A. Mann. Philip Hall's 'Curious' Formula for Non-Abelian Groups.

In 1938 Hall published the following remarkable identity, which he described as 'rather curious'. Fix a prime p, and let G vary over all finite abelian p-groups. Then

$$\sum 1/|G| = \sum 1/|\operatorname{Aut}(G)|.$$

Hall's proof is combinatorial. Several later proofs were given, combinatorial, group theoretical, and number theoretical. We will discuss identities of a similar type, which hold for families of non-abelian *p*-groups. It will be seen that Hall's identity is the simplest, and most striking, of infinitely many related results. These identities were also anticipated by Hall, but he did not write them explicitly, nor gave proofs. A proof was published by J. Tappe in 1980. Our proof applies ideas and results of subgroup growth. We will also comment on other proofs of Hall's formula and its analogues.