LOSSLESS CODING OF HYPERSPECTRAL IMAGES WITH PRINCIPAL POLYNOMIAL ANALYSIS

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ABSTRACT

The transform in image coding aims to remove redundancy among data coefficients so that they can be independently coded, and to capture most of the image information in few coefficients. While the second goal ensures that discarding coefficients will not lead to large errors, the first goal ensures that simple (point-wise) coding schemes can be applied to the retained coefficients with optimal results. Principal Component Analysis (PCA) provides the best independence and data compaction for Gaussian sources. Yet, non-linear generalizations of PCA may provide better performance for more realistic non-Gaussian sources. Principal Polynomial Analysis (PPA) generalizes PCA by removing the non-linear relations among components using regression, and was analytically proved to perform better than PCA in dimensionality reduction. We explore here the suitability of reversible PPA for lossless compression of hyperspectral images. We found that reversible PPA performs worse than PCA due to the high impact of the rounding operation errors and to the amount of side information. We then propose two generalizations: Backwards PPA, where polynomial estimations are performed in reverse order, and Double-Sided PPA, where more than a single dimension is used in the predictions. Both yield better coding performance than canonical PPA and are comparable to PCA.

Index Terms— Principal Component Analysis, Principal Polynomial Analysis, hyperspectral image coding, decorrelation, entropy

1. INTRODUCTION

In the last years, a number of techniques have been proposed to exploit spectral and spatial redundancy to encode hyperspectral images. The popular approaches usually combine a 1-D spectral transform followed by a 2-D spatial transform. The spectral decorrelation for multicomponent images has proven to be crucial for compression due to the large amount of inter-component correlation/redundancy, with PCA/KLT [1] transform and its extensions [2] being widely applied due to its high coding efficiency and matrix invertibility.

Recently, several non-linear generalizations of PCA have been proposed [3–5]. These generalizations go beyond linearity and exploit both linear and non-linear correlation between data. Also, their kernel versions [6] have become a powerful tool to extract non-linear features. However, the reconstruction problem has so far restrained their application in compression schemes [7]. Invertibility of the transform is requested for lossless compression, as it leads to perfect reconstruction and provides better understanding of the transform. For lossless coding applications, it is therefore necessary to consider non-linear generalizations of PCA that still satisfy: (1) invertibility to achieve perfect reconstruction, and (2) variance minimization along the considered directions to yield competitive coding performance.

In this paper, we analyze the lossless coding efficiency of an invertible non-linear generalization of PCA, the Principal Polynomial Analysis (PPA) [8, 9], originally proposed for dimensionality reduction. PPA is a deflationary algorithm based on drawing a sequence of Principal Curves that address one dimension at a time [10]. These Principal Curves are analytic and each step in the sequence consists of two basic operations. The first operation is based on PCA, i.e., it is a projection onto the orthogonal direction that maximizes the variance. The second operation estimates and removes the conditional mean of the data in the orthogonal subspace. The estimation is carried out by using a polynomial regression. This provides a better estimation of the conditional mean than the straight line, given the eventual non-linear relation between features.

As opposed to *lossy* compression, direct application of PPA to *lossless* compression may be hampered by the impact of the side information and the necessary lifting scheme applied in each step for integer mapping. In this paper, we specifically analyze these issues: how to handle PPA side information required for the reconstruction, how to reduce the impact of integer mapping error and how to exploit the energy compaction property. Two generalizations of PPA are proposed. The first one, Backwards PPA, works as the original algorithm but in reverse order, i.e., starting from the last components of a PCA, so that the Principal Component, highly responsible for the bitrate budget, is better handled. The second one, Double-Sided PPA, can be seen as a further generalization, a sequential algorithm that simultaneously considers the first and last components of a PCA, uses more dimensions in the prediction, proceeds inwards, and performs less iterations to alleviate the integer mapping penalization.

The paper is organized as follows. Section 2 reviews the elements of PPA transform. Section 3 addresses fundamental issues for the use of PPA in coding: its inverse, the side information, and the proposed generalizations for lossless coding. Section 4 illustrates the application of PPA to hyperspectral images and draws the integer reversible implementation. In section 5 we show the experimental settings and results. Finally, section 6 concludes.

2. PRINCIPAL POLYNOMIAL ANALYSIS

Let $\mathbf{X}_0 \in \mathbb{R}^{d\times n}$ be the input data matrix, where rows represent components/dimensions and columns different realizations (or samples). Note that PCA can be seen as a deflationary method (or a sequence of $d-1$ elementary transforms $\mathbf{R}_p$) that maps $\mathbf{X}_0$ to a response domain $\mathcal{R} \subseteq \mathbb{R}^{d\times n}$. Each elementary transform describes a single curvilin-

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ear dimension of the input by computing one single component of the response:

$$\begin{pmatrix} \\ \mathbf{X}_0 \\ \\ \end{pmatrix} \xrightarrow{R_1} \begin{pmatrix} \boldsymbol{\alpha}_1 \\ \\ \mathbf{X}_1 \end{pmatrix} \xrightarrow{R_2} \begin{pmatrix} \boldsymbol{\alpha}_1 \\ \boldsymbol{\alpha}_2 \\ \\ \mathbf{X}_2 \end{pmatrix} \cdots \xrightarrow{R_{d-1}} \begin{pmatrix} \boldsymbol{\alpha}_1 \\ \boldsymbol{\alpha}_2 \\ \vdots \\ \boldsymbol{\alpha}_{d-1} \\ \mathbf{X}_{d-1} \end{pmatrix}$$

In each step p of the sequence, two basic operations are applied:

$$\boldsymbol{\alpha}_p = \mathbf{e}_p^\top \mathbf{X}_{p-1} \quad (1)$$

$$\mathbf{X}_p^{\text{PCA}} = \mathbf{E}_p^\top \mathbf{X}_{p-1} \quad (2)$$

where (1) is the projection of the data coming from the previous step, $\mathbf{X}_{p-1}$, onto the unit norm vector $\mathbf{e}_p \in \mathbb{R}^{d-p\times 1}$ that maximizes the variance of the projected data and (2) is the projection onto the orthonormal subspace $\mathbf{E}_p \in \mathbb{R}^{(d-p+1)\times(d-p)}$.

PPA follows the same deflationary scheme but it improves the second operation by predicting and removing the conditional mean $\hat{\mathbf{m}}_p$:

$$\boldsymbol{\alpha}_p = \mathbf{e}_p^\top \mathbf{X}_{p-1} \quad (3)$$

$$\mathbf{X}_p^{\text{PPA}} = \mathbf{E}_p^\top \mathbf{X}_{p-1} - \hat{\mathbf{m}}_p \quad (4)$$

The conditional mean is estimated by using polynomial regression to fit the non-linear relationship between the first component $\boldsymbol{\alpha}_p$ and the remaining components $\mathbf{X}_p$ of the projected data coming from the first operation (PCA). In matrix notation, the model can be written as $\hat{\mathbf{m}}_p = \mathbf{W}_p\mathbf{V}_p$:

$$\hat{\mathbf{m}}_p = \begin{pmatrix} w_p^{11} & \dots & w_p^{1(\gamma+1)} \\ \vdots & \vdots & \vdots \\ w_p^{(d-p)1} & \dots & w_p^{(d-p)(\gamma+1)} \end{pmatrix} \cdot \begin{pmatrix} 1 \\ \vdots \\ \boldsymbol{\alpha}_p^\gamma \end{pmatrix}$$

where $\mathbf{V}_p$ is the Vandermonde matrix of order γ and $\mathbf{W}_p$ is the matrix of polynomial coefficients. The least squares solution for the coefficients is $\mathbf{W}_p = \mathbf{E}_p^\top \mathbf{X}_{p-1}\mathbf{V}_p^\dagger$, where $\mathbf{V}_p^\dagger$ stands for the pseudo-inverse of $\mathbf{V}_p$.

3. SIDE INFORMATION AND CODING-ORIENTED GENERALIZATIONS

3.1. Inverse and side information

The PPA transform is invertible, thus leading to perfect reconstruction. The inverse transform is, like the forward transform, also a sequence of two elementary inverse operations. The original data $\mathbf{X_0}$ is obtained by applying the following transform recursively on the given transformed data:

$$\mathbf{X}_{p-1} = \begin{pmatrix} \mathbf{e}_p | \mathbf{E}_p \end{pmatrix} \begin{pmatrix} \boldsymbol{\alpha}_p \\ \\ \mathbf{X}_p + \mathbf{W}_p\mathbf{V}_p \end{pmatrix}, \quad p = d-1, \dots, 1$$

In practice, the polynomial coefficients, $\mathbf{W}_p$, and the vector $\mathbf{e}_p$ are necessary side information since they are required at each step of the inversion. On the contrary, there is no need to store $\mathbf{V}_p$ and $\mathbf{E}_p$. On the one hand, $\mathbf{V}_p$ is generated from the data. On the other hand, any method to compute an orthogonal complement from $\mathbf{e}_p$ is fine to obtain $\mathbf{E}_p$ since the reconstruction error does not depend on the selected basis [8, 9].

According to this, the number of elements in the side information corresponding to each elementary transform $\mathbf{R}_p$ is: $(\gamma + 1) \times (d - p)$ (from the $\mathbf{W}_p$'s) plus $d-p$ (from the $\mathbf{e}_p$'s). Taking into account that the size of side information for PCA is $\frac{d^2+d}{2}$, PPA side information is $2d^2-d-1$ when using order 2 for the polynomial regression.

3.2. Backwards and Double-sided generalizations of PPA

When using dimensionality reduction techniques, the components/coefficients that retain most of the signal energy may have a wide dynamic range. Each PPA step is designed to reduce the dynamic range of the residual signal (prediction error after nonlinear regression). The particular choice of which dimensions are visited in the deflationary scheme will determine the resulting dynamic ranges. While the canonical formulation of PPA follows the *largest-variance-first* criterion (as in PCA), this particular choice may not be appropriate for signal coding.

In order to obtain coefficients with smaller dynamic range, here we propose two variations of the canonical PPA: Backwards PPA and Double-Sided PPA.

In Backwards PPA, the $\mathbf{e}_p$ vector at each step is the one that minimizes the variance of the projected data. This implies retaining the lowest dynamic range projections to reduce the variance of the highest dynamic range ones. The sequence of the Backwards PPA is as follows:

$$\begin{pmatrix} \\ \mathbf{X}_0 \\ \\ \end{pmatrix} \xrightarrow{R_1} \begin{pmatrix} \mathbf{X}_1 \\ \\ \boldsymbol{\alpha}_d \end{pmatrix} \xrightarrow{R_2} \begin{pmatrix} \mathbf{X}_2 \\ \\ \boldsymbol{\alpha}_{d-1} \\ \boldsymbol{\alpha}_d \end{pmatrix} \cdots \xrightarrow{R_{d-1}} \begin{pmatrix} \mathbf{X}_{d-1} \\ \boldsymbol{\alpha}_2 \\ \boldsymbol{\alpha}_3 \\ \vdots \\ \boldsymbol{\alpha}_d \end{pmatrix}$$

The side information for the Backwards generalization is essentially the same as in the canonical PPA.

The Double-Sided variation of PPA uses more than one component in each prediction. At each step it uses the largest variance component as in the canonical PPA plus k components from the lowest dynamic range end. This scheme reduces both complexity and side information, since the number of steps needed is $N{=}\lfloor \frac{d}{k+1} \rfloor$ instead of $N{=}(d-1)$. The sequence of the Double-Sided PPA is:

$$\begin{pmatrix} \\ X_0 \\ \\ \end{pmatrix} \xrightarrow{R_1} \begin{pmatrix} \boldsymbol{\alpha}_1 \\ X_1 \\ \boldsymbol{\alpha}_{d-k+1} \\ \vdots \\ \boldsymbol{\alpha}_{d-1} \end{pmatrix} \cdots \xrightarrow{R_i} \begin{pmatrix} \boldsymbol{\alpha}_1 \\ \boldsymbol{\alpha}_2 \\ \vdots \\ X_{i-1} \\ \vdots \\ \boldsymbol{\alpha}_{d-1} \end{pmatrix}$$

The side information for the Double-Sided generalization is $(\gamma(k+1)+1)\sum_{i=0}^{N}(d-i(k+1))$ due to $\mathbf{W}'_p s$, and $k(\sum_{i=0}^{N-1}(d-i(k+1))$ because of $(\mathbf{e}'_p s)$, where $N = \lfloor \frac{d}{k+1} \rfloor$ is the number of steps.

4. PPA FOR HYPERSPECTRAL IMAGE CODING

Hyperspectral imaging leads to 3D spectral-spatial arrays. Even though the d-dimensional input to PPA may come from arbitrary arrangements of the hyperspectral arrays, here we will illustrate the use of this transform in a *spectral-first pipeline* (Fig. 1). Current coding standards include PCA in the spectral stage [1, 2]. In our experiments we will stick to this standard scheme and will replace PCA by PPA.

4.1. Reversible integer PPA transform

For lossless compression, it is mandatory to consider reversible implementations of the redundancy reduction transforms that produce integer outputs as close as possible to the real coefficients. Hao and Shi [11] proposed a reversible integer KLT that maps integers to integers based on a $\mathbf{PLUS}$ factorization of the transform matrix $\mathbf{A}$. $\mathbf{PLUS}$ is a product of unit Triangular Elementary Reversible Matrices (TERMs), where $\mathbf{P}$ is a permutation matrix, $\mathbf{L}$ and $\mathbf{S}$ are lower TERMs and $\mathbf{U}$ is upper TERM. This factorization is possible, if and only if, $|det(\mathbf{A})| = 1$ [12].

Galli and Salzo [13] improved this factorization by proposing a quasi-complete pivoting. This improvement minimizes the elements' magnitudes of the matrices $\mathbf{L}$ and $\mathbf{U}$, therefore, it reduces the error between the integer implementation and the original transform, which could affect the energy compaction capability.

Going back to the PPA transform and given that it is based on two main operations (PCA projection and conditional mean removal), we propose an integer reversible PPA transform for lossless compression based on two integer reversible operations:

1. Integer reversible PCA/KLT (RPCA/RKLT) based on $\mathbf{PLUS}$ factorization using quasi-complete pivoting.
2. Removal of the quantized conditional mean:

$$\mathbf{X}_p = \mathbf{X}_{p-1} - Q(\hat{\mathbf{m}}_p)$$

The needed conditional mean quantization is performed through a simple rounding operation, $Q(\cdot)$, although any other quantization operation could be applied.

5. EXPERIMENTAL SETTINGS AND RESULTS

In this section, we evaluate the proposed reversible integer transforms (PPA, Backwards PPA and Double-Sided PPA) on a set of 5 radiance AVIRIS images: Yellowstone scene 0, Cuprite scene 1, Jasper Ridge scene 2, Moffet Field scene 1 and Low Altitude scene 1, available at *http://aviris.jpl.nasa.gov/data/index.html*. All the images were cropped to 512×512 spatial resolution and 224 components and have a bit-depth of 16 bits per pixel per component (bpppc).

To account for reproducibility, basic Matlab source code for PPA is available online [14], and the proposed generalizations can be found at [15]. The coding system pipeline is shown in Fig. 1 and explains the compression process, where the proposed transforms based on PPA are applied in the spectral domain followed by a DWT 2-D transform 5/3 (5 levels) in the spatial domain and finally a lossless compression using the standard JPEG2000 with the KAKADU software v6.0.

Tables 1–5 provide the lossless coding performance for the proposed transforms as well as for PCA. We report the needed bitrate to encode the images and the size of the requested side information (SI), also measured in bpppc. For Double-Sided PPA with k=80, the full matrix ($\mathbf{e}_p\mathbf{E}_p$) has been used as side information, while for $k < 80$ only the unit norm vector $\mathbf{e}_p$ was used. Coefficients for

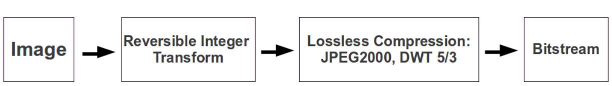

Fig. 1: Coding pipeline: 1D transform in the wavelength domain followed by JPEG2000 standard.

the side information have used a precision of 32 bits and have been compressed with LZMA. Order 2 has been used for the polynomial regression.

Table 1: Lossless coding results for Yellowstone scene 00 (bpppc).

Method	Image	SI $\mathbf{W}_p$'s	SI $\mathbf{e}_p$'s	Total
PCA	3.99	0	0.025	4.02
PPA	5.01	0.038	0.012	5.06
Backwards	4.66	0.037	0.012	4.71
D-Sided $k = 1$	4.57	0.032	0.006	4.61
D-Sided $k = 7$	4.33	0.026	0.001	4.42
D-Sided $k = 80$	3.97	0.017	0.036	4.02

Table 2: Lossless coding results for Cuprite scene 01 (bpppc).

Method	Image	SI $\mathbf{W}_p$'s	SI $\mathbf{e}_p$'s	Total
PCA	4.99	0	0.025	5.02
PPA	5.61	0.038	0.012	5.66
Backwards	5.17	0.03	0.012	5.22
D-Sided $k = 1$	5.19	0.032	0.006	5.23
D-Sided $k = 7$	5.09	0.026	0.001	5.12
D-Sided $k = 80$	4.93	0.017	0.035	4.98

Table 3: Lossless coding results for Jasper Ridge scene 02 (bpppc).

Method	Image	SI $\mathbf{W}_p$'s	SI $\mathbf{e}_p$'s	Total
PCA	4.99	0	0.025	5.02
PPA	5.61	0.038	0.012	5.66
Backwards	5.17	0.037	0.012	5.22
D-Sided $k = 1$	5.19	0.032	0.006	5.23
D-Sided $k = 7$	5.11	0.028	0.003	5.14
D-Sided $k = 80$	4.91	0.017	0.035	4.96

Table 4: Lossless coding results for Moffet Field scene 01 (bpppc).

Method	Image	SI $\mathbf{W}_p$'s	SI $\mathbf{e}_p$'s	Total
PCA	5.01	0	0.025	5.03
PPA	5.72	0.038	0.012	5.77
Backwards	5.27	0.038	0.012	5.77
D-Sided $k = 1$	5.36	0.031	0.006	5.32
D-Sided $k = 7$	5.25	0.029	0.001	5.28
D-Sided $k = 80$	5.01	0.017	0.036	5.07

Table 5: Lossless coding results for Low Altitude scene 01 (bpppc).

Method (bpppc)	Image	SI $\mathbf{W}_p$'s	SI $\mathbf{e}_p$'s	Total
PCA	5.33	0	0.025	5.35
PPA	5.91	0.038	0.012	5.96
Backwards	5.47	0.037	0.012	5.52
D-Sided$k$=1	5.58	0.031	0.006	5.62
D-Sided$k$=7	5.51	0.029	0.001	5.54
D-Sided$k$=80	5.31	0.017	0.035	5.36

The reported results indicate that a reversible implementation of PPA does not yield better coding performance than PCA, with a bitrate penalization of at most 0.7 bpppc. This suggests that the sequential rounding error due to the integer mapping on the one hand,

and the conditional mean quantization on the other hand penalize the compression performance. In addition, PPA asks for a larger side information than PCA. Our two PPA-based alternatives are aimed at alleviating these issues.

On the one hand, note that the Backwards PPA is subject to the same rounding process and conditional mean quantization as PPA, and it has the same complexity and side information too; however, it is usually able to improve the results of PPA by about 0.4 bpppc, since in this case the first principal component also has its conditional mean subtracted, decreasing its dynamic range and benefiting the final performance. On the other hand, the Double-Sided PPA outperforms both canonical PPA and Backwards PPA, especially for higher values of parameter k (the number of components from the right-most end that are transformed together with the first component in each iteration). First, the impact of rounding is reduced due to the lower number of iterations. Besides, the predictive power is higher since it uses more than one component ($k+1$), giving rise to a better estimation of the conditional mean. In addition, the computational complexity and side information are smaller in Double-Sided PPA (cf. Section 3.2). According to the reported results, the coding performance of Double-Sided PPA with $k = 80$ is comparable to that of PCA.

6. CONCLUSIONS

Lossless compression of hyperspectral images is largely benefited from a 1D spectral transform, which is traditionally performed through a wavelet or a PCA transform, the latter yielding increased performance. PPA is a non-linear generalization of PCA originally proposed in the framework of dimensionality reduction. Here we investigated the suitability of PPA for spectral redundancy reduction in hyperspectral image compression. Coding performance of integer reversible PPA is worse than that of PCA due to the rounding operations error and to the increased size of the side information. For a better adaptation of PPA to the signal at hand, we proposed two generalizations. Backwards PPA reverses the order in which the transform is applied, proceeding from the lower dynamic range hand to the higher dynamic range components. Double-Sided PPA extends canonical PPA by using several components in the regressions and performing less iterations than canonical PPA, thus reducing the penalization of the rounding error. Double-Sided PPA improves PPA performance and reaches PCA performance for lossless compression. These proposals improved the results, and could eventually be used in other nonlinear invertible schemes in the future, not necessarily based on PPA.

Several questions are now open for further research. From a theoretical point of view, it is worth analyzing Double-Sided PPA generalization and investigate whether there is any bound on the number of iterations that still provide some gain. Also, the regression used in PPA to remove the non-linear relations among components has been herein carried out with a simple second-order polynomial regression; other approaches can be devised to improve the estimation and, additionally, to decrease the size of the side information they entail. One such approach could be to apply dedicated polynomial regression in tiles or clustered data, while other *sparse* regression schemes could be explored. It does not escape our notice that PPA could be beneficial compared to PCA in lossy schemes, given that PPA better captures and preserves the relevant image features, and the good results observed previously in dimensionality reduction problems.

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