On the Prediction of Visibility for Deep-Sky Objects

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Abstract

Knowing in advance the visibility of a given object is important when planning an observation. New amateurs usually search only bright objects since they present smaller difficulties for finding and observing them, except perhaps under severe conditions (e.g., city, full moon). Nevertheless, amateurs gradually extend their observations to more challenging targets, fainter and more elusive. These objects, being more difficult, can offer dramatically different images depending on the telescope aperture and sky quality. Therefore, to predict with certain accuracy the difficulty that an observation will involve may become a puzzling task. Most observers estimates the possibilities of observing a celestial object in a subjective way, depending on experience in handling their telescopes and knowledge on deep-sky objects. It would be however convenient to have a more objective tool to decide whether a given object will be visible or not.

Keywords: prediction of visibility, surface brightness, background surface brightness, contrast, limiting magnitude, visibility

Introduction

For a long time, the prediction of visibility for deep-sky objects has been one of my favorite astronomical topics. The same way as many amateurs, I knew that many widely extended ideas in this field, stated as true, were indeed quite unreliable. Points as the convenience of using telescopes with short focal ratios and very low magnifications, able to concentrate light of diffuse objects, and questions as the binocular giving the best performance, or the faintest star visible at a given telescope. Some few works appeared where shyly the dominant trends —imposed by astronomers of unquestionable authority, as Sidgwick for instance— were contradicted. However, nobody dare to disagree.

When Roger Clark's *Visual Astronomy of the Deep Sky* (VASDS) was published, many of the thoughts mentioned above became tangible. VADS represented almost a revolution, a breaking point in visual observational techniques. In this excellent book, Roger Clark smartly explains all those ideas that deep-sky amateurs have in mind, by mere intuition, as we know better our telescope and we learn to observe. If any of you haven't yet read VADS, I strongly recommend to do it.

Almost all the published methods to calculate the visibility of deep-sky objects are based on the concept of *surface brightness*. The surface brightness (*SB*) is defined as the brightness, measured in magnitudes, that a given angular surface pertaining to the studied object presents, supposed uniform, and is given by the following expression:

$$\mathrm{SB} = \mathrm{m} + 2.5\log\left(\frac{\mathrm{a\,b\,\pi}}{4}\right) \tag{1}$$

In Equation 1, *m* represents the object's visual magnitude, and *a*, *b* are the major and minor axes, measured in convenient angular units. For example, the Crab Nebula, which is 6' × 4' sized and shines with the 8.4 visual magnitude, would have a surface brightness of 11.6 mag×arcmin^{-2} (or 20.5 mag×arcsec^{-2}, depending on the units used for expressing the axis size). One can use arcminutes for *a* and *b*, and however obtain *SB* in mag×arcsec^{-2}, just adding 8.89 to the result expressed in mag×arcmin^{-2}. In the present work we'll use mag×arcsec^{-2} for two reasons. One is that these units are more convenient taking into account the ordinary diameters of deep-sky objects, avoiding thus negative numbers resulting from the logarithm in Equation 1, which is translated into a *SB* greater than the object's magnitude. The second reason is related to the Airy's disk given by a telescope, whose diameter is quite smaller than one arcmin[2], but closer in magnitude order to one arcsec[2], which is useful to compare with stellar thresholds. Table 1 list some surface brightness values for common astronomical objects:

Table 1— Typical surface brightness values for astronomical objects.

Object	Size	Visual magnitude	*SB* mag×arcmin^{-2}	*SB* mag×arcsec^{-2}
Sun	32'	–26.8	–19.5	–10.6
Venus	9.8"	–3.9	–8.1	0.8
Full moon	32'	–12.6	–5.3	3.6
Júpiter	45"×42"	–2.5	–3.2	5.4
Nautical twilight	all sky	meaningless	5.0	14.0
Metropolitan site, *LM*=4.5	all sky	meaningless	7.0	16.0
Ring Nebula, M 57	86"×62"	8.8	9.0	17.9
Crab Nebula, M 1	6'×4'	8.4	11.6	20.5
Dark sky background, *LM*=6.5	all sky	meaningless	12.5	21.5
Helix Nebula, NGC 7293	12'×10'	7.3	12.2	21.1
M 51 main (NGC 4594)	9'×7'.5	8.4	12.7	21.6
M74	10'×9'.6	9.2	13.9	22.8
Planetary nebula Abell 72	132"×121"	13.8	15.2	24.0
Rural site background, ML=7.5	all sky	meaningless	15.3	24.2
Sculptor Dwarf	75'	10.5	19.6	28.5

To take into account the object's brightness and size is a good starting point, but it is not enough. It is also necessary to consider the brightness of the sky background where the object is projected on. Sky background severely disturbs our capability to distinguish subtle details. In the following discussion, we will not consider irregular factors as Milky Way glow, or zodiacal light. The residual glow of the sky dome, namely the *background surface brightness* (SB_0), can be evaluated from calibrates. These calibrates are prepared measuring the fading apparent diameter of a set of defocused stars with known magnitudes. The visual angle at which stars are confounded with the sky background is usually taken between 6 and 10 arcminutes for weak images (see Figure 1).

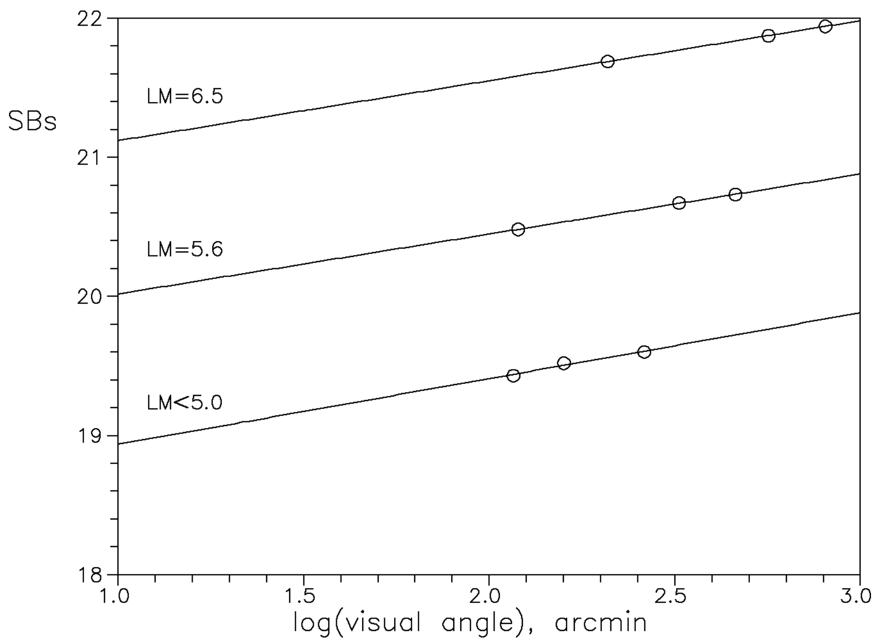


Figure 1

Faintest surface brightness (SB_s) visible through a 68 mm refractor on several background conditions. Data shown were experimentally measured by the author defocusing stars of known magnitude. The background surface brightness can be evaluated reading SB from the figure at the critical visual angle (a value in the 6'-10' range for the background illumination conditions examined), log angle = 0.8-1.0.

Despite that, precise enough values can be obtained based on the faintest star visible at the naked eye, through the following polynomial approach:

$$SB_0 = 24.19 - 2.814LM + 0.3694LM^2 \qquad (2)$$

This expression is valid for limiting magnitudes at the naked eye (*LM*) greater than 5.0. If *LM* is smaller than 5.0 there is no rodopsine (or *visual purple*: the pigment responsible of the greater eye sensitivity during the night) activation, and our eyes will work as under daylight conditions, but with a more dilated pupil. The upper limit (8.5 *LM*) corresponds to a 27 mag×arcsec^{-2} background surface brightness: the eye detection threshold. Two drawbacks, however, prevent us to reach such a faint surface brightness at the naked eye. One is that the sky is never dark enough; the other is that faint stars are never really isolated: we are dazzled by their neighboring stars. From best rural sites we can see up to 7.8 magnitude stars at the naked eye. Under these conditions, the celestial dome presents a residual brightness of about 24.3 mag×arcsec^{-2}, with small emission lines at the visible window due to ionized atmospheric gases under very low partial pressures (sodium, hidroxile, oxigene, etc.). The maximum background darkness in the visible light domain is located at 410 nm.

In most cases, the background glows more even in rural sites, reaching values often in the 21.5 to 22.5 mag×arcsec^{-2} range. Therefore, we are unable to reach 7.8 magnitude stars at the naked eye, but quite more modest values: 6.5 to 7.0. Going beyond 7.8 magnitude is only possible in laboratory experiments, but there is a more exciting exception: using a telescope. With the aid of a telescope, we make the background darker and we separate at greater angular distances the disturbing stars, closing to those ideal conditions which are required to reach the 8.5 *LM*. Of course, we'll not see stars of the 8.5 magnitude, but much more fainter ones due to the improvement in light gathering made with the telescope. Closer to cities the situation becomes harder: new emission lines appear as a result of artificial lights produced by gas lamps, such as high pressure sodium or mercury, that strongly disturb us. These lines can be eventually removed through interference filters.

The literature mentions an easy rule of thumb to decide whether an object is visible or not. The idea is to establish a five per cent of the sky's background brightness as a visibility threshold. Applying that criterion, we would expect to see objects with a surface brightness up to 3.25 magnitudes above the background brightness:

$$SB_{lim} = SB_0 + 2.5\ \log\frac{100}{5} = SB_0 + 3.25 \qquad (3)$$

Suppose a good sky, where 6.5 magnitude stars could be seen at the naked eye. Substituting this value in Equation 2 yields 21.5 mag×arcsec^{-2} as the value of SB_0. A further substitution of SB_0 in Equation 3 yields a visibility threshold about 24.8 mag×arcsec^{-2}.

Too simple to be really good. That criterion doesn't take into account neither the telescope diameter nor the object's apparent size, which are two very important factors. Given two objects having the same surface brightness, the greatest one will result easier to be seen. This fact is related with our vision system, which definitively is not a simple passive light gatherer, as a photographic film. We don't observe with a telescope only: we do it through our telescope, our eye(s), and our brain, which are associated in order to process signals to produce meaningful data. In that way, as we increase the object's apparent size varying magnification, both the object and the background become darkened in the same amount. But, as we are enlarging the image, more retinal cells are contributing to confirm the perception of faint signals. With all that, a significant image enhancement is achieved although less light is arriving to a given retinal cell. Always? Well, not exactly: the improvement is only true below a critical magnification, where background and object are too darkened to obtain any enhancement in perceptibility via increasing the object's visual angular size. We can benefit of these properties of our vision system to see faint objects.

We can illustrate this fact in a dramatic way: let's calculate the surface brightness of an Airy's disk produced by a star just at the threshold. Suppose we are using a 68 mm refractor, which shows stellar images about 1.7 arcseconds in diameter. Under moderate seeing conditions these images are clearly upon the visual limits constrained by a steady atmosphere. This tiny instrument can reach stars a little fainter than the 13th magnitude, seven magnitudes weaker than those values measured at the naked eye, but one can read in many books that the *TLM* for this instrument should be the 11th magnitude. Why this difference?

The telescopic limiting magnitudes (*TLM*) published in the literature are certainly quite conservative. This deserves some more comments. Traditionally, *TLM* values are calculated for a background brightness similar to that visible at the naked eye. That is, when the eyepiece used at the telescope presents the same exit pupil than the human eye: 7.5 mm, so there is only a geometrical gain in light gathering. The equation for traditional values of *TLM* is just based on the calculation of telescope's aperture to eye's pupil surface ratio:

$$\mathrm{TLM}_{\mathrm{trad}} = \mathrm{LM} + 5\log\left(\frac{\mathrm{DIAM}}{7.5}\right) \tag{4}$$

where *LM* is the magnitude of the faintest star visible at the naked eye, and *DIAM* is the telescope's aperture in millimeters. Under such conditions the observer's vision capability is severely restricted, and the telescope cannot develop its true power. For instance, a 11.3 *TLM* is obtained for a R68 telescope when 6.5 stars are visible at the naked eye; too modest to be good.

Fortunately, the outlined situation is rarely found in practice: through many eyepieces the sky background glows substantially less, allowing us to see fainter stellar objects. As an example, in a common night (6.5 *LM*), the mentioned 68 mm little refractor can easily show stars fainter than 12.5 magnitude through a wide range of eyepieces. Let's consider that when the magnification is ×150, a R68 telescope is able to show 13.2 magnitude stars in a 6.5 *LM* night. The surface brightness of an stellar source at the threshold (13.2 magnitude, 1.7 arcseconds wide) would be then 14 mag×arcsec^{-2}. At this point, I recommend you to have a look to Table 1 again, to familiarize a bit more with usual *SB* values for deep sky objects.

The reader should conclude that something is wrong, since if *SB* threshold actually is 14 mag×arcsec^{-2}, we virtually should expect to see no diffuse object!. No Messier object has such a high average surface brightness. The Messier object having the highest *SB*, M57, is only 18 mag×arcsec^{-2}. However, it can be verified that using this instrument, images presenting 22 mag×arcsec^{-2} are easily perceived, *1500 times fainter than the faintest stellar image we are able to see*. Even more: just changing the eyepieces to get optimum magnifications, we'll see that this process can be carried out further, up to *SB* above 24 mag×arcsec^{-2}. Therefore, we are forgetting to include something that is very essential in our treatment. We will tackle later the problem of extended objects, but first let's finish the discussion on the visibility of stellar objects (*e.g.*, stars, smaller planetary nebulae, and farther galaxies with bright cores).

The true limiting stellar magnitude we can reach with a given telescope (*TLM*), at a given magnification (*MAG*), when stars of *LM* magnitude can be seen at the naked eye, is easily calculated through the next two formulae sequentially substituted, that can be easily obtained combining the concepts of darkening, surface ratio between telescope and eye, and the polynomial approach for predicting background surface darkness (Equation 2):

$$\mathrm{SB}_{0\mathrm{T}} = 28.57 - 2.814\mathrm{LM} + 0.3694\mathrm{LM}^2 + 5\log\left(\frac{\mathrm{MAG}}{\mathrm{DIAM}\sqrt{\mathrm{t}}}\right) \tag{5}$$

$$\mathrm{TLM} = -22.81 + 1.792\mathrm{SB}_{0\mathrm{T}} - 0.02949\mathrm{SB}_{0\mathrm{T}}^2 + 2.5\log\left(\mathrm{DIAM}^2 \times \mathrm{t}\right) \tag{6}$$

Equation 5 yields the background surface brightness as seen through the telescope, that is, darkened by effect of magnification, whereas Equation 6 gives us the faintest star visible at the eyepiece on that artificially darkened background. There is a new variable in Equations 5 and 6, namely the *transmission factor* (*t*), that represents the percentage of gathered light we benefit from. The transmission factor takes into account not only light diminishes due to central obstructions (secondary mirrors), glass absorption, undesired reflections or light scattering, but also gains due to the experience level and the ability of the observer. In this work, we will take $t = 0.9$ for refractors and $t = 0.7$ for reflectors.

Since maximal *TLM* happens when the sky background presents a 27 mag×arcsec^{-2} surface brightness, one can derive an equation similar to traditional *TLM* from Equation 4, but this time more realistic:

$$\mathrm{TLM}_{\max} = 4.12 + 2.5\log\left(\mathrm{DIAM}^2 \times \mathrm{t}\right) \quad (7)$$

The magnification necessary to get this darkening (*i.e.*, the magnification that better help us to see stellar sources), can now be derived from Equation 5:

$$\mathrm{MAG}_{27} = 0.1333 \times \mathrm{DIAM} \times \sqrt{\mathrm{t}} \times 10^{\frac{2.81 + 2.814\mathrm{LM} - 0.3694\mathrm{LM}^2}{5}} \quad (8)$$

As mentioned, the above expressions are only valid for stellar or semi-stellar objects. But when an extended object is observed, the prediction of visibility fails. For this kind of problem, it must be taken into account, moreover the surface brightness of both the object and background, the object's size (or, if it is round, its global magnitude, being two variables mathematically linked through the definition of *SB*). Some years ago I successfully used empirical calibrates obtained through a little refractor. Nevertheless, the theory developed by Roger Clark has been proved to be more suitable, at least in some cases.

Now we'll introduce the Roger Clark's method for predicting the visibility of extended objects. An analogous method derived by the author, that even seems more reliable, will be explained further.

The Clark's Method

In *Visual Astronomy of the Deep Sky*, Roger Clark proposed a method for predicting the visibility of non-stellar objects at a given telescope, as well as to calculate the optimal magnification. The method is based on a previous paper due to Blackwell and published decades ago, where medical measurements on the eye's performance for detecting glowing objects on differently illuminated backgrounds, were tabulated. Clark transformed those data to plot a graphic relating the object's contrast with the apparent angular size required to see the object at the threshold, under different background illumination conditions. We'll call *Blackwell/ Clark's surface* to this plot (see Figures 3 and 4).

Some very interesting concepts have been introduced in VADS. Perhaps the most important one is the *optimum magnified visual angle* (OMVA): the angular size which allows to see an object under optimal conditions according to the eye physiology. By intuition we know that deep sky objects can be optimally seen through certain magnifications. Indeed, as we increase the eyepiece power, many objects become easier to see. However, this is only true up to a certain magnification, beyond which the image diminishes quickly than the eye's performance improves. Based on the response surface mentioned above, Roger Clark plotted a second graphic that allows a quick estimation of the OMVA, arguing that all the points in the Blackwell/ Clark's surface where the first derivative is –1 represent perceptibility maximums. Since all these points draw a curve on the surface, the OMVA estimation can be expedited; this is the aim of the mentioned second plot (Figure 2, next page). Once the OMVA is obtained for a given object and observational conditions, just dividing this angle by the object's minor axis leads to the *optimum detection magnification* (ODM).

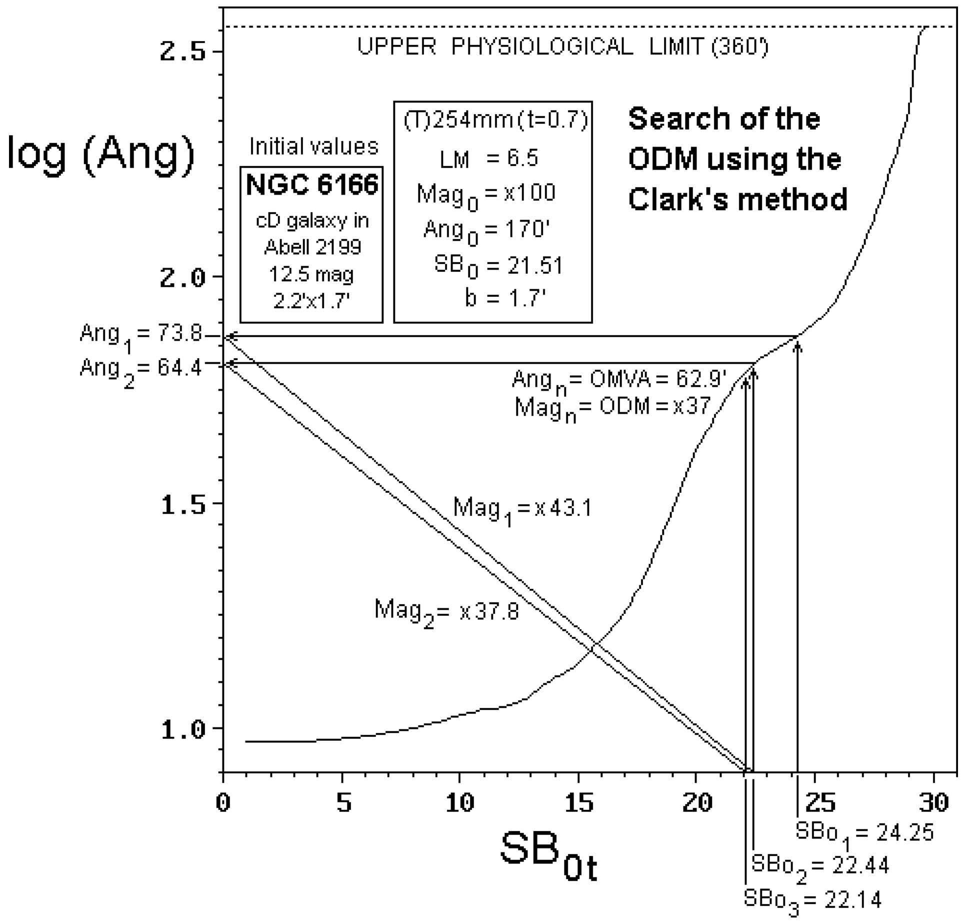
Perhaps the reader should appreciate now an example to understand better this theory. The calculation of Clark's ODM consists of an iterative process. Let's illustrate it finding the ODM for NGC6166, using a 254 mm telescope ($t = 0.7$) in a night where 6.5 magnitude stars could be seen at the naked eye. NGC6166, the main member of the galaxy cluster Abell 2199, is an huge cD galaxy, shining with 14.0 photographic magnitude (about 12.5 visual), and measuring 2'.2 × 1'.7 at the telescope. Study carefully now Figure 2, whereas you are reading the following explanations, to understand how the algorithm works. Two initial parameters are required: the object's minor axis (b = 1'.7) and the sky's background surface brightness at the naked eye (SB_0 = 21.51, obtained with Equation 2 for LM = 6.5). The algorithm starts assuming an arbitrary

Figure 2

Calculation of the optimum magnification for NGC6166 (12.5m, 2'.2 × 1'.7) using the Clark's iterative algorithm described in the text.

This simulation corresponds to a hypothetical night at which 6.5 magnitude stars could be seen at the naked eye, using a 254 mm telescope with a transmission factor t = 0.7.

Apparent background values of surface brightness SB_{0i} at each iteration step are calculated via expressions (1) and (2) shown in Table 2. The optimum detection magnification (ODM) is ×37, according to this method.



initial magnification (MAG_0 = ×100), from which the darkening is calculated (= +2.74, see Table 2, Note 1). Darkening is now added to SB_0 to obtain the background surface brightness as seen through the telescope: SB_{01} (= 24.25), and we should locate that value on the X-axis. From this darkened background (SB_{01}), a value for an optimal angular size (Ang_1 = 73.8) is measured from the projection on the Y-axis of the point located on the OMVA curve having the SB_{01} value previously calculated. Dividing Ang_1 by the object's minor axis, *b*, a new value of magnification is obtained (MAG_1 = 73.8/1.7 = ×43.1), usually different from the previous value (MAG_0 = ×100). If MAG_1 does not fit with MAG_0, the full process is subsequently repeated, using the MAG_1 value just obtained as the new initial magnification at the current iteration. The process is repeated as many times as required up to obtain no significant difference between MAG_i and MAG_{i+1}, that is, between two consecutive iterations. When this happens, we have ODM = MAG_i = MAG_{i+1}. In the case studied, ODM is ×37. Finally, we check whether the object is visible or not, interpolating in the Blackwell/Clark's surface; we will consider later how is this done. In the example, NGC6166 is actually visible using the optimal magnification found (×37).

Reasonable results are obtained in many cases. But, surprisingly, if the method is applied to predict the visibility for objects which are both bright but very small, it leads to inconsistent values. To illustrate it, let's consider a little planetary nebula 2" wide and shining with the 12.5 magnitude. Through a 254 mm reflector (t = 0.7) in a 6.3 *LM* night, experience and common sense tell us that it should be an easy object, since it's almost a stellar source and we should be able to see stars three magnitudes fainter. To avoid a too large image, which could be even greater than the rods area of our retina, we'll impose a six degrees constraint on the object's apparent size for limiting possible magnifications. Nevertheless, if we apply the Clark's Method, we conclude that ×10800 is the optimum magnification!. The object, in fact, was perfectly visible at a smaller magnification, but we have enlarged it so much that it can not be perceived at all. Some tricks can be proposed to avoid a so large magnification, but, in any case, the visibility cannot be guaranteed at the end of the calculation process.

Such kind of result introduces severe doubts over the procedure used to calculate the optimum magnification. The Clark's Method seems predict useless magnifications when we want to see very small objects or almost stellar sources, such as planetary nebulae or small galaxies. And, which is more important, the results do not agree with those obtained when we predict the visibility of stars of similar magnitude, that seem to be completely reliable.

Equations 5 and 6 predict for instance that using a 254 mm telescope in a $LM = 6.3$ night, we are able to see stellar objects shining with 12.5 magnitude from to ×14. The maximum visibility will happen when the background is darkened up to 27 mag×arcsec^{-2}; that is over ×390 (Equation 8). One should expect a smooth transition between both procedures (*i.e.*, Clark's Method and Stellar Method, that is, Equations 5, 6, 7 and 8) as the object's size increases, with a certain range of coexistence: an interval where nearly identical predictions should be obtained with both methods. But that does not happen.

The Threshold Method

An alternative approach for searching the optimum magnification

Clark's Method performs only an *angular optimization*, not considering the object's visibility until the end of the calculation. It is quite surprising to separate angular optimization from visibility optimization in an algorithm designed for predicting the visibility, since we certainly want to find magnifications where the object could be seen!. According to Roger Clark, the optimum magnification is obtained considering only the object's apparent dimensions. If an easy object is not visible at the Clark's ODM, perhaps we require to define another kind of optimal magnification that balances perceptibility with apparent angular size. We need to introduce ourselves deeper into the visibility concept.

At this point it's convenient to take a few time explaining the Blackwell/Clark's surface (see Figures 3 and 4), which, as commented, gives the critical circumstances for observing any kind of object at the threshold. This figure is extremely important, and you must spend some time to understand it properly.

The Blackwell/Clark's graphic shows thus a three-dimensional response surface seen from above, plotting the lost dimension (height, Z-axis) on the X-Y plane as some contour lines. Only two axes are thus conserved:

1. The X-axis is the object's apparent angular diameter (minor axis multiplied by magnification): the object's size as seen through the eyepiece.

2. The Y-axis is the *object's contrast*, which we'll define right now.

In both X and Y axes, logarithms are used to extend the linear behavior as possible. The Z-axis, which is lost due to the projection, is the background surface brightness required for perceiving at the threshold an object with a given size (X-axis) and surface brightness (Y-axis). Several lines have been overlaid, each one corresponding to a given background surface brightness, in the range from 4 to 27 mag×arcsec^{-2}. In this way, if an object with a certain apparent size were projected on a background of a certain surface brightness, this graphic would yield the critical surface brightness beyond which the object would be visible.

The Y-axis involves a new concept, called *contrast*: the object-to-background surface brightness ratio, given by:

$$C = 10^{\frac{SB_0 - SB}{2.5}} \tag{9}$$

When the logarithm of C is taken, units are obtained that grow at the same rate that the visual stimulus. This is just the *Fechner's Law*, a well-known rule for any deep-sky or variable-star observer, which justifies the relation between magnitudes and visual stimulus. Contrast tells us how faint an object should be in order to be visible at the threshold, taken as zero the background brightness. Or, if you prefer, contrast allows the calculation of the limiting surface brightness we can reach: $SB_{lim} = SB_0 - 2.5 \log C$.

Let's suppose a given object observed at certain sky conditions. We can calculate the limiting surface brightness (SB_{lim}) at several magnifications. When we plot the results on the Blackwell/Clark's surface, a curve results that we'll call *limit of detection*. For the Clark's Method to work, minimums for all possible limit of detection curves should be located always over the OMVA curve. Surprisingly, this fact does not seem to occur.

Plotting a limit of detection curve is not so hard, since we can easily calculate for each magnification, the logarithm of the object's apparent size (*X*) and the darkened background surface brightness as seen through the telescope at that magnification (*Z*):

$$X = \log\left(b \times MAG\right) \tag{10}$$

$$Z = SB_0 + 5\log\left(\frac{7.5\,MAG}{DIAM \times \sqrt{t}}\right) \tag{11}$$

In Equation 10, *b* is the length of the object's minor axis measured in arcminutes. The minor axis is used instead of the major dimension for the reason that the former is actually the limiting factor when observing a faint image. The above expressions give us the coordinates *Z* (darkened background) and *X* (apparent size).

The third coordinate (*i.e.*, the Y-axis, or logarithm of contrast), should be evaluated from those (*Z* and *X*) values using the corresponding contour line. As commented, the minimum visible contrast for the considered object at a certain magnification is represented by the Y-axis, whose coordinates are interpolated from the Blackwell/Clark's graphic starting from *X* and *Z* values. When successive points (each one corresponding to a different magnification) are plotted together and joined, a curve is obtained: the above introduced *limit of detection*. The minimum of that curve yields the angular size which allows to gather the faintest possible surface brightness in the studied object. Dividing the so-calculated angular size by the object's minor axis, the optimum magnification according to both visibility and apparent angular size is obtained. We'll call it OptDM to emphasize that it is a different concept to Clark's ODM.

Numerically speaking, a risk arises when working with stellar sources: if zero is used for the object's apparent size, log(0) and divisions by zero will appear. To avoid it, the diameter of the Airy's disk produced by the telescope should be used. In fact, that is what we actually see for a really small object.

It should be pointed out that at increasing magnifications, despite of the fact that both the object and the background are darkened, the ratio between *SB* and SB_0 remains constant: the logarithm of the object's contrast does not depend on magnification. That's why the graphical representation of the object's contrast is just a straight line, that we'll call *object contrast line*. Do not confuse this concept with the smaller visible contrast represented by the limit of detection, that actually depends on magnification.

For deciding the condition range where the object is visible, we need to plot two lines overlaid on the Blackwell/Clark's graphic: (1) the limit of detection curve, and (2) the object's contrast line. If the limit of detection (Equations 10 and 11) is completely located above the object's contrast line, then we can conclude that the object will not be visible at any magnification. For the object to be seen, the limit of detection should be located below the object's contrast line for the whole range of available magnifications, or at least for some portion of it. The magnification corresponding to the maximum of visibility will be located at the point where a maximum distance exists between the object's contrast line and the limit of detection, as far as the object is visible, of course.

At this point, an example will help you to understand the full process. The considered object will be again NGC6166, as in the Clark's example (2'.3 × 1'.7, 12.5 visual magnitude). We will take a 254 mm reflector ($t = 0.7$) as the working instrument, and the naked-eye limiting magnitude will be also $LM = 6.5$ ($SB_0 = 21.44$, see Equation 2). Experience tells us that no difficulty should arise in this observation. All the algorithm is explained in detail in Table 2 and Figure 3. To draw the limit of detection we should plot the points of columns 3 (X-axis) and 6 (Y-axis) of Table 2 on the Blackwell/Clark's surface. It should be remarked again that using the Blackwell/Clark's surface, both size and visibility are simultaneously optimized: angular optimization is unnecessary, since it is implicitly included in the Blackwell/Clark's surface.

Table 2— Visibility of NGC 6166 (2.2' x1.7', 12.5m) in a 6.5 *LM* night, using a 254 mm telescope. The process of building a threshold curve is step by step given.

Instrument	(T)254, t = 0.7
Object's size and visual magnitude	2'.2×1'.7, 12.5m
Object's surface brightness	SB = 22.56 mag×arcsec^{-2}
Limiting magnitude at the naked eye	LM = 6.5
Background surface brightness	SB_0 = 21.44 mag×arcsec^{-2}
Object's contrast line	$Y = \log C_0 = -0.4\ (22.56 - 21.44) = -0.45$

Magnification	Darkening *1	SB_{0app} *2	log(axis) *3	log C *4	SB_{lim} *5	log(C_0/C) *6
40	0.75	22.18	1.83	–0.62	22.98	+0.17
50	1.23	22.67	1.93	–0.63	23.01	+0.18
60, optimum	**1.63**	**23.07**	**2.01**	**–0.63**	**23.02**	**+0.18**
70	1.96	23.40	2.08	–0.63	23.01	+0.18
80	2.25	23.69	2.13	–0.62	22.99	+0.17
90	2.51	23.95	2.18	–0.62	22.97	+0.17
120	3.13	24.57	2.31	–0.58	22.88	+0.13
150	3.62	25.05	2.41	–0.53	22.77	+0.08
180	4.01	25.45	2.49	–0.50	22.67	+0.05
250, not visible	4.73	26.16	2.63	–0.41	22.45	–0.04
300, not visible	5.12	26.56	2.71	–0.33	22.26	–0.12
400, not visible	5.75	27.18	2.83	–0.15	21.80	–0.30
500, not visible	6.23	27.67	2.93	+0.08	21.23	–0.53

*1 Darkening = –5 log (0.1116 × *DIAM* / *MAG*), where *DIAM* is the telescope diameter in mm, and *MAG* is the magnification.

*2 Apparent background surface brightness, as is seen through the eyepiece: $SB_{0app} = SB_0$ + (Darkening), where SB_0 = 21.44 in LM = 6.5, see Equation 2.

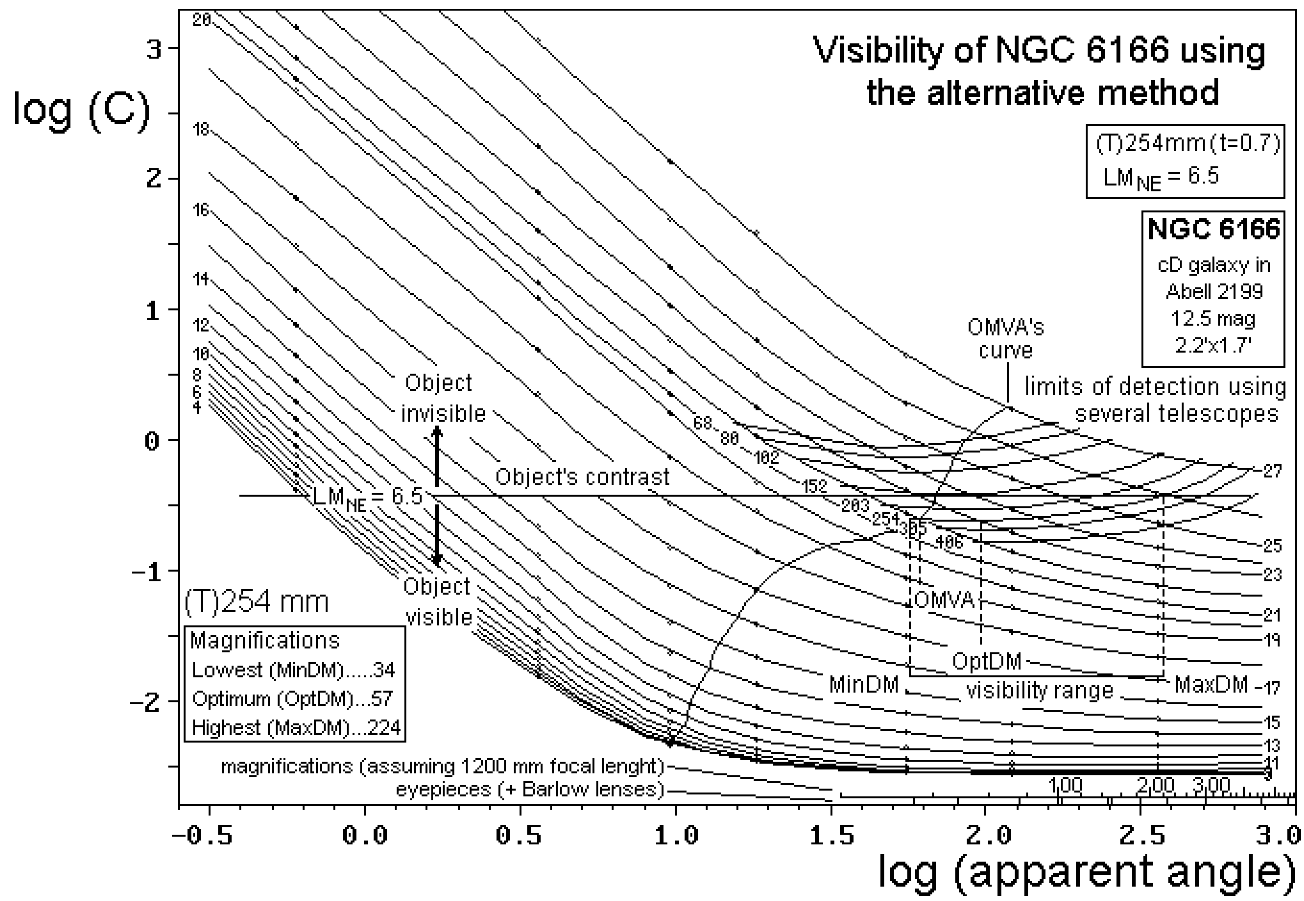
*3 Logarithm of the apparent minor axis (as seen magnified with the telescope), in arc-minutes. Thus, for NGC6166, whose minor axis is 1'.7, log(axis) = log(1.7 *MAG*).

*4 Obtained interpolating from Blackwell/Clark's surface (Figures 3, 4) the values of SB_{0app} and the logarithm of the magnified minor axis.

*5 Minimum surface brightness that can be detected in a 254 mm telescope using the current magnification $SB_{lim} = SB_0 - 2.5 \log C$.

*6 Visibility, from which the object's contrast has been subtracted: Visibility = $Y - \log C$. The object can be seen if this value is greater than zero. The greater this value, the easier to see the object.

Figure 3
Calculation of the optimum detection magnification for NGC6166, according to the Threshold Method explained in the text and illustrated in Table 2, using the same conditions than those for Figure 1. The minimum magnification required to see this galaxy (MinDM) is that necessary to obtain a 7.5 mm exit pupil, that is, 254/7.5 = ×34. From this value, one can increase telescope's power up to arriving to a visibility maximum. That is the optimum detection magnification (OptDM), which is ×57 for a 254 mm telescope. The galaxy could be seen at progressively higher power, up to a magnification at which it is confounded with the background: the maximum detection magnification (MaxDM), ×224 in this case. The visibility curves corresponding to different apertures are overlaid on the same graphic. It's easy to realize that smaller optimum detection magnifications are obtained for smaller telescopes. However, smaller telescopes also require smaller exit pupils. All these results contradict the Clark's theory, but fits better with the user experience on deep-sky. Thus, for telescopes with apertures of 68, 80, 102, 152, 203, 305 and 406 mm, the optimum magnification increases: 32, 33, 37, 48, 52, 61 and 67, respectively, whereas the exit pupil also increases: 2.1, 2.4, 2.8, 3.2, 3.9, 5.0 and 5.7 mm, respectively.



With values from the last column of Table 2 (previous page), we can introduce three new observational parameters:

OptDM/OptDA — *Optimum detection magnification/angle*, at which the object can be seen under the most favorable conditions, accessing to faintest details or, what is equivalent, more similar to the background (smaller contrasts). In Figure 3, we can see that OptDM corresponds to the minimum of each limit of detection plotted, and that these points are not generally located on the OMVA curve. In the example of Table 2, OptDM is slightly greater than ×50. ODM, as calculated with the Clark's procedure, results ×40. OptDM, as well as OMVA ODM, tend to be similar in magnitude for objects having appreciable dimensions, what explains why Clark's OMVA works in many cases.

MaxDM/MaxDA — *Maximum detection magnification/angle*, at which the object's apparent dimensions have been enlarged up to coincide with the limit of detection. If the object has internal details, they will be easier to detect under these conditions. This parameter corresponds to the crossing between the object's contrast line and the limit of detection. Due to the horizontal expansion in the X-axis (logarithmic scale), the values for this parameter are relatively uncertain, so they should be taken with care. In Table 2, MaxDM is about ×250, a value fully concordant with the experience.

MinDM/MinDA — It can be defined also, although of smaller interest, a *minimum detection magnification/angle*. It's no more than the minimum magnification required to just start seeing the object. In this work, MinDM has been considered at least *DIAM*/7.5, to avoid losses of light gathering by using too low powers — exit pupils must be smaller than 7.5 mm for all cases. For weak and small objects, MinDM is greater than *DIAM*/7.5.

An observer can design a working strategy based on the parameters we have just defined. MinDM will lead to the minimum useful magnification. OptDM will give the most suitable one in order to detect the whole object. Magnifications higher than OptDM, but near to MaxDM, will be useful to distinguish internal details. The evolution of the limit of detection, by the other hand, will show how critical the optimum found will be, letting us to know in advance how much we can depart from OptDM without important losses of performance.

In the Clark's Method the object is assumed to be *optimally* visible at the OMVA. Clark states that the OMVA curve is obtained from the Blackwell/Clark's plot, joining all those points where slope is equal to –1. Perhaps that's true, but only for a given object observed with a specific telescope under some particular sky conditions. The minima of the limits of detection are actually due to some irregular spacing existing between the Blackwell/Clark's contour lines, so there is no reason for them to be located at the points where the first derivatives are –1. This fact can be illustrated (Figure 3) through the same example we saw referring to the

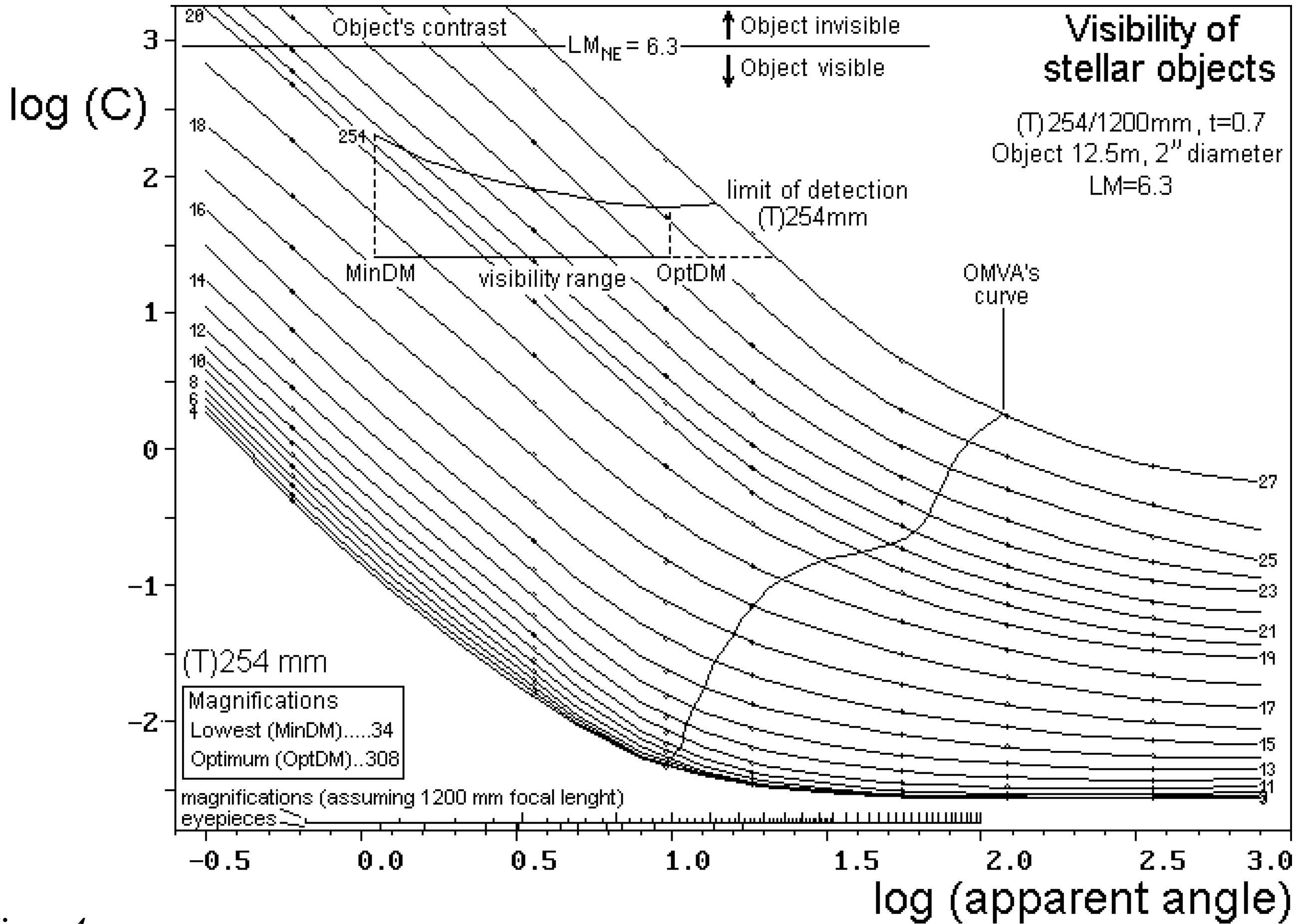


Figure 4

Detection of a stellar object (2" diameter, 12.5 visual magnitude), through a 254 mm telescope (t = 0.7), when 6.3 magnitude stars could be seen at the naked eye. With the Threshold Method, the prediction points that the object is visible at any reasonable magnification, but optimally at ×308. Clark's Method predicts an optimum magnification at ×10800. At such a high power, however, the object is not visible. The stellar method predicts a maximum visibility using ×390 (see text), which is fully congruent with the Threshold Method: ×390 is the limit of detection corresponding to a $SB_0 = 27$ *mag×arcsec*$^{-2}$*; as the object studied is not punctual, but little extended, the optimum magnification is somewhat smaller: ×308.*

Clark's Method: NGC6166. Figure 3 also shows limits of detection corresponding to several telescopes, calculated via the Threshold Method. As can be easily seen, only the curve for one telescope (approximately 125 mm, not drawn) would present its minimum located just on the OMVA curve. Consequently, although the Clark's optimum magnified visual angle (OMVA) has a defined position on the Blackwell/Clark's surface, there is no reason to think that this line will contain all the points of maximum visibility under all conditions.

What would happen with the hypothetical planetary nebula (2" sized, 12.5 magnitude, (T)254, *LM* = 6.3) introduced in the Clark's Method, if treated with the Threshold Method? We find that this nebula is visible at any reasonable magnification (see Figure 4, previous page), but there is a maximum of perceptibility at ×310 (OptDM) with a visibility of +1.2, in a point far away from the OMVA curve. Under good conditions, the object could be observed even at ×1000 of more, a circumstance that would permit to observe eventual internal details, increasing the magnification as far as the atmosphere allows us. This coincides with the predictions made via the stellar method and, what is more important, fits better with what an expert deep-sky observer would may expect.

Main Conclusions

1. With the Threshold Method, higher magnifications are not necessarily predicted for smaller telescopes to see a given object. Instead of that, smaller exit pupils are predicted for smaller telescopes. A smaller telescope must make a proportionally greater effort in order to see a given object. That means that smaller telescopes need to darken the sky's background more with the purpose of outstanding better the object. However, the smaller the exit pupil, the harder to see the object.

2. Another result that agrees with common sense: smaller exit pupils are also predicted for bad sky conditions.

3. Although optimal values of contrast are predicted, there is no reason for they to lay on the Clark's OMVA curve. Differences are specially significant for small objects.

4. The position and shape of the limits of detection depend on the sky's background surface brightness. These curves do not lay in a permanent position on the Blackwell/Clark's surface, as they are also a function of the limiting magnitude; Clark's Method underlay on the idea that they are independent.

5. The Threshold Method is fully congruent with predictions of visibility for stellar or nearly stellar objects. Modifying Equation 2, which gives the limiting magnitude, visibility can be successfully predicted even for stars under twilight conditions.

Some Considerations on Real Deep-Sky Objects

Due to the fact that real object's images are simplified as elliptical homogeneous glows, inner light-distribution variations may lead to different results to those expected. In this way, a Sc galaxy seen from above usually results harder to see than an elliptical one, with a more outstanding core. A faint, high-gradient globular cluster is easily seen, whereas a brighter, but with a flatter light profile globular cluster —as often happens with Palomar clusters—, will result harder to see. Observers take into account these factors in order to enhance their evaluations of visibility. Identically, one should consider these non-idealities in numerical approaches, to judge prediction adequately. That is, the corrections in the visibility prediction when we cross the line from an ideal object to a real deep-sky object. Parameters as the Hubble or Vacouleurs type for a galaxy, or the Shapley gradient class for a globular cluster, should be considered as correctors to adapt predictions to reality, but anyway, mean object brightness, size and background brightness are indeed the most essential features. Obviously, the more even the object, the more adequate the prediction. Although an homogeneous distribution of light has been assumed and this is not true for real deep-sky objects, observation of internal details, as have been mentioned, will be favored if we use magnifications between OptDM and MaxDM. For some cases, it can occur that even magnifications higher than MaxDM allow to see better small parts of the object.

Another important factor to be taken into account relates to the data used for predictions. Magnitude and size entries differ significantly among catalogues. What catalogue should be the best? In many of them, compiled data correspond to extrapolations in the object's light profile, which is technically correct, but not useful for our purposes, since we'll never arrive to see those regions in the object. In other catalogues data correspond to photographic measures. Crossing data helps us to validate them, discarding some sources and averaging others. I prefer to use, when available, visual observations of qualified observers as magnitude and size sources, because those values use to be closer to standard images than extrapolated measurements. Anyway, one should take into account the variability of data sources to estimate the confidence of the predictions. An optimist and fatalistic pair size-magnitude can help us to bracket our predictions and yield a more accurate estimation of our possibilities to see the object using our telescope.

How to Apply Easily the Threshold Method

For field work, I prepared a set of special graphics that reduces all the results of the Threshold Method for some usual telescopes. One only needs to get the corresponding graphic for his/her telescope, to know the magnitude, size and surface brightness of the object he/she wants to see, and that's all. No calculation required! Figure 5 is given as an example: this is a universal graphic to predict the visibility, and to calculate the OptDM, for a 254 mm reflector under *LM* greater than 5.5. Worth information can be obtained from the plot:

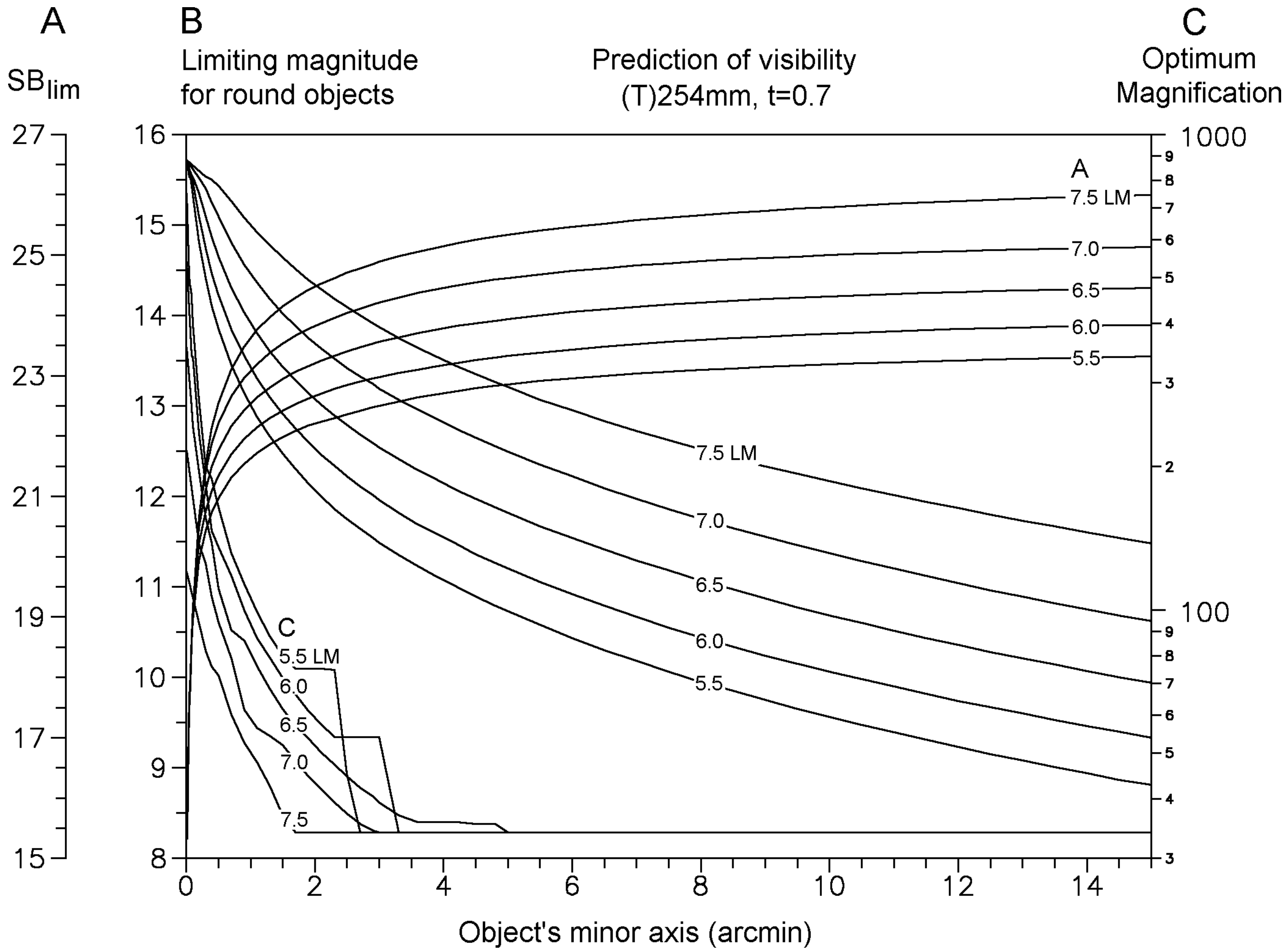
1. For a given object, the process starts locating the size (length of the smaller axis) on the graphic's X-axis. Next, we find the minimum surface brightness, SB_{lim} (A-scale, and associated "A" curves). This is done by plotting a vertical straight line from the X-axis up to intersect the "A" curve corresponding to the limiting magnitude perceptible at the naked eye, and then a second, perpendicular straight line from this point to the left, up to crossing the A-axis. If the object's surface brightness (Equation 1) is smaller than the SB_{lim} value just obtained, then the object will be visible.

2. For near-circular objects, visibility can be predicted without needing to calculate its surface brightness, by using the B-scale and associated "B" curves, in a similar procedure to that just explained: after finding the object's minor axis on the X-axis, a vertical line is plotted up to intersect the "B" curve corresponding to the current limiting magnitude at the naked eye. Then, a second line plotted from that intersection to the right, up to crossing the B-Axis, where the telescopic limiting magnitude (*TLM*) can be read. If the object's magnitude is smaller than the obtained value, it will be visible.

3. Finally, the same graphic and procedure can be repeated to find out the optimum detection magnification. Now the C-axis and associated "C" curves should be used, following an analogous procedure to that one explained above for A and B curves. However, some additional comments deserve to be pointed out, especially the reason for those odd and flat regions in the curves. Why does it happen?. The explanation is somewhat complex since several factors concur into it. The first consideration is that limits of detection can be nearly flat and the calculation of the optimum detection magnification is therefore not very accurate. The second one is that Blackwell/Clark's surface presents regions with varying accuracy level; those regions with smaller slope lead to limits of detection poorly defined, which happens for objects with noticeable dimensions. Finally, when we examine a collection of images presenting all the same surface brightness but an increasing size, the way they are perceived suddenly changes: up to a given point the object becomes too easy to see, and we should use the minimum magnification, the magnification that most concentrates the light. This produces a sudden drop. Immediately below that magnification, the limit of detection is flat and no improvement is achieve increasing the eyepiece power.

Figure 5

An example of universal graphic constructed for a 254 mm telescope (t = 0.6), that can be used for predicting the visibility of astronomical objects as well as to evaluate their optimum magnification.



The author would be very interested in knowing the opinions of those who decide to use the Threshold Method. It is not too hard to write a computer program that can perform all the calculation process in a quick and efficient way. For instance, all the figures presented in this work have been generated through a computer program that was written by the author, which can operate with the Clark's Method and the Threshold Method as well. At least according to my experience in practical astronomy, results obtained until now with the Threshold Method seem to be correct and reliable.

Valencia, February 13th 2000

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