spot exchange rates away from PPP are persistent, consistent with a unit root or near-unit root behavior of the real exchange rates. The persistent divergence from equilibrium causes that linear PPP-based fundamentals exchange rates models do not perform well in predicting or explaining future or past exchange rate movements (Frankel and Rose, 1995, Taylor, 1995, Sarno and Taylor, 2002). Other authors still believe that some form of PPP does in fact hold at least as a long run relationship (MacDonald, 1999, 2004). The issue of whether the real exchange rate tends to revert towards a long-run equilibrium has been a topic of considerable debate in the literature (e.g. Lothian and Taylor, 1996, Lothian and Taylor, 1997, and Taylor and Taylor 2004, among others). Panel unit root and long-run studies have reported favourable evidence to parity reversion (see Taylor, 1995, for a survey), however, as pointed out by Rogoff (1996), it is impossible to reconcile the high short-term volatility of real exchange rates with the slow rate at which shocks in the real exchange rate appear to die out in those studies. This conclusion, known as the PPP-Puzzle, constitutes one of the most controvertial issues related to real exchange rates.

The relatively recent literature on nonlinearities and exchange rates can be considered a possible solution to such puzzles. Taylor, Peel and Sarno (2001) and Kilian and Taylor (2003) argue that allowing for nonlinearities in real exchange rate adjustment are key both to detect mean reversion in the real exchange rate and to solve the PPP-puzzle. Following their argument, the further away the real exchange rate is from its long-run equilibrium, the stronger will be the forces driving it back towards equilibrium. Another way to reconsider the linear PPP-based models is to integrate in this basic model the impact of shocks coming from real variables. Thus, persistent shocks might be supply-related and incorporate, for example, the Harrod-Balassa-Samuelson (HBS) effect which postulates that productivity shocks affect the equilibrium real exchange rate. From the empirical evidence, it seems that the productivity differential plays a very important role in explaining some real exchange rate movements. Alquist and Chinn (2002) find supporting evidence for the productivity differential as the most important fundamental that explains the behavior of the real dollar-euro exchange rate since the mid 1980s. Furthermore, they argue that the magnitude of the correlation
between the two variables is much larger than what would predict the HBS effect. Camarero, Ordóñez and Tamarit (2002) have also estimated a long-run model for the euro-dollar exchange rate, finding that the main factor explaining the dynamic adjustment in the error correction model is again the productivity differential. In contrast, Schnatz et al. (2004) find that although the productivity differential is an important determinant of the real dollar-euro exchange rate, its ability to explain the real depreciation of the euro in the late nineties is very limited.

In this paper we focus on testing for and estimating some form of nonlinear adjustment in the real dollar-euro exchange rate towards the productivity fundamental. The purpose of the analysis is to check the ability of the productivity differential to capture the euro-dollar exchange rate behavior. With this aim, we consider a nonlinear model that incorporates the productivity differential as a transition variable for the real exchange rate. As a previous step, we compare the productivity differential against other fundamentals to assess whether the former is the main factor explaining the dollar-euro real exchange rate dynamics.

The remainder of this paper is organized as follows. Section 2 briefly describes the methodology used in the empirical analysis. In Section 3 we present the data, as well as the estimated linear model, which will constitute the starting point of the modelling methodology. In Section 4 we describe the estimated nonlinear model and analyze the adjustment of the real exchange rate towards its productivity fundamental. The last section concludes.

2 Methodology

A number of authors have reported evidence of nonlinear adjustment in the real exchange rate\(^1\). Such nonlinearities can be modelled using a smooth transition autoregressive (STAR) process, proposed by Granger and Teräsvirta (1993). In his model, adjustment takes place in every period at a speed of adjustment that varies with the extent of deviation from equilibrium. STAR models can be formulated as follows:

\[
y_t = (\alpha + \sum_{i=1}^{p} \phi_i y_{t-i})[1 - G(\gamma, y_{t-d} - c)] + (\tilde{\alpha} + \sum_{i=1}^{p} \tilde{\phi}_i y_{t-i})G(\gamma, y_{t-d} - c) + \varepsilon_t
\]

\(^1\)See for example Taylor (2006) for a recent overview of the real exchange rate and purchasing power parity debate.
where $\alpha$, $\tilde{\alpha}$, $\gamma$ and $c$ are constant terms; $\varepsilon_t$ is an i.i.d. error term with zero mean and constant variance $\sigma^2$. The transition function $G(y_{t-d}; \gamma, c)$ is continuous and bounded between 0 and 1.

The STAR models can be considered regime-switching models that allow for two regimes associated with the extreme values of the transition function $G(y_{t-d}; \gamma, c) = 1$ and $G(y_{t-d}; \gamma, c) = 0$, where the transition between these two regimes is smooth.

A very popular transition function used to model real exchange rates is the exponential function suggested by Granger and Teräsvirta (1993):

$$G(y_{t-d}; \gamma, c) = 1 - \exp\{-\gamma(y_{t-d} - c)^2\}, \quad \gamma > 0 \quad (2)$$

where $c$ is the equilibrium level of $\{y_t\}$ and $\gamma$ the transition parameter, which determines the speed of transition between the two extreme regimes, with higher values of $\gamma$ implying faster transition.

Putting (1) into (2) we obtain an exponential STAR or ESTAR model. The exponential function is symmetric and U-shaped around zero. The ESTAR model collapses to a linear AR($p$) model for either $\gamma \to 0$ or $\gamma \to \infty$, and it is therefore useful to capture symmetric adjustment of the endogenous variable above and below the equilibrium level. Symmetric adjustment is a necessary condition to hold in order to model real exchange rates, because otherwise it is difficult to think of any economic reasons why the speed of adjustment of the real exchange rate should vary according to whether one currency is overvalued or undervalued.

In our research, we will use the procedure suggested by Granger and Teräsvirta (1993) and Teräsvirta (1994) for the specification and estimation of parametric STAR models. Their technique consists of the "specific-to-general" strategy for building nonlinear time series models suggested by Granger (1993) and, as indicated by van Dijk (2002), it comprises the following steps: (a) specify a linear AR model of order $p$ for the time series under investigation; (b) test the null hypothesis of linearity against the alternative of STAR nonlinearity; (c) if linearity is rejected, select the appropriate transition variable and the form of the transition function; (d) estimate and evaluate the model; (e) use the model for descriptive or forecasting purposes.

In order to test for linearity against STAR alternatives Luukkonen et al. (1988) and Teräsvirta (1994) suggest a sequence of tests based on the
estimation of the following auxiliary regression:

\[ y_t = \beta_0 + \sum_{i=1}^{p} \beta_{1i} y_{t-i} + \sum_{i=1}^{p} \beta_{2i} y_{t-i} - d + \sum_{i=1}^{p} \beta_{3i} y_{t-i}^2 - d + \sum_{i=1}^{p} \beta_{4i} y_{t-i}^3 - d + \epsilon_t \]  

(3)

with \(1 \leq d \leq p\). Equation (3) is obtained by replacing the transition function in the STAR model by a suitable Taylor series approximation. The null of linearity against a STAR model corresponds to: \(H_0: \beta_{2i} = \beta_{3i} = \beta_{4i} = 0\) for \(i = 1, 2, \ldots, p\). The test statistic, denoted \(LM_3\) in van Dijk et al. (2002), has an asymptotic \(\chi^2\) distribution with \(3(p+1)\) degrees of freedom under the null (of linearity).\(^2\)

Furthermore, in order to test linearity against an ESTAR alternative, Escribano and Jordá (2001) suggest a fourth-order Taylor approximation. They argue that by adding a fourth order regressor the test provides better results when the data are mainly in one of the regimes, as well as when there is uncertainty about the lag length of the autoregressive part. The corresponding LM test statistic, denoted \(LM_4\) in van Dijk et al. (2002) has an asymptotic \(\chi^2\) distribution with \(4(p+1)\) degrees of freedom under the null of linearity.

Once nonlinearities are proved to be significant, the adequacy of the estimated STAR model can be evaluated using the tests suggested by Eitrheim and Teräsvirta (1996). They proposed three LM tests for the hypotheses of no error autocorrelation, no remaining nonlinearity and parameter constancy.

### 3 Data and linear estimation

The data is quarterly and covers the period 1970:Q1 to 2006:Q4. We use the series from Camarero, Ordóñez and Tamarit (2005) for the nominal (synthetic) dollar-euro exchange rate data for the period 1970:Q1 to 1997:Q4 and from the European Central Bank Monthly Bulletin for the rest of the sample. The real dollar-euro exchange rate is computed using consumer price indices. CPI-data are obtained from the OECD Main Economic Indicators.

\(^2\)van Dijk et al. (2002) also proposed another linearity test based on a first order Taylor approximation of the transition function around \(\gamma = 0\). This test statistic is denoted \(LM_1\) and has an asymptotic \(\chi^2\) distribution with \((p+1)\) degrees of freedom under the null of linearity. These authors also suggest another version of the \(LM_3\) statistic, which can be obtained by augmenting the auxiliary regression in the first-order Taylor expansion with regressors \(y_{t-d}^2\) and \(y_{t-d}^3\). The resultant test, denoted \(LM_3^e\), has an asymptotic \(\chi^2\) distribution with \((p+3)\) degrees of freedom under the null of linearity.
database for the US and from the European Central Bank Monthly Bulletin for the EMU. The productivity differential is proxied by labor productivity differential, computed as GDP per employed person. Employment and GDP data are taken from the OECD Main Economic Indicators database with the exception of the German labor data for the period 2001:Q1 to 2006:Q4 which have been obtained from the German Statistisches Bundesamt. European productivity is a weighted average based on the relative GDP of the four largest euro-area economies, with fixed weights and base year 2005.\(^3\)

Figure 1 displays both the logarithm of the real dollar-euro exchange rate and the logarithm of the productivity differential. As pointed out by Alquist and Chinn (2002) there seems to be a strong negative correlation between these two series. For this reason, it would be interesting to analyze how movements in the productivity differential affect the dynamics of the real dollar-euro exchange rate.

In order to do so and to start with the estimation and evaluation procedure, we first test whether the real exchange rate behavior can be properly captured by a linear autoregressive model. To this purpose, we estimate an AR model with a lag length of one year.\(^4\) Table 1 contains the estimated model, as well as its corresponding diagnostic tests. From the results, the model does not present any specification problems and therefore, it still cannot be discarded as a possible alternative for modelling the real dollar-euro exchange rate.

### 4 Nonlinear estimation results

The next step in the modelling process is to select the transition variable. Table 2 presents the test statistics for testing for linearity against STAR nonlinearity. We present linearity test for five exchange rates fundamentals to assess which one is the most suitable as a transition variable. The suggested fundamentals are the labor productivity differential between the US and the euro-zone (\(\text{prod}_t\)), the differential in public expenditure (\(\text{exp}_t\)), the differential in foreign asset position (\(\text{nfa}_t\)), the relative oil dependence (\(\text{oil}_t\)) and, finally, the real interest rate difference (\(\text{int}_t\))\(^5\). In order to choose

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\(^3\)The choice of the countries used for aggregation is mainly due to problems of data availability. However, even if we consider only four countries, Germany, France, Italy and Spain account for over 80% of the euro-area GDP.

\(^4\)The order of the lag has been determined by excluding the statistically insignificant lags of higher order, starting with a lag of 8.

\(^5\)Data consists of an updated version of those used in Camarero, Ordóñez and Tamarit (2002). A detailed description of the variables can be found in appendix A.
the transition variable within these five fundamentals, we follow the procedure proposed by Teräsvirta (1994). According to him, the appropriate transition variable in the STAR model can be determined by computing the \( \text{LM}_3 \) statistic for some candidate transition variables, and selecting the one with the lowest \( p \)-value. In general, the productivity differential presents for all lags the lowest \( p \)-values with both the standard linearity test and its heteroskedasticity corrected version. We therefore select the productivity differential as our transition variable. Next we need to choose the lag length for this variable. Using the same procedure as before we select \( \Delta \text{prod}_{t-3} \) as the transition variable for the model.

Table 3 presents the estimated ESTAR model as well as a series of misspecification tests suggested by Eitrheim and Teräsvirta (1996). The results show the existence of two real exchange rate regimes, one of which is stationary, with the real exchange rate error correcting, whereas in the other one the real exchange presents a near unit root behaviour. Both regimes are conditioned to different values of the transition function, where the stationary one comprises the values \( 0 \leq G(\Delta x_{t-3}; \hat{\gamma}, \hat{c}) \leq 0.7 \), whereas the near unit root regime is defined for the values \( 0.7 \leq G(\Delta x_{t-3}; \hat{\gamma}, \hat{c}) \leq 1 \). Figure 2 shows both the logarithm of the real dollar-euro exchange rate and the estimated transition function \(^6\). Here, one can observe that the transition function stays most of the time in the near unit root regime, so that the real exchange rate shows a high degree of persistence. However, large real exchange rate deviations from its fundamental, in either direction, activate the transition mechanism, which impinges the real exchange rate towards its long run equilibrium level.\(^7\) The high value of the estimated transition parameter \( \hat{\gamma} \) (20.132) indicates that this adjustment occurs rapidly. It also becomes stronger as the deviation from equilibrium increases, and palliates once the series approaches its long run equilibrium level, that is, when it enters the near unit root regime.

The results of the misspecification tests in Table 3 suggest that the model is well specified since there is no evidence of either autocorrelation or heteroskedasticity. The adequacy of the model is also proved through the evaluation tests proposed by Eithreim and Teräsvirta (1996). The statistics \( \text{LM}_{C1}, \text{LM}_{C2} \) and \( \text{LM}_{C3} \) test for parameter constancy under a parametric alternative, which explicitly allows the parameters to change smoothly over time. The \( \text{LM}_{C1} \) and \( \text{LM}_{C2} \) statistics allow for a smooth monotonic and a non-

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\(^6\)The horizontal line in Figure 3 represents the value of 0.7 for the transition function.

\(^7\)The long run equilibrium level coincides with the estimated equilibrium level \( \hat{c} = 0.045 \), in the neighborhood of which the real exchange rate dynamics are close to a unit root process.
monotonic change respectively, whereas $LM_{C3}$ considers both possibilities. According to the results, all possible nonconstancies in the parameters have been properly captured by our model. In addition, the tests for non remaining nonlinearity indicate that there are no nonlinearities of STAR type in the data which have not been captured by the model. Finally, similar tests have also been applied to the variance, showing satisfactory results for the two types of misspecification.

To conclude with the results of the estimation procedure, we test whether the estimated ESTAR model can beat the ability of the estimated linear model in terms of out-of-sample forecasting. The relative forecast performance can also be used as a model selection criterion and thus, as a way to evaluate the estimated models.

We use the data from 2004:Q2 up to 2006:Q4 to evaluate the forecasting performance of the estimated AR and ESTAR models. The results of the forecasting procedure are shown in Table 4. We compute 1 to 4 steps ahead forecasts from the estimated linear AR model and the ESTAR model. The forecast evaluation criteria shown in Table 4 are based upon the entire forecast period, that is, they are not conditioned to the value of the transition function $G(x_{t-3}, \hat{\gamma}, \hat{c})$. The forecasting accuracy is evaluated using the mean prediction error (MPE) and the mean squared prediction error (MSPE). The results of both criteria show that the ESTAR model offers better forecast performance than the AR model and thus, our estimated ESTAR model is preferred to the linear one for the estimation of the real dollar-euro exchange rate.

Finally, with the estimated transition function it is possible to obtain the degree of over- or undervaluation of the dollar relative to the euro according to the productivity fundamental. Taylor and Peel (2000) propose a series of transformations to the transition function, which allow to assess whether the dollar is overvalued or undervalued.\footnote{They argue that the transition function itself cannot be used as an indicator of either overvaluation or undervaluation, as it is only a measure of the importance of the deviation from equilibrium regardless of the sign.} Figure 3 displays the time series plot of the transformed transition function, as well as the logarithm of the real dollar-euro exchange rate. Values above the horizontal axis indicate dollar undervaluation and those below it show dollar overvaluation. From our results, it appears that the dollar has been most of the time undervalued against the (synthetic) euro. However, following the path described by the logarithm of the real dollar-euro exchange rate there are two cases of strong dollar appreciation, where our estimated model also indicates dollar overvaluation. The first one starts in 1981 and reverts in 1986 with the Plaza
Agreement (1985) and the Louvre Accord (1987). The other one runs from 1999 to 2002, coinciding with the first three years after the introduction of the euro and giving rise to an interesting debate in the empirical literature of exchange rates (De Grauwe (2000), Meredith (2001), Alquist and Chinn (2002), Schnatz et al. (2004)). The fact that our model captures well these two important episodes, which characterized the real dollar-euro exchange rate during our sample period, highlights the importance of the nonlinear models against the linear ones, as well as the robustness of our estimated model.

5 Conclusion

In this paper we estimate a nonlinear model for the real dollar-euro exchange rate determination based on the productivity differential. As demonstrated in the empirical literature on exchange rates, nonlinear models offer satisfactory results in dealing with some of the real exchange rate puzzles, so that they are able to explain the persistent behavior of the real dollar-euro exchange rate.

Using quarterly data on the dollar-euro exchange rate and the associated productivity differential for the period 1970:Q1 to 2006:Q4, we find evidence of nonlinearities in the dynamics of the exchange rate. These nonlinearities, which are of the form of an exponential smooth transition model, allow real dollar-euro exchange rate deviations from equilibrium to be consistent with the productivity fundamental, despite the apparent persistent behavior of the series. We have also considered other economic fundamentals proposed in the real exchange rate literature. However, the productivity differential turned out to be the adequate transition variable. Our results also indicate that the nonlinear model offers better forecasting performance than the linear one, and thus, it must be preferred to the linear model in order to estimate the real exchange rate.

In addition, the transformed transition function is able to capture the well-established dollar overvaluation in the mid 1980s and the weakness of the euro after its introduction in 1999. This fact reinforces the idea that the productivity differential is an adequate explanation of the behavior of the real dollar-euro exchange rate and that volatility in exchange rates should not be directly associated with disconnection from fundamentals.
References


Meredith, G. (2001): “Why has the euro been so weak?”, Working Paper 01/155, IMF.


Figure 1: Real dollar-euro exchange rate and productivity differential
Figure 2: Estimated transition function and real dollar-euro exchange rate
Figure 3: Estimated dollar overvaluation relative to the productivity fundamental
Table 1: A linear model for the real dollar-euro exchange rate

Estimated model:

$$\Delta q_t = 0.007 + 0.239 \Delta q_{t-1} + \text{Seasonal Dummies} + \varepsilon_t$$

Diagnostic tests:

- Autocorrelation 1-5: $F(5, 136) = 0.27842$ [0.9243]
- ARCH 1-4: $F(4, 133) = 1.2171$ [0.3066]
- Normality: $\chi^2(2) = 3.5046$ [0.1734]
- Heteroskedasticity, $F_{X_iX_j}$: $F(8, 132) = 0.69976$ [0.6913]
- Heteroskedasticity, $F_{X_i^2}$: $F(5, 135) = 0.52353$ [0.7581]
- Modelo specification, RESET test: $F(1, 140) = 2.1281$ [0.1469]

Note: The OLS standard errors are in parentheses below the estimates and p-values are in square brackets. Autocorrelation 1-5 stands for the autocorrelation tests for the residuals up to 5 lags; ARCH 1-4 stand for autoregressive conditional heteroskedasticity tests (ARCH) up to order 4. The normality test is the Jarque-Bera. Heteroskedasticity are tests for residual heteroskedasticity due to omission of cross products of regressors and/or squares regressors, $F_{X_iX_j}$ and $F_{X_i^2}$ respectively. RESET is the standard regression specification test by Ramsey.
## Table 2: Linearity vs. STAR nonlinearity

<table>
<thead>
<tr>
<th>Transition variable</th>
<th>Standard Tests</th>
<th>Heterosked. Robust Tests</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$LM_1$</td>
<td>$LM_3$</td>
</tr>
<tr>
<td>Trend</td>
<td>0.666</td>
<td>0.872</td>
</tr>
<tr>
<td>$\triangle prod_{t-1}$</td>
<td>0.022</td>
<td>0.017</td>
</tr>
<tr>
<td>$\triangle prod_{t-2}$</td>
<td>0.027</td>
<td>0.027</td>
</tr>
<tr>
<td>$\triangle prod_{t-3}$</td>
<td>0.012</td>
<td>0.064</td>
</tr>
<tr>
<td>$\triangle prod_{t-4}$</td>
<td>0.0151</td>
<td>0.058</td>
</tr>
<tr>
<td>$\triangle prod_{t-5}$</td>
<td>0.086</td>
<td>0.063</td>
</tr>
<tr>
<td>$\triangle exp_{t-1}$</td>
<td>0.255</td>
<td>0.133</td>
</tr>
<tr>
<td>$\triangle exp_{t-2}$</td>
<td>0.236</td>
<td>0.113</td>
</tr>
<tr>
<td>$\triangle exp_{t-3}$</td>
<td>0.237</td>
<td>0.281</td>
</tr>
<tr>
<td>$\triangle exp_{t-4}$</td>
<td>0.194</td>
<td>0.169</td>
</tr>
<tr>
<td>$\triangle exp_{t-5}$</td>
<td>0.039</td>
<td>0.034</td>
</tr>
<tr>
<td>$\triangle nfa_{t-1}$</td>
<td>0.120</td>
<td>0.184</td>
</tr>
<tr>
<td>$\triangle nfa_{t-2}$</td>
<td>0.143</td>
<td>0.189</td>
</tr>
<tr>
<td>$\triangle nfa_{t-3}$</td>
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<tr>
<td>$\triangle nfa_{t-4}$</td>
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<tr>
<td>$\triangle nfa_{t-5}$</td>
<td>0.015</td>
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<tr>
<td>$\triangle oildep_{t-1}$</td>
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<td>0.128</td>
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<tr>
<td>$\triangle oildep_{t-2}$</td>
<td>0.035</td>
<td>0.063</td>
</tr>
<tr>
<td>$\triangle oildep_{t-3}$</td>
<td>0.076</td>
<td>0.232</td>
</tr>
<tr>
<td>$\triangle oildep_{t-4}$</td>
<td>0.138</td>
<td>0.380</td>
</tr>
<tr>
<td>$\triangle oildep_{t-5}$</td>
<td>0.025</td>
<td>0.047</td>
</tr>
<tr>
<td>$\triangle int_{t-1}$</td>
<td>0.033</td>
<td>0.007</td>
</tr>
<tr>
<td>$\triangle int_{t-2}$</td>
<td>0.041</td>
<td>0.008</td>
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<tr>
<td>$\triangle int_{t-3}$</td>
<td>0.127</td>
<td>0.136</td>
</tr>
<tr>
<td>$\triangle int_{t-4}$</td>
<td>0.097</td>
<td>0.124</td>
</tr>
<tr>
<td>$\triangle int_{t-5}$</td>
<td>0.134</td>
<td>0.187</td>
</tr>
</tbody>
</table>

Note: p-values of F variants of the LM-type tests for STAR nonlinearity for labor productivity differential between the US and the euro-zone ($prod$), the public expenditure differential ($exp$), the foreign assets differential ($nfa$), the relative oil dependence ($oil$) and the real interest rate differential ($int$). For a brief description of the test statistics see Section 2.
Table 3: Estimated ESTAR model for the real dollar-euro exchange rate

Estimated model:

\[ \Delta q_t = 0.010 + 0.032 D + (-0.159 q_{t-1} + 0.289 \Delta q_{t-2}) \times [1 - G(\Delta x_{t-3}; -20.132, 0.045)] + \\
(0.393 \Delta q_{t-1} + 0.227 \Delta q_{t-4} - 0.146 \Delta q_{t-5}) \times G(\Delta x_{t-3}; -20.132, 0.045) + SD + \varepsilon_t \]

where \( G(\Delta x_{t-3}; -20.132, 0.045) = \left[1 - \exp\{-20.132(\Delta x_{t-3} - 0.045)^2\}\right] \)


Diagnostic tests:

<table>
<thead>
<tr>
<th>Test</th>
<th>Statistic</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Autocorrelation 1-5</td>
<td>0.729</td>
<td>[0.602]</td>
</tr>
<tr>
<td>ARCH 1-4</td>
<td>6.726</td>
<td>[0.151]</td>
</tr>
</tbody>
</table>

Test for constancy of parameters:

- \(LM_{C1} = 0.233 [0.989]\)
- \(LM_{C2} = 0.685 [0.819]\)
- \(LM_{C3} = 0.826 [0.707]\)

Test for non remaining nonlinearity:

- Quadratic terms: 1.282 [0.259]
- Cubic terms: 1.422 [0.145]
- Fourth powers: 1.036 [0.431]

Test for constancy of variance:

- \(LM_{C1} = 0.028 [0.867]\)
- \(LM_{C2} = 0.053 [0.948]\)
- \(LM_{C3} = 0.147 [0.931]\)

Test for non remaining nonlinearity in variance:

- Quadratic terms: 0.008 [0.929]
- Cubic terms: 0.039 [0.961]
- Fourth powers: 0.333 [0.801]

Note: The standard errors are in parentheses below the estimates whereas the \(p\)-values are in square brackets. \(D\) is a Dummy variable set in 1990:Q1 in order to capture the reunification in Germany. \(SD\) are seasonal dummies. Autocorrelation 1-5 stands for the autocorrelation tests for the residuals up to 5 lags; ARCH 1-4 stand for autoregressive conditional heteroskedasticity tests(ARCH) up to order 4. The normality test is the Jarque-Bera. \(LM_{C1}, LM_{C2}\) and \(LM_{C3}\) test for parameter constancy in ESTAR models under a parametric alternative which explicitly allows the parameters to change smoothly. \(LM_{C1}\) allows a monotonic parameter change, \(LM_{C2}\) considers a nonmonotonic change and \(LM_{C3}\) allows monotonically as well as nonmonotonically changing parameters (see Eitrheim and Teräsvirta (1996) for details).
Table 4: Forecast evaluation of the estimated AR and ESTAR models

<table>
<thead>
<tr>
<th>h</th>
<th>AR</th>
<th>ESTAR</th>
<th>Better</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.00069</td>
<td>0.00038</td>
<td>ESTAR</td>
</tr>
<tr>
<td>2</td>
<td>-0.00432</td>
<td>-0.00387</td>
<td>ESTAR</td>
</tr>
<tr>
<td>3</td>
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<td>-0.00617</td>
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<tr>
<td>4</td>
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<table>
<thead>
<tr>
<th>h</th>
<th>AR</th>
<th>ESTAR</th>
<th>Better</th>
</tr>
</thead>
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<td>ESTAR</td>
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<td>4</td>
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<td>0.00139</td>
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</tbody>
</table>

Note: The forecast period runs from 2004:Q2 to 2006:Q4. The evaluation criteria are unconditional upon any values of the transition function $G(x_{t-3}; \hat{\gamma}, \hat{c})$. MPE is the mean prediction error and MSPE is the mean squared prediction error. $h$ is for the steps ahead for which the forecasts are computed. The column Better indicates which model offers a best prediction.