Employment protection and the non-linear relationship between the wage-productivity gap and unemployment

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Abstract

This paper studies the relationship between the wage-productivity gap and unemployment in the OECD countries during the last 25 years. In particular, we investigate whether the differences across countries in the Employment Protection Legislation (EPL) can affect the link between these two variables. We find that the elasticity of unemployment with respect to the wage-productivity gap switches from a positive value to a negative one when the EPL is above a certain threshold. From a theoretical point of view, we argue that this result is related to the set of labor market reforms introduced in many OECD countries since the middle of the 1980’s, which have affected the relative strictness of the EPL on fixed-term and permanent contracts.

Keywords: Employment protection, wage-productivity gap, unemployment, matching model, nonlinearities, smooth transition regression model.

JEL Classifications: E24, J31, J41, C23.

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1 Introduction

For many years, one of the major issues in macroeconomics has been the link between unemployment, wages and productivity. Indeed, in their influential work, Bruno and Sachs (1985) suggested that the rise in the real wage gap, defined as the proportion of real wages in excess of the full employment marginal productivity of labor, was one of the main contributors to the rise in the OECD unemployment rate during the 1970's. However, ten years after their finding, Gordon (1995) pointed out that an examination of the wage gap data for Europe and the US became obsolete during the 1980's. He showed that there was no cross-country correlation between the increase in unemployment and the increase in the manufacturing wage gap during this period. Together with these two studies, there was an abundant literature dealing with the link between the wage-productivity gap and unemployment, showing a small positive effect or even the lack of support for the wage gap as a determinant of unemployment\(^1\).

Yet, it is still a question of controversy whether unemployment is the result of real wages which are “too” high. For example, using various panel data techniques on 22 OECD countries over the period 1960 to 1993, Junankar and Madsen (2004) find a statistically significant, but economically insignificant, positive relationship between unemployment and the wage gap. In turn, applying a set of unit root and cointegration tests with non-linear error correction mechanism, Pascalau (2007) finds mixed evidence on the response of unemployment to increments on the wage productivity gap. Moreover, these studies also reflect heterogeneous results across countries.

In addition, there is still an important question that remains unanswered: What can explain the differences observed in the relationship between the wage gap and unemployment across countries? In this paper we propose that the influence of institutions on the performance of the labor market can help us to answer this question\(^2\). In particular, the employment protection legislation can play a central role behind the observed behavior between the wage gap and unemployment.

\(^1\)See for example, Madsen (1994), McCallum (1986), Myatt and Septhon (1990).
\(^2\)See Layard et al. (1991)
Indeed, since the middle of the 1980s, a set of labor market reforms introduced in many OECD countries have affected the relative strictness of the Employment Protection Legislation (EPL) on fixed-term and permanent contracts. According to the OECD (2004), most of these reforms have alleviated the relative strictness of EPL for fixed-term contracts relative to the one on permanent contracts. As a result, two types of labor market characteristics can be found in many OECD economies. The first type shows a small degree of EPL in regular contracts, and no limitations on the renewal and duration of temporary contracts\(^3\). The second type of countries combines a high degree of employment protection in the regular segment with a limited flexibility in the use of temporary contracts\(^4\).

Therefore, this paper studies the relationship between the wage-productivity gap and unemployment in the OECD countries during the last 25 years. In particular we investigate whether the differences across countries in the EPL can affect the link between unemployment and the wage gap. First, using a matching model with two type of jobs we explore how employment protection in regular jobs and limited flexibility in the use of temporary contracts affects the comovement between these two variables. We find that under this scenario, the wage of a permanent worker increases by a higher magnitude than average labour productivity while the wage of a temporary one increases in a much lower magnitude. This result takes place because with a relatively higher implicit bargaining power, permanent workers can negotiate extra wage adjustments to the productivity shock in good times. In contrast, temporary workers have lower bargaining power because firing costs are not operational at this level. Thus, firms can discount to these workers the extra wage adjustments in the side of regular contracts. According to this result, the relationship between unemployment and the wage gap in countries characterized by different degrees of EPL can be positive or negative depending on how temporary and permanent workers interact in each particular labour market.

\(^3\)Some of them are the well-known Anglo-Saxon economies; Australia, Canada, Ireland and US.

\(^4\)For example, in Spain fixed term contracts are restricted to 3 the maximum number of successive contracts with a top accumulated duration of 2 years (OECD, 2004). Other countries with limited duration and renewal process of temporary contracts are Belgium, Denmark, France Germany, Italy, Korea, Netherland, Norway and Sweden.
Along this line, we also show empirical evidence that in many OECD countries the unemployment rate does not react linearly to alterations between real wages and productivity gains. On the one hand, in countries where the labour market is highly protected, unemployment and the wage gap exhibit a negative relationship. On the other hand, we observe a positive relationship in those economies with low employment protection in permanent contracts and with no restriction in the use of temporary contracts. We get these results using different techniques of estimation. First, we explore the conditional comovement between the wage-productivity gap and the unemployment rate using the correlation coefficients of a vector autoregressive (VAR) forecast errors as proposed in den Haan (2000). Second, we rely in panel data techniques and then use nonlinear smooth transition (PSTR) models to explain such relationships. The PSTR models show that the elasticity of unemployment with respect to the wage-productivity gap switches from a positive to a negative value when EPL increases above certain threshold, which is around 2.2 depending on the estimated model\textsuperscript{5}. 

The rest of the paper is organized as follows. In the next section we present the matching model with temporary and permanent jobs. Section 2 describes the data and deals with the methodological aspects. Section 3 presents the results. Finally, section 4 gives the main conclusions.

2 The model

Given that our model is basically the same as the one used in Silva and Toledo (2008), its presentation is reduced to the minimum. The economy is integrated by a continuum of risk-neutral, infinitely-lived workers and firms. Workers have linear utility over consumption of a homogeneous good. Workers and firms discount future payoffs at a common and constant rate $\delta$, and capital markets are perfect. In addition, time is discrete.

\textsuperscript{5}The EPL index scores from 0 to 6 with higher values representing stricter regulation. However, the highest level reached by our group of countries was 3.82 in Spain between 1985 and 1993. On the contrary, the USA had 0.21, the lowest value, during the whole period.
There is a time-consuming and costly process of matching workers and job vacancies, captured by a constant-return-to-scale matching function

\[ m(u_t, v_t) = \frac{u_t v_t}{(u_t^\varphi + v_t^\varphi)^{1/\varphi}}, \quad \varphi > 0, \]  

where \( u_t \) denotes the unemployment rate and \( v_t \) are vacancies. From the properties of the matching function, unemployed workers and vacancies meet to each other at the rates \( \frac{m(u_t, v_t)}{u_t} = f(\theta_t) \) and \( \frac{m(u_t, v_t)}{v_t} = q(\theta_t) \), respectively. Due to the CRS assumption, these rates only depend on the vacancy-unemployment ratio \( \theta_t \). The higher the number of vacancies with respect to the number of unemployed workers, the easier is for each of these workers to find a job \( f'(\theta_t) > 0 \), and the more difficult is for a firm to fill its vacancy \( q'(\theta_t) < 0 \).

Workers can be either unemployed or employed. Unemployed workers get \( b \) units of the consumption good each period, which could be understood as the value of leisure, home production, or unemployment benefits. Those who are employed can be either temporary (\( T \)) or permanent employees (\( P \)). The productivity of the match is a function of aggregate productivity \( A_t \), and a term \( z_t \) idiosyncratic to the match. There is a firm-specific productivity term independent and identically distributed across firms and time, with a cumulative distribution function \( G(z) \) and support \([0, \bar{z}]\).

We also assume that \( \log A_t \) follows a Markovian stochastic process.

In turn, firms have a constant-return-to-scale production technology that uses only labor. A job can be either filled or vacant. Job creation takes place when a firm and a worker meet and agree on a temporary contract. However, before a position is filled, the firm has to open a job vacancy with flow cost \( c \). Each filled job yields instantaneous profit equal to the difference between labor productivity and the wage, which is either \( A_t z_t - w^T_t \) for a temporary position or \( A_t z_t - w^P_t \) for a job filled with a permanent employee.

Firms face firing costs when a permanent match is endogenous destroyed. In particular, firms lose \( \gamma \) when a match with an permanent worker is terminated by the firm. This cost is assumed to be fully wasted and not a transfer, reflecting dismissal protections imposed by the government. Due to legal restrictions temporary positions are bounded to convert it into permanent ones with
different values are given by the following Bellman equations: 

\[ U_t = b + \delta E_t \left[ f(\theta_t) \int_{z_{t+1}^T}^{\bar{z}_t} W_{t+1}^T(z) dG(z) + [1 - f(\theta_t)(1 - G(\bar{z}_{t+1}^T))]U_{t+1} \right], \]  

(2) 

\[ W_t^T(z_t) = w_t^T(z_t) + \delta (1 - \phi) E_t \left[ \left( \int_{z_{t+1}^T}^{\bar{z}_t} W_{t+1}^P(z) dG(z) + G(\bar{z}_{t+1}^T)U_{t+1} \right) \right] + \delta \phi E_t U_{t+1}, \]  

(3) 

\[ W_t^P(z_t) = w_t^P(z_t) + \delta E_t \left[ (1 - \phi) \left( \int_{z_{t+1}^P}^{\bar{z}_t} W_{t+1}^P(z) dG(z) + G(\bar{z}_{t+1}^P)U_{t+1} \right) + \phi U_{t+1} \right]. \]  

(4) 

\[ V_t = -c + \delta E_t \left[ q(\theta_t) \int_{\bar{z}_{t+1}^T}^{\bar{z}_t} J_{t+1}^T(z) dG(z) + [1 - q(\theta_t)(1 - G(\bar{z}_{t+1}^T))]V_{t+1} \right], \]  

(5) 

\[ J_t^T(z_t) = A_t z_t - w_t^T(z_t) + \delta (1 - \phi) E_t \left[ \left( \int_{z_{t+1}^C}^{\bar{z}_t} J_{t+1}^P(z) dG(z) + G(\bar{z}_{t+1}^C)V_{t+1} \right) \right] + \delta \phi E_t V_{t+1}, \]  

(6) 

\[ J_t^P(z_t) = A_t z_t - w_t^P(z_t) + \delta (1 - \phi) E_t \left[ \left( \int_{z_{t+1}^P}^{\bar{z}_t} J_{t+1}^P(z) dG(z) + G(\bar{z}_{t+1}^P)U_{t+1} - \gamma \right) \right] + \delta \phi E_t V_{t+1}, \]  

(7) 

where \( z_{t+1}^j, j = \{T, C, P\} \), are match-specific productivity thresholds defined such that nonprofitable matches (i.e., with negative surplus) are severed. These thresholds must satisfy the following conditions:

\[ J_t^T(z_{t+1}^T) - V_t = 0, \]  

(8) 

\[ J_t^C(z_{t+1}^T) - V_t = 0, \]  

(9) 

\[ J_t^P(z_{t+1}^T) - V_t + \gamma = 0. \]  

(10)
Expressions (8) and (10) define the reservation productivity for temporary and permanent workers, respectively, whereas (9) refers to those temporary workers on the verge of becoming permanent.

It follows that the permanent and temporary workers separate with probabilities

\[
\begin{align*}
s^P_t & = \phi + (1 - \phi)G(\tilde{z}^P_t), \\
s^T_t & = \phi + (1 - \phi)\left[(1 - \iota)G(\tilde{z}^T_t) + \iota G(\tilde{z}^C_t)\right].
\end{align*}
\]

Moreover, job creation takes place with probability \( q(\theta_t)(1 - G(\tilde{z}_{t+1}^T)) \) when a firm and a worker meet and agree on a temporary contract. Similarly, unemployed workers find a job with probability \( f(\theta_t)(1 - G(\tilde{z}_{t+1}^T)) \).

We close the model by introducing two more assumptions. One is the free entry condition for vacancies. Therefore, in equilibrium we must have

\[
V_t = 0.
\]

The other assumption is that wages are set through Nash bargaining. The Nash solution is the wage that maximizes the weighted product of the worker’s and firm’s net return from the job match. The first-order conditions for temporary and permanent employees yield the following two conditions,

\[
\begin{align*}
(1 - \beta)(W_t^T(z_t) - U_t) & = \beta (J_t^T(z_t) - V_t), \\
(1 - \beta)(W_t^P(z_t) - U_t) & = \beta (J_t^P(z_t) - V_t + \gamma),
\end{align*}
\]

where \( \beta \in (0, 1) \) denotes workers bargaining power relative to firms.

Using the equations above, we can now solve for the equilibrium wages as a function of the aggregate state variables \( A_t \) and \( \theta_t \),

\[
\begin{align*}
w_t^T(z_t) & = (1 - \beta)b + \beta \theta_t c + \beta A_t z_t - \delta \beta \iota (1 - \phi)(1 - G(\tilde{z}^C_{t+1}))\gamma, \\
w_t^P(z_t) & = (1 - \beta)b + \beta \theta_t c + \beta A_t z_t + \beta [1 - \delta (1 - \phi)] \gamma.
\end{align*}
\]
Introducing firing costs in permanent jobs decreases the wage of a temporary worker (16) by a fraction of the separation costs. In contrast, the permanent wage (17) is higher because separation costs are now operational, which increase his bargaining power.

To fully characterize the dynamics of the model economy, we need to define the law of motion for the unemployment rate $u_t$, and the mass of temporary and permanent workers, $n_t^T$ and $n_t^P$, respectively. These evolve according to the following difference equations:

$$ u_t = u_{t-1} + s_t^T n_{t-1}^T + s_t^P n_{t-1}^P - f(\theta_{t-1})(1 - G(\bar{z}^T_t))u_{t-1}, \quad (18) $$

$$ n_t^T = n_{t-1}^T + f(\theta_{t-1})(1 - G(\bar{z}^T_t))u_{t-1} - s_t^T n_{t-1}^T - (1 - \phi)(1 - G(\bar{z}^T_t))n_{t-1}^T, \quad (19) $$

$$ n_t^P = n_{t-1}^P + (1 - \phi)(1 - G(\bar{z}^C_t))n_{t-1}^T - s_t^P n_{t-1}^P, \quad (20) $$

$$ 1 = u_t + n_t^T + n_t^P. \quad (21) $$

The average separation probability is equal to

$$ s_t = \frac{s_t^T n_{t-1}^T + s_t^P n_{t-1}^P}{(1 - u_{t-1})}. \quad (22) $$

Finally, total output $y_t$ is equal to

$$ y_t = A_t \bar{z}^P n_t^P + A_t \bar{z}^T n_t^T - cv_t, \quad (23) $$

where $\bar{z}^j = E[z | z \geq \bar{z}^j]$.

### 2.1 Calibrated parameters

Our benchmark model is the one-type of job model without employment protection, such as the US economy. Thus we calibrate the model at quarterly frequency in order to match four targets for this economy between 1985 and 2007. Following Blanchard and Diamond (1990) we set an average unemployment rate at $u = 11\%$ (target 1). Following Shimer (2005) we target a steady-state job separation probability $s$ equal to 0.10 per quarter (target 2), and an elasticity of the matching function with respect to unemployment of $\varepsilon_{m,u} = 0.72$ (target 3). As mentioned in Silva and Toledo (2008), the 1982 Employment Opportunity Pilot Project and the 1992 Small
Business Administration Surveys estimate total hiring costs to be about 4.3 percent of the quarterly compensation of a new hired worker. Therefore, we set \( c \) such that in the steady state it is equal to \( 0.043w^T \) (target 4).

We set the discount factor \( \delta = 0.99 \), which implies a reasonable quarterly interest rate of nearly 1 percent. We normalize the aggregate labor productivity \( A \) to 1. The logarithm of this variable follows an AR(1) process such that \( \log A_t = \rho \log A_{t-1} + \varepsilon_t \). The values of the autoregressive parameter and the standard deviation of the white noise process, \( \rho = 0.96 \) and \( \sigma_A = 0.01 \), have been calibrated to match the cyclical volatility (2.0 percent) and persistence (0.88) of the average US labor productivity \( y_t/(1-u_t) \) between 1985 and 2007.

Regarding the exogenous separation probability \( \phi \), we follow den Haan et al. (2000) by interpreting exogenous separations as worker-initiated separations. Hence, since only endogenous separations are associated with the layoff rate, firms do not incur in firing costs when separations are exogenous. According to the evidence from the Job Opening Labor Turnover Survey (JOLTS) layoffs represent on average about 35% of total separations. Thus, the value for \( \phi \) is 0.065.

Since our baseline parametrization describes the US labor market, we assume there are no firing costs restrictions in this economy. Thus, we set \( \gamma = 0 \). This implies the existence of just one type of job since temporary and permanent become perfect substitutes. Therefore, the job conversion probability \( \iota \) becomes irrelevant. However, since we also consider the temporary-permanent scenario where a number temporary positions are bounded to convert into permanent ones, we fix \( \iota = 0.25 \).

The idiosyncratic productivity \( z_t \) is assumed to be log-normally distributed with parameters \((\mu;\sigma_z)\). As in in Walsh (2005), we choose the mean and the standard deviation of log \( z_t \) to be \( \mu = 0 \) and \( \sigma_z = 0.13 \), respectively. Finally, the hiring cost parameter \( c \), is calibrated together with the matching technology parameter \( \varphi \), the workers’ bargaining power \( \beta \), and the employment opportunity cost \( b \). We select these parameters such that the steady-state equilibrium satisfies our four calibration targets. This yields \( c = 0.071, \varphi = 1.891, \beta = 0.348, \) and \( b = 0.931 \).
2.2 Simulated results

We simulate the model presented above 10,000 times. Each time we simulate the economy for 1,072 quarters and throw away the first 1,000 of them in order to obtain the U.S. period between 1985-2003. We calculate the correlation matrix of the temporary and permanent wage gaps with respect to unemployment (express both in logs) for different levels of firing costs $\gamma$. When modifying this parameter, we hold all the other ones constant and compute the new equilibrium values of the endogenous variables in the steady state. Table 1 shows the simulated correlations.

<table>
<thead>
<tr>
<th>$\gamma$</th>
<th>corr($\log(w^T_t) - \log(y^T_t/n_t), \log(u_t)$)</th>
<th>corr($\log(w^P_t) - \log(y^P_t/n_t), \log(u_t)$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.973</td>
<td>0.973</td>
</tr>
<tr>
<td>0.05</td>
<td>0.998</td>
<td>-0.454</td>
</tr>
<tr>
<td>0.10</td>
<td>0.995</td>
<td>-0.815</td>
</tr>
<tr>
<td>0.25</td>
<td>0.992</td>
<td>-0.902</td>
</tr>
<tr>
<td>0.50</td>
<td>0.993</td>
<td>-0.903</td>
</tr>
</tbody>
</table>

Notes: Average labor productivity is equal to $\frac{w}{n_t} = \frac{A^Tz^T_t n^T_t + A^Pz^P_t n^P_t - cv_t}{(1-u_t)}$ where $z^T_t$ and $z^P_t$ are, respectively, the average idiosyncratic productivity shocks across entrants and insider jobs.

It is interesting to observe that in the benchmark case, there is an almost perfect positive correlation (0.985) between the log of wage gap and unemployment. The value added of this exercise, however, lies in the second case, with the presence of firing costs. As shown in Table 1, the correlation coefficient between the log of the permanent wage gap, $\log(w^P_t) - \log(y^P_t/n_t)$, and the log unemployment, $\log(u)$, becomes negative (-0.454) when firing costs and limited restriction in the use of temporary contracts (captures through $\gamma$ and $\iota$, respectively) are introduced in the model.
In contrast, the correlation between the log of the temporary wage gap, $\log(w^T_t) - \log(y^T_t)$, and the log unemployment (0.998) resembles very much the one of the pure deregulated labor market with no firing costs (0.975).

To understand this result we can look to the wage response to a productivity shock with $\gamma = 0.5$. Figure 1 shows the response of the conditional mean of the temporary and permanent wages, $E[w^T_t(z)|z \geq \tilde{z}^T_t]$ and $E[w^P_t(P)|z \geq \tilde{z}^P_t]$, to a 1 percent increase in aggregate labor productivity ($A_t$).

One quarter after the impact, the average labor productivity increases in 1.3%, the permanent wage increases by 1.5% while the temporary wage increases in a lower magnitude (1.0%). Intuitively, with a relatively higher implicit bargaining power, permanent workers can negotiate an additional wage increase to a favorable productivity shock. In contrast, temporary employees have lower bargaining power because firing costs are not operational at this type of job. Thus, firms can discount to these workers the extra wage adjustments in the permanent job side. This is the reason why we observe a negative correlation between the permanent wage and unemployment. The relationship between the temporary wage gap and unemployment remains positive because these workers are the relevant ones to the marginal employment decision. It is not profitable to keep the level of hired workers when the temporary wage is increasing more than the average labor productivity.

3 The wage-productivity gap and unemployment in OECD countries. An empirical analysis

Our previous model states that in economies with high level of employment protection in permanent jobs (captures through the firing costs parameter $\gamma$) and limited restriction in the use of temporary positions (capture trough a positive job conversion probability $\iota > 0$), the relationship between the wage-productivity gap and unemployment may be negative due to the presence of permanent workers who have the power to negotiate wage adjustments higher than the increment on labor productivity.
The purpose of this section is to conduct a detailed empirical analysis for the link between the unemployment rate and the wage gap in 18 OECD countries. In order to do so, we have adopted two complementary approaches. As a first approximation, we explore the conditional comovement between the wage-productivity gap and the unemployment rate using the correlation coefficients of a vector autoregressive (VAR) forecast errors as proposed in den Haan (2000).

Second, according to our previous model, the relationship between the wage gap and unemployment depends on different states of the world or regimes that prevail at any point in time. It is therefore a nonlinear relation. That is, unemployment’s reaction to real wages being too high (or too low) with respect to the productivity is likely to depend on institutional characteristics of the labour market. Thus, there is no reason to expect the same coefficient in countries characterized by a low labour regulation (as the United States) than in countries with stricter regulations (as Spain). We capture this nonlinearity by means of a panel smooth transition model (PSTR) using a measure of employment regulation as a threshold variable.

3.1 Data sources and definition of the variables

The data sample consists of annual observations for the period 1985-2003, collected from different sources. Data for the unemployment rate has been taken from the OECD’s Main Economic Indicators Database. Productivity has been defined as the volume GDP to the employment index (both 2000=100) from the IMF’s Financial Statistics and the OECD’s Main Economic Indicators Database respectively. Real wages correspond to the IMF’s data on hourly or monthly wages deflated either by the consumer price index (CPI) or by the GDP deflator (DEF).

Based on the previous information, we consider two different measures of the wage-productivity

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6Notice that we are not trying to measure the direct effects of the labour protection on the unemployment rate. Rather, we want to consider it indirectly when estimating the relationship between the wage gap and the unemployment.

7Except Germany, in which case data starts in 1991.

8For some countries where constant GDP is not available, the industrial production index has been used instead (Belgium, Finland, Ireland, Korea, Norway and Sweden).
The consumption wage-productivity gap is the difference between the wage rate measured in terms of consumption goods (the nominal wage divided by the price of consumption goods), and the real GDP per worker.

The product wage-productivity gap is the wage rate in terms of output (the nominal wage divided by the price of output), and the real GDP per worker.

Now, our purpose is not only to analyze the relationship between the unemployment and the wage productivity gap but also to see how this relationship changes according to two institutional characteristics of the labour market: a) the firing costs and, 2) the relative weight of permanent and temporary workers.

However, it is not straightforward to compile a reliable measure for the previous “institutional” variables for an extended period of time. Perhaps the best indicator that we can count with is the Employment Protection Legislation Index (EPL from now on) by the OCDE\textsuperscript{9}. This index measures regulations concerning hiring and firing, even if they are not grounded primarily in the law, but originate from the collective bargaining of the social partners.

This index is computed along 18 basic items, which can be classified in three main areas: a) Employment protection of regular workers against individual dismissal; b) Specific requirements for collective dismissals; and c) Regulation of temporary forms of employment. The EPL also considers strictness of regulation for regular contracts, temporary contracts, and collective dismissals. Therefore, it combines the two aspects that we are interested in and provides a useful framework for analyzing the relationship between unemployment and the wage gap in the presence of different institutional variables that vary both across countries and over time.

Finally, included countries are Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Italy, Ireland, Japan, Korea, Netherlands, Norway, Spain, Sweden, the UK and the USA.\textsuperscript{9}Unfortunately, the EPL series covers only from 1985 to 2003 which is why our models are estimated just for this period.
3.2 Conditional co-movements

The co-movement with VAR forecast errors can be briefly described in the following terms.

\[ X_t = A_0 + \sum_{i=1}^{m} A_i X_{t-1} + v_t \]

where \( X_t \) is an \( n \)-vector of random variables that may include both stationary and integrated processes; \( A_0 \) is an \( n \)-vector of constant terms or a matrix of deterministic coefficients; \( i \) are \( n \times n \) matrices of coefficients; \( v_t \) is an \( n \)-vector of error terms, and \( m \) is the total number of lags included.

Denote the \( k \) – period ahead forecast of variable \( y \) by \( E_t y_{t+k} \) and its forecast error by \( y_{fe,t+k} \). The same applies to variable \( x \). Denote the covariance between \( x_{fe,t+k} \) and \( y_{fe,t+k} \) by \( COV(k) \) and the correlation coefficient between these two variables by \( CORR(k) \). One way to construct estimates of these covariance and correlation coefficients is to construct time series for the forecast errors using the difference between subsequent realizations and their forecasts. The constructed time series are then used to generate covariance and correlation coefficients. We estimate a set of bivariate VARs between the wage-productivity gap and unemployment rate expressed both in logs.

The VARs were estimated without imposing the unit-root restriction and considering linear and quadratic trend if it is necessary\(^{10}\). The lag length in the VAR, as well as the deterministic components were chosen by the Akaike Information Criterion. Table 2 summarizes the correlation coefficients for a forecast horizon of 5 years for both the product wage gap and the consumption wage gap, respectively.

Clearly, we can identify two types of countries. On the one hand, France, Italy, Germany, Korea, Netherland Norway, Spain and Sweden display a negative correlation coefficient between the wage gap and unemployment. On the other, the rest of the countries show a positive relationship. Thus, when real wage exceed labor productivity unemployment tends to increase in some countries and to decrease in others.

\(^{10}\)The matlab program was writing by Steve Sumner and can be download at Den Haan’s web page http://faculty.london.edu/wdenhaan/
Table 2: Correlation coefficients between the wage-productivity gap and unemployment from 1985 to 2005. k-period ahead forecast error

<table>
<thead>
<tr>
<th>Forecast Horizon</th>
<th>Product wage gap</th>
<th>Consumption wage gap</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Australia</td>
<td>0.394</td>
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<td>Austria</td>
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<td>Germany</td>
<td>-0.176</td>
<td>-0.233</td>
</tr>
<tr>
<td>Ireland</td>
<td>0.484</td>
<td>0.698</td>
</tr>
<tr>
<td>Italy</td>
<td>0.102</td>
<td>0.011</td>
</tr>
<tr>
<td>Japan</td>
<td>0.728</td>
<td>0.793</td>
</tr>
<tr>
<td>Korea</td>
<td>-0.233</td>
<td>-0.436</td>
</tr>
<tr>
<td>Neth.</td>
<td>-0.186</td>
<td>-0.238</td>
</tr>
<tr>
<td>Norway</td>
<td>-0.156</td>
<td>-0.170</td>
</tr>
<tr>
<td>Spain</td>
<td>-0.339</td>
<td>-0.447</td>
</tr>
<tr>
<td>Sweden</td>
<td>-0.212</td>
<td>-0.257</td>
</tr>
<tr>
<td>UK</td>
<td>0.003</td>
<td>0.108</td>
</tr>
<tr>
<td>US</td>
<td>0.320</td>
<td>0.285</td>
</tr>
</tbody>
</table>

The relationship between the average EPL index between 1985 and 2003 and the average estimated correlation coefficients between the wage gap and unemployment is captured in Figure 2. This figure also differentiates between those countries with and without restrictions on the duration or renewal use of temporary contracts (black and white points, respectively). Most of the countries
(except Belgium and Denmark) with negative correlation between the wage-productivity gap and unemployment present high levels of EPL with limited flexibility in the use of temporary contracts\textsuperscript{11}. In contrast, those economies (except the UK) characterized by a small degree of EPL in regular contracts, and no limitations on the renewal and duration of temporary contracts show a positive correlation between the wage-productivity gap and unemployment.\textsuperscript{12}

Finally, notice that, in general, the sign of the correlation coefficients remains invariable for a forecast horizon of up to five years, suggesting a stable relationship between unemployment and the wage gap.

4 The nonlinear relationship between unemployment and the wage-productivity gap

In this section we empirically analyze the relationship between the unemployment and the wage gap in the same group of OECD countries that in the previous part. However, we adopt a different approach. First of all, we rely in panel data techniques. Second, we use nonlinear smooth transition (PSTR) models to explain such a relationship.

We use panel data techniques because we believe that these countries share important similarities usually lost in country-by-country estimations. Yet, we also need a model that allows the regression coefficient (the elasticity) to vary over time and across cross-sectional units. Finally, we are particularly interested in models which coefficients that are functions of other exogenous variables (i.e, the EPL).

To introduce this regime-switching behaviour in the model, we employ the PSTR model developed by Gonzales, et. al (2004) and Fok, et. al. (2005). The PSTR model has several interesting features that make it suitable for our purposes. First, regression coefficients can take on a small number of different values, depending on the value of another observable variable. In other words,
the observations in the panel are divided into a small number of homogenous groups or “regimes”,
with different coefficients in different regimes. Second, regression coefficients are allowed to change
gradually when moving from one group to another. That is, PSTR is a regime-switching model that
allows for a small number of extreme regimes associated with the extreme value of a transition func-
tion and where the transition from one regime to the other is smooth rather than discrete. Finally,
individuals are allow to change between groups according to variations in the so-called “threshold
variable”.

4.1 Methodology

The basis of our empirical approach consists in estimating the parameter of a function which relates
the unemployment rate with the wage gap as the explanatory variable. Let us consider a balanced
panel with \( t \) indicating time and \( i \) the individual. Denoting \( u_{it} \) the dependent variable (the log
unemployment rate), \( \mu_i \) the individual fixed effects, \( w_{git} \) the exogenous variable (the wage gap
expressed in logarithm), the model can be expressed as follows:

\[
\begin{align*}
    u_{it} &= \mu_i + \beta_1 w_{git} + \varepsilon_{it} \\
\end{align*}
\]  

(24)

In equation (24), the parameter \( \beta_1 \) denotes the elasticity of the unemployment rate with respect
to the wage gap. However, if expressed like that, equation (24) is a linear model that does not allow
for parameter heterogeneity. In order to take into account the nonlinear features of the model, we
propose to estimate the relationship by means of a PSTR model as follows:

\[
\begin{align*}
    u_{it} &= \mu_i + \beta_{1,0} w_{git} + \beta_{1,1} w_{git} g \left( q_{it}; \gamma, c \right) + \varepsilon_{it}, \\
\end{align*}
\]  

(25)

where \( g \left( q_{it}; \gamma, c \right) \) is the transition function, normalized and bounded between 0 and 1, \( \gamma \) is the
speed of transition, \( c \) denotes the threshold parameter and \( q_{it} \) is the transition variable which, for our
particular case, is the EPL. Therefore, in the previous model the two extreme values are associated
with regression coefficients \( \beta_{1,0} \) and \( \beta_{1,0} + \beta_{1,1} \) depending on whether the threshold variable is lower
or larger than the threshold \( c \). Yet, it allows for a smooth and gradual transition between the two regimes.

Following Granger and Terasvirta (1993) in the time series context and Gonzales, et. al (2004) in the panel framework, we consider the following function:

\[
g(q_{it}; \gamma, c) = \left(1 + \exp \left(-\gamma \prod_{j=1}^{m} (q_{it} - c_j)\right)\right)^{-1}
\]

with \( \gamma > 0 \) and \( m = 1, 2 \). Therefore, when \( m = 1 \), the function is a logistic one which implies that the two extreme regimes are associated with low and high values of the transition variable with a single monotonic transition of the coefficients from \( \beta_{1,0} \) to \( \beta_{1,0} + \beta_{1,1} \), and the change is centered around \( c_1 \). The other possibility that we consider here is when \( m = 2 \), the quadratic logistic function, in which case the transition function has its minimum at \((c_1 + c_2)/2\) and attains the value 1 both at low and high values of \( q_{it} \).

Based in the methodology used in the time series context, Gonzales, et. al (2004) suggest a three step strategy to apply to PSTR models: (i) specification, (ii) estimation and (iii) evaluation. The aim of the identification step is to test for homogeneity against the PSTR alternative. The estimation step, nonlinear least squares are used to obtain the parameter estimates, once the data have been demeaned\(^1\). Finally, the evaluation step consist of applying misspecification tests in order to check the validity of the estimated PSTR model and determining the number of regimes\(^2\).

### 4.2 Empirical results

We first estimated single country and panel data linear equations for the relationship between unemployment and the wage gap relationship (equation 24). The equation has been estimated for

\(^1\)For more details, the reader is referred to Gonzales, et. al (2004).

\(^2\)It should be noted that demeaning the data is not straightforward in a panel context (see Hansen (1999), and Gonzales (2004))
both the wage gap defined in terms of the CPI and the wage gap in terms of the deflator\textsuperscript{16}. Results from this estimations are presented in Table 3 below.

Our country estimates show that there are two groups of countries depending on the sign of the coefficient (even though they are not significant in some cases). In the first group, characterized by a positive relationship, we find Australia, Austria, Belgium, Canada, Denmark, Ireland and the USA. On the other, Finland, France, Germany, Italy, Japan, Korea, Netherlands, Norway, Spain, Sweden and the UK. In general, this pattern is the similar for both the consumption and the product wage-productivity gap, except for the cases of Denmark and Japan. The panel data results, however, show a positive and significant elasticity of the unemployment rate to the wage gap.

Yet, the drawbacks of the previous linear relationship are at least the following. First, the panel data estimates are assume, probably wrong, to be common to all countries. Second, it does not provide any guidance of the reason for the different elasticities among countries. Finally, equation (24) does not consider the effects that the employment regulation has on the unemployment-wage gap relationship.

As it was advanced before, we believe that it is the dissimilar labour regulation the factor behind this different behaviour, a fact that can be capture by means of a nonlinear model. Then, as a first approximation, we show in Figure 3 the $\beta_1$ coefficient from equation (24) against the average EPL index between 1985-2003. As shown, roughly speaking we observe that countries with a higher employment protection are associated with a negative elasticity of the unemployment rate with respect to the wage gap. On the contrary, countries with low protection usually have a positive

\textsuperscript{16} There is an extensive but not conclusive discussion regarding the order of integration of the unemployment rate. On the one hand, the so-called “natural” rate of unemployment or NAIRU, characterizes unemployment dynamics as a mean reverting process. On the other, the “hysteresis” hypothesis states that cyclical fluctuations have permanent effects on the level of unemployment. In this paper, we abstract from this issue and estimate the equation in levels. However, as noticed by Colletaz and Hurlin (2006), the consequences of the non-stationarity in linear and nonlinear panel are not equivalent to those generally pointed out in a time series context, providing then consistent estimates of some long-run regression coefficients.
Table 3: The relationship between the unemployment rate and the wage gap. Country-by-country and panel linear estimates, 1985-2003

<table>
<thead>
<tr>
<th>Country</th>
<th>Consumption wage gap</th>
<th>Product wage gap</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\beta_1$</td>
<td>$t$-stat</td>
</tr>
<tr>
<td>Australia</td>
<td>2.77</td>
<td>1.67</td>
</tr>
<tr>
<td>Austria</td>
<td>2.74</td>
<td>2.21</td>
</tr>
<tr>
<td>Belgium</td>
<td>0.18</td>
<td>0.16</td>
</tr>
<tr>
<td>Canada</td>
<td>2.20</td>
<td>4.86</td>
</tr>
<tr>
<td>Denmark</td>
<td>3.87</td>
<td>1.68</td>
</tr>
<tr>
<td>Finland</td>
<td>-1.54</td>
<td>-2.48</td>
</tr>
<tr>
<td>France</td>
<td>-1.13</td>
<td>-1.98</td>
</tr>
<tr>
<td>Germany</td>
<td>-3.90</td>
<td>-3.28</td>
</tr>
<tr>
<td>Italy</td>
<td>-0.21</td>
<td>-1.23</td>
</tr>
<tr>
<td>Ireland</td>
<td>1.53</td>
<td>9.54</td>
</tr>
<tr>
<td>Japan</td>
<td>-2.03</td>
<td>-0.82</td>
</tr>
<tr>
<td>Korea</td>
<td>-1.36</td>
<td>-2.40</td>
</tr>
<tr>
<td>Netherlands</td>
<td>-6.35</td>
<td>-5.98</td>
</tr>
<tr>
<td>Norway</td>
<td>-2.08</td>
<td>-3.67</td>
</tr>
<tr>
<td>Spain</td>
<td>-5.68</td>
<td>-3.25</td>
</tr>
<tr>
<td>Sweden</td>
<td>-4.61</td>
<td>-1.71</td>
</tr>
<tr>
<td>UK</td>
<td>-5.53</td>
<td>-2.37</td>
</tr>
<tr>
<td>USA</td>
<td>1.10</td>
<td>3.17</td>
</tr>
</tbody>
</table>

Individual fixed effects: 4.75 4.23 5.44 5.27
Individual random effects: 4.89 4.41 5.43 5.40
Individual and time fixed effects: 3.97 3.48 4.66 4.53

Notes: (1): Due to data availability, the panel excludes Germany.

elasticity.
Before turning to the estimation of the PSTR model, equation (24) has been tested for linearity\textsuperscript{17}. The linearity test results presented in Table 4 clearly lead to the rejection of the null hypothesis of linearity in the relationship between the unemployment rate and the wage gap. Thus, we proceeded to the estimation of the panel nonlinear models for both the consumption and the product wage gap. Also, we estimated the models with both the logistic and the quadratic logistic functions as transition functions.

Table 4: Lagrange multiplier tests for linearity in equation (24), 1985-2003

<table>
<thead>
<tr>
<th>Consumption wage gap</th>
<th>Product wage gap</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>F</strong></td>
<td><strong>p-value</strong></td>
</tr>
<tr>
<td>Wald test</td>
<td>64.733 0.000</td>
</tr>
<tr>
<td>Fisher test</td>
<td>76.446 0.000</td>
</tr>
</tbody>
</table>

Notes: (1): In both tests, \( H_0 \): Linear Model; \( H_1 \): PSTR model

Table 5 provides the estimation results of equation (25) for the panel of countries. As it can be seen, the thresholds, given by the EPL, divide a regime for which the relationship is positive to another regime where this relationship changes to a negative one\textsuperscript{18}. Let us first comment on the results for the consumption wage gap with a logistic transition function. In this case, when the EPL is below 2.21, an increase in the wage gap (real wages grow faster than productivity) induces an increase in the unemployment rate. However, once the index is above this threshold, the coefficient turns to a negative one\textsuperscript{19}. Similar results are found for the unemployment and production wage gap relationship.

Regarding the quadratic logistic specification, while countries with and EPL between 2.22 and 3.57 are characterized by a negative coefficient, others with values below and above this threshold

---

\textsuperscript{17}See Colletaz and Hurlin (2006) for a complete description of the linearity tests.

\textsuperscript{18}The estimated parameters \( \beta_{1,0} \) and \( \beta_{1,1} \) in equation (25) cannot be directly interpreted as elasticities, but their signs can be. See Colletaz and Hurlin (2006) for further details.

\textsuperscript{19}Notice that a country can change from a positive to a negative relationship (or the contrary) according to the evolution of its labour regulation.
have the opposite sign\textsuperscript{20}.

Table 5: The relationship between the (log) unemployment rate and the (log) wage gap.

Panel smooth transition estimates, 1985-2003

<table>
<thead>
<tr>
<th></th>
<th>Consumption wage gap</th>
<th></th>
<th>Product wage gap</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>F= logistic</td>
<td>F= logistic quadratic</td>
<td>F= logistic</td>
<td>F= logistic quadratic</td>
</tr>
<tr>
<td></td>
<td>Coefficient</td>
<td>t-stat</td>
<td>Coefficient</td>
<td>t-stat</td>
</tr>
<tr>
<td>$\beta_{1,0}$</td>
<td>1.468</td>
<td>10.32</td>
<td>-2.072</td>
<td>-10.27</td>
</tr>
<tr>
<td>$\beta_{1,1}$</td>
<td>-2.992</td>
<td>-13.13</td>
<td>3.568</td>
<td>14.19</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>30.9212</td>
<td>17.218</td>
<td>37.759</td>
<td>30.355</td>
</tr>
<tr>
<td>$c_1$</td>
<td>2.210</td>
<td>2.224</td>
<td>2.196</td>
<td>2.190</td>
</tr>
<tr>
<td>$c_2$</td>
<td>-</td>
<td>3.568</td>
<td>-</td>
<td></td>
</tr>
</tbody>
</table>

Notes: Idem table (3).

Even though the coefficients cannot be interpreted directly as elasticities, we can compute, for each country of the sample and for each date, the time varying elasticity of the unemployment rate with respect to the wage gap as\textsuperscript{21}:

$$e_{it} = \beta_{0,1} + \beta_{1,1}g(q_{it}; \gamma, c)$$

These individual elasticities for both the logistic and the logistic quadratic functions for both the consumption and product wage gap are shown in Figure 4. As it can be seen, countries that have remained with a positive elasticity in the whole period are Australia, Canada, Ireland, Japan, the UK and the USA (most of them, the so-called “Anglo-Saxon economies”). On the contrary, France, Norway, Spain and Sweden show a constant negative relationship. As we can see, broadly speaking this is the same pattern that we found with the correlation coefficients in the previous section.

\textsuperscript{20}The quadratic logistic function has a u form. Therefore, the function $g(q_{it}; \gamma, c)$ is equal to 1 for $EPL < 2.22$ and $EPL > 3.568$ and equal to 0 in between.

\textsuperscript{21}See Colletaz and Hurlin (2006) for details
We also observe that in some countries the sign of the elasticity has changed from a negative value to a positive one. One of this “switching” countries is Belgium, which, according to the logistic specification, shows a change from -1.5 between 1985 and 1996 to 1.0 every since then\(^{22}\). Along this line, the EPL index was reduced from 3.15 to 2.15 in 1997, when restrictions on temporary work agencies were substantially reduced and fixed term contracts were made renewable. Similar changes in the elasticity from a negative value to a positive one can be observed in Finland (1991), Netherlands (in 1999) and Italy (in 2001), coinciding also with important reductions in the restriction of the use of temporary contracts.

5 Conclusions

In this paper we have analyzed why some economies present a negative relationship between the wage gap and unemployment during the last 25 years.

Looking to the conditional correlation coefficients of a vector autoregressive (VAR) forecast errors as proposed in den Haan (2000), and using the nonlinear smooth transition (PSTR) models to panel data we have found an important difference between economies with a low degree of employment protection legislation and countries with high levels of protection in regular contracts and limited flexibility in the use of fixed-term contracts. Indeed, while the first group is characterized by a positive relationship between unemployment and the wage gap, countries belonging to the second group react differently. According to our theoretical model, in some countries, such as France and Spain, firing costs and the important weight of permanent workers give workers a market power that they use to push up their wages in a higher magnitude than the increment of labour productivity during good times. In contrast, since temporary workers have lower bargaining power, firms can discount to these workers the extra wage adjustments in the side of regular contracts. As a result, in this type of labour market unemployment can be reduced even though the average wage is increasing.

\(^{22}\)Following the quadratic logistic specification, the reversion in the sign of the elasticity was in 1997, changing from -2.06 to 0.95 in those years.
more than labour productivity.

We have also identified some countries like Belgium, Netherland and Italy, which have changed the elasticity of unemployment with respect to the wage-productivity gap from a negative to a positive value. According to our results, the fact that at the end of the sample period in most of the countries an increase in the wage gap encourages an increase in the unemployment rate is due to reductions observed in the EPL, especially in the size of temporary contracts. In other words, persistent reductions in the labour regulations, that characterized many OECD countries can help to explain the positive relationship between unemployment and the wage-productivity gap observed in most OECD economies nowadays.

References


Figure 1: Impulse response in the model with temporary and permanent jobs to an aggregate productivity shock \( A \).
Figure 2: EPL index and the correlation between the wage-productivity gap and unemployment.

Figure 3: Coefficient $\beta_1$ in equation (24) versus average EPL, 1985-2003 (a higher value implies more strict labour protection)
Figure 4: Elasticity of the unemployment with respect to the consumption wage gap. Logistic and quadratic logistic functions.