TOPIC 2: OLIGOPOLISTIC MODELS

2.1 Consider a N firms industry. These N firms compete "a la Cournot". The inverse demand function for this industry is given by:

$$P(q) = a - b\sum_{i=1}^{n} q_i$$

and the cost function for each one of the firms is $C_i(q_i) = cq_i$

a) Obtain the reaction function of firm 1 (remember that it is a function of the level of production of all the other firms in the market)

b) Let us assume symmetry ($q_i = q^N$ for all i). Work out the equilibrium solutions for p and q^N. What happens with p when the number of firms in the industry tends to infinity? And with q^N?

2.2 Work out and explain how the number of firms in an industry affects production and price in the Cournot model.

2.3 Explain and comment the following argument. "The oligopoly theory predicts a positive relationship between industry concentration and price cost margin".

2.4 The inverse demand function of wheels is given by $p(Q) = 100 - 2Q$, and the cost function for any firm operating in this industry is $C(Q) = 4Q$

a) Which is the marginal cost of the firms in the industry?

b) If the wheel industry were a perfectly competitive industry, which would its production level? , which would be the price of the wheels?

c) Let us assume that there are only two firms in the industry and they compete "a la Cournot". Work out the reaction functions for both firms. Which are the equilibrium price and production levels? Which is the profit of both firms?

d) Draw (in a figure) the reaction functions of both firms and indicate the Cournot equilibrium.

e) If both firms reach a collusive agreement. Which will be production and market price?

f) Let us assume that both firm collude (collusive agreement) and share the market evenly (both firms sell and produce the same quantity). If firm 1 assumes that firm 2 will not react if it changes its own production, which is the optimal production for firm 1 if firm 2 produces 12 units of product?. Has firm 1 any incentive to maintain the collusive agreement.

g) Let us assume that both firms behave "a la Stackelberg" being firm 1 the leader and firm 2 the follower. Which will be production and market price? Which will be the production of firms 1 and 2? Which will be the profit of each one of the firms

h) Using isoprofit curves draw and compare the situations in Cournot, Stackelberg and Collusive cases.

2.5.- Let us consider an industry with two firms (A and B) that produce an homogeneous good. The linear inverse demand function is given by:

$$p = 1 - Q \qquad Q = q_A + q_B$$

where p y Q are market price and production and q_A y q_B are the productions of firms A and B, respectively. Marginal costs of A and B are equal to zero, i.e., $c_A = c_B = 0$. Additionally, let us assume that A an B strategic variable is production (they choose the quantities q_A y q_B that maximize their profits).

a) Calculate market price and production, and profits of firms A and B when both firms choose output simultaneously (Cournot's model).

b) Calculate market price and production, and profits of firms A and B when firms choose optimal output sequentially. Firstly, A acting as a leader, choose its optimal output. Second, B (acting as follower,) chooses its optimal output. (Stackelberg's model).

c) Analyse the differences in firms' profits and consumers' surplus between cases a) and b) (Cournot and Stackelberg models).

2.6.- Let us consider and industry with two firms (A and B) that produce an homogeneous good. The linear demand function is given by $p=20-3Q$, where p is the market price and Q is the market production, with $Q=q_A + q_B$. q_A y q_B are the firm A and B productions respectively. Marginal costs of A and B are constant and identical, i.e, $c_A=c_B=2$. There are no fixed costs. Firms A and B choose production to maximize profits (their strategic variable is production).

a) Calculate market price and production and profits of firms A and B when firms cooperate, maximize their joint profit and share demand and profits evenly (collusive behaviour).

b) Calculate market price and production and profits of firms A and B when firms behave in non-cooperative way and they decide simultaneously their repective productions (Cournot).

c) Calculate, comment and show graphically the differences in firms' profits in cases a) and b). Do firms have incentives to maintain a collusive behaviour?

2.7.- Let us consider an industry with two firms, L y F, that produce an homogeneous good. Firm L is the industry leader and so both firms behave according to the Stackelberg model: L behaves as Stackelberg's leader and F behaves as a Stackelberg's follower.
The inverse demand function is, $p(Q)$, is linear an given by $p=1-q_L-q_F$, where q_L y q_F are the leader and follower productions respectively, and p is the market price. Further, assume that the costs function is $C(q_i)=q_i/2$, $i=L,F$.
a) Calculate firms L and F equilibrium productions and market price.
b) Compare this situation with the one we would obtain under the assumptions of the Cournot's model and comment the differences between these two models.

2.8.- Which are the only equilibrium prices in an industry with two firms (A and B) that choose prices to maximize profits if the marginal costs of A and B are c_A y c_B, with c_A lower than c_B? Show that there exists a unique equilibrium. Which are the sales of firms A and B at the equilibrium prices if the market demand is $D(p)$? Which are their profits?

2.9.- Consider the following non-cooperative game between two firms that can choose whether behaving collusively (C) or competing "a la Cournot" (non-collusive behaviour, NC). In every one of the possible situations, the profits of both firms are the following:

		Firm 2	
		C	NC
Firm 1	C	8,8	2,10
	NC	10,2	3,3

a) Find out the dominant strategies of both players, if they exist. Find out the set of strategies of the Nash equilibrium.

b) Consider an infinite repetition of the game. Describe the pair of "trigger" strategies in the supergame and explain how the pair of strategies (C,C) could become a Nash equilibrium. Consider now a finite repetition of the game and analyze whether collusion is or not a Nash equilibrium.

2.10.- Consider the following non-cooperative game between two firms. Firms choose simultaneously between competing " a la Cournot" or to produce the collusive quantity (C). Profits in each case are the following:

		Player 2	
		C	NC
Player 1	C	160,160	40,200
	NC	200,40	60,60

a) Find out the dominant strategies for both players (in case they exist). Find out the Nash equilibrium of the game. Is the Nash equilibrium pareto-efficient?
b) Consider a finite repetition of the game. Which would be the Nash equilibrium in this case? Explain your answer.
c) Consider a finite repetition of the game. Define the pair of trigger strategies of the supergame. Explain whether it is possible to get the pair of strategies (C, C) as a Nash equilibrium of the infinite horizon game.