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Normalization in TOPSIS-based approaches with data of different nature: application to the ranking of mathematical videos

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Abstract

YouTube is the largest Internet video-sharing site in the world and in the last years it has become an important learning resource making educational contents accessible for hundreds of millions of people around the world, from developed and developing countries, allowing students to watch contents on demand. The utility of the performance assessment and ranking of educational videos available in You Tube goes beyond the simple control of the correctness and precision of the instructional contents. It requires considering other important didactical features as waste of time in the exposition, empathy with the user and the degree of adaptation of the contents to the educational context.

In this paper a ranking method for instructional videos will be proposed, taking into account decision criteria of different nature: precise and imprecise and a reference solution (*ideal solution*). The decision matrix describing the assessment of videos with respect to each criterion will be formed by data of diverse nature: real numbers, intervals on the real line and/or linguistic or sets of categorical variables.

Classical normalization procedures do not always take into account situations where the different nature of the data of the decision matrix could make the ranking of the alternatives quite unstable. A new normalization method will be proposed allowing us to mitigate this problem. Through this normalization procedure, the nature of the transformed normalized data will reflect the similarity of each alternative with the reference solution becoming thus, the decision matrix of homogeneous nature.

Keywords: MCDM, TOPSIS, normalization, reference solution, mathematical videos, educational performance.

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1. Introduction

The Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) (Hwang and Yoon 1981) ranks decision alternatives based on a set of decision criteria choosing firstly the alternatives that simultaneously have the shortest distance from the positive ideal solution (PIS) and the farther distance from the negative-ideal solution (NIS). The positive ideal solution maximizes criteria of the type "the more, the better" and minimizes criteria of the type "the less, the better", whereas the negative ideal solution maximizes "the more, the better" criteria and minimizes "the more, the better" criteria.

Main decisions to be made in any TOPSIS-based approach are related to the type of the data, the selected normalization process, the considered ideal solution, the decision maker preferences and the distance function used to select those alternatives closer to the positive ideal solution and simultaneously farther to the negative ideal solution. All these decisions will depend on the particular circumstances of each decision making context.

With regards to the data, in this paper we are interested in real-world problems involving human judgements characterized by fuzziness, high complexity and uncertainty. In these situations Fuzzy Sets Theory becomes a useful tool and many extensions of the classical TOPSIS approach have been proposed which take into account imprecision, uncertainty, lack of information and/or vagueness (see Dymova et al. 2013, Wang 2014; Cables et al. 2016).

One of the first steps in any ranking Multiple Criteria Decision Making (MCDM) method consists of the normalization of the criteria. However, classical normalization procedures do not always take into account situations where the different nature of the data of the decision matrix (real numbers, intervals, sets linguistic variables…) could make the ranking of the alternatives quite unstable. In order to address this situation, in this paper we will propose a normalization method which will be based on the similarity with the reference solution and will permit us to construct a new decision matrix composed of the similarity degrees of each alternative to the benchmark or ideal for each criterion. In this way, and thanks to this normalization procedure, the nature of the transformed normalized data will be homogeneous.

The transformed normalized decision making matrix will be then used in a new TOPSIS-based approach, Ideal Similarity TOPSIS (IS-TOPSIS) which constitutes a generalization of the method proposed by Cables et al. (2016), the Reference Ideal Method based on TOPSIS (RIM-TOPSIS). Our method also relies on the idea that the ideal solution, instead of being an optimal solution, can take any value between the minimum and maximum values of the range of the criteria. However, the method proposed by Cables et al. (2016) only allows the consideration of decision matrices composed of real numbers and our method will permit any king of data in this matrix (real numbers, intervals on the real line, linguistic variables...).

In order to illustrate the suitability of the proposed methodological approach to the resolution of real decision making problems we will apply IS-TOPSIS to the ranking of mathematical educational videos in You Tube. YouTube could well be considered as one of the most important educational tools of our time. This free-video platform have more than 1.5 billion monthly visits from more than 61 countries around the world with more than 100 hours of content in terms of videos being published every minute (https://www.youtube.com/intl/en-GB/yt/about/press/). Educators have recognized this phenomenon and they have moved to a new educational model where free-online

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educational contents are created and made accessible for hundreds of millions of people around the world, from developed and developing countries, allowing students to watch those educational contents on demand. An increasing number of universities recognize the transformative way video can impact on teaching and learning and promote its use among educators and students with initiatives as the *flipped classroom model* which allows students to digest lecture content at their pace and explore content more deeply during class time. Many educators, especially at Universities, are worried about the hard constraint imposed by time in the educational processes taking place in the classrooms (see Hughes 2012; Lage et al. 2000; Zappe et al. 2009; Sankey and Hunt 2013). In this situation, the recording of video contents available online for the students can be used to engage them before the class (and at any time), letting then more time during the classes for deeper explanations or activities allowing a better exploration of the content. It can also give opportunities for students to review, discuss, and investigate during and after the class.

In this situation, and with more than 10 million videos tagged as educational in YouTube, the evaluation and ranking of educational videos in terms of their quality is crucial. However, in this context, quality is a multidimensional concept which needs to consider important dimensions related not only to the quality of the contents from an educational point of view but to other aspects related to the quality of the instructional process, the quality of the production of the videos or the authority of the authors of the videos.

In the following section we will review the main characteristics of the classical TOPSIS approach highlighting the main questions that need to be solved regarding the steps of the process. In section 3, we address the question related to normalization processes in presence of high heterogeneity of the criteria or non-compensatory nature. Section 4 integrates the proposed normalizing approach into a TOPSIS-based method, the ideal similarity TOPSIS, IS-TOPSIS. Finally, the advantages of the proposed approach are illustrated with a real case where mathematical videos are ranked based on their quality from a broad educational perspective.

2. The classical TOPSIS

In this section we will briefly present the main characteristics of the classical TOPSIS approach. TOPSIS makes full use of the attribute information, provides a cardinal ranking of alternatives, and does not require the attribute preferences to be independent (Chen and Hwang 1992; Yoon and Hwang 1995). The main steps of this technique can be summarized as follows:

STEP 1. Determine the decision matrix X, where the number of alternatives is m and the number of criteria is n, $X = \left(x_{ij}\right)_{mxn}$, being x_{ij} real numbers.

STEP 2. Construct the normalized decision matrix,

$$R = \left(r_{ij}\right)_{mxn}$$

STEP 3. Determine the weighted normalized decision matrix. Given, $w_j \in [0,1]$ with $w_1+w_2+\ldots+w_n=1$, we calculate

$$v_{ij} = w_j r_{ij}, \quad i = 1,\ldots,m, \quad j = 1,\ldots,n.$$

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STEP 4. Determine the positive ideal A^{+} (PIS) and negative ideal A^{-} solutions (NIS),

$$A^{+}=\left\{ v_1^{+},\ldots,v_n^{+} \right\}=\left\{ \left(\max_i v_{ij}, j\in F^{+}\right)\left(\min_i v_{ij}, j\in F^{-}\right) \right\} \quad i=1,2,\ldots,m$$

$$A^{-}=\left\{ v_1^{-},\ldots,v_n^{-} \right\}=\left\{ \left(\min_i v_{ij}, j\in F^{+}\right)\left(\max_i v_{ij}, j\in F^{-}\right) \right\} \quad i=1,2,\ldots,m$$

where F^{+} is associated with "the more, the better" criteria and F^{-} is associated with "the less, the better" criteria.

STEP 5. Calculate the separation measures with respect to the PIS and NIS,

$$S_i^{+}=\left(\sum_{j=1}^{n}\left(v_{ij}-v_j^{+}\right)^2\right)^{1/2}, \quad S_i^{-}=\left(\sum_{j=1}^{n}\left(v_{ij}-v_j^{-}\right)^2\right)^{1/2}, \quad 1\le i\le m.$$

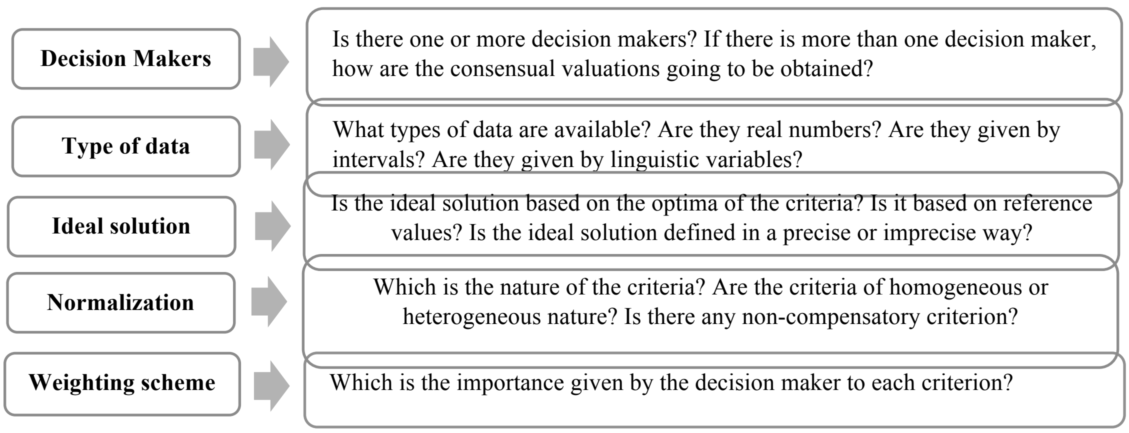
STEP 6. Calculate the relative proximity to the ideal solution using the relative index

$$R_i=\frac{S_i^{-}}{S_i^{+}+S_i^{-}}, \quad 1\le i\le m.$$

STEP 7. Rank the best alternatives according to R_i in descending order.

Figure 1 summarizes the key questions to be answered in any TOPSIS-based approach. As we can observe the main decisions derived from these questions will mainly be related to the number of decision makers, the type of the data, the type of ideal solution, the decision context and the decision maker preferences, all these factors being susceptible of change depending on the circumstances.

Figure 1. Key questions in any TOPSIS-based approach



Source: own elaboration.

A large number of approaches can be found in the literature dealing with the previous questions and giving rise to different TOPSIS-based approaches (see Behzadian et al. 2012; Zyoud and Fuchs-Hanusch 2017). In this paper, we focus on the discussion about the normalization process and the ideal determination, questions which in our approach are related. In the following sections we will discuss how these questions have been addressed and we will propose a ranking method based on TOPSIS, the Ideal Similarity TOPSIS (IS-TOPSIS) which will allow us to normalize heterogeneous criteria which in certain cases will be of non-compensatory nature. As we will show, in this case a

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suitable approach is to try to normalize taking into account the degree of similarity with the ideal. In the next section we will present our normalization proposal which will be integrated in the IS-TOPSIS approach in Section 4.

3. Normalization process

One of the key questions in the classical TOPSIS approach it that related with the normalization of criteria of different nature (see for example, Pavlicic 2001). Several works can be found in the literature dealing with this problem (see Table 1). However, the discussion about this topic mainly focuses on the avoidance of a common TOPSIS problem, the rank reversal (see García-Cascales and Lamata 2012) and does not take into account an important problem in many multiple criteria decision making problems: heterogeneity of the criteria. In what follows we propose an alternative to traditional normalization in TOPSIS that takes into account the degree of similarity of each criterion with its ideal. Aggregation is then addressed for criteria that have been transformed into homogeneous or compensatory as they now reflect the degree of similarity with the ideal valuation.

Table 1. Commonly used normalization methods in TOPSIS

Normalization Method	*Formulae*	*Sample of References*
(i) Vector Normalization	$r_{ij}=\frac{x_{ij}}{\lVert x_j\rVert}=\frac{x_{ij}}{\sqrt{\sum_{k=1}^{\#X}x_{kj}^2}}$ if $j\in F^+$ $r_{ij}=\frac{1/x_{ij}}{\lVert 1/x_j\rVert}=\frac{1/x_{ij}}{\sqrt{\sum_{k=1}^{\#X}(1/x_{kj})^2}}$ if $j\in F^-$	Amiri et al. (2009), Isiklar and Büyüközkan (2007), Jahanshahloo et al. (2006), Li (2010), Peng et al. (2011), Secme et al. (2009), Shyur (2006), Wu et al. (2009), Wu et al. (2010), Yu et al. (2011), Zandi and Tavana (2011), Zhang et al. (2010)
(ii) Linear Scale Transformation (Max-Min)	$r_{ij}=\frac{x_{ij}-\min_{k=1,\dots,\#X}x_{kj}}{\max_{k=1,\dots,\#X}x_{kj}-\min_{k=1,\dots,\#X}x_{kj}}$ if $j\in F^+$ $r_{ij}=\frac{\max_{k=1,\dots,\#X}x_{kj}-x_{ij}}{\max_{k=1,\dots,\#X}x_{kj}-\min_{k=1,\dots,\#X}x_{kj}}$ if $j\in F^-$	Lin et al. (2010), Bai et al. (2014)
(iii) Linear Scale Transformation (Max)	$r_{ij}=\frac{x_{ij}}{\max_{k=1,\dots,\#X}x_{kj}}$ if $j\in F^+$ $r_{ij}=1-\frac{x_{ij}}{\max_{k=1,\dots,\#X}x_{kj}}$ if $j\in F^-$	Ertugrul and Karakasglu (2008), KarimiAzari et al. (2011), Sun (2010), Sun and Lin (2009), Vahdani et al. (2012)
(iv) Linear Scale Transformation (Sum)	$r_{ij}=\frac{x_{ij}}{\sum_{k=1}^{\#X}x_{kj}}$ if $j\in F^+$ $r_{ij}=\frac{1/x_{ij}}{\sum_{k=1}^{\#X}(1/x_{kj})}$ if $j\in F^-$	Huang and Peng (2012)

Source: Ouenniche et al. (2017).

Given a criterion c_j we consider its variation range as $[A_j,B_j]=[\min_i x_{ij},\max_i x_{ij}]$. We can interpret that any of the normalizations in Table 1 reflect, with a value in [0,1], the similarity of x_{ij} with B_j (when the objective is to maximize the criterion) or with A_j (when the objective is to minimize the criterion). This idea can be generalized when an ideal exists for criterion c_j given by an interval $[a_j,b_j]\subseteq[A_j,B_j]$. With this aim, we will need to introduce the concept of Fuzzy Set.

The main idea of Fuzzy Set Theory is quite intuitive and natural: instead of determining the exact boundaries as in an ordinary set, a fuzzy set allows for no sharply defined boundaries because of the generalization of a characteristic function to a membership

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function. By letting X denote a universal set, a fuzzy set $\tilde{A}$ of X can be characterized as a set of ordered pairs of element x and the grade of membership of x in $\tilde{A}$, $\mu_{\tilde{A}}(x)$, and it is often written as

$$\tilde{A} = \left\{ \left(x, \mu_{\tilde{A}}(x) \right) / x \in X \right\} \tag{1}$$

Note that the membership function is an obvious extension of the idea of a characteristic function of an ordinary set because it takes values between 0 and 1, not only 0 and 1. A membership level equal to zero means no membership, a membership value equal to one means Boolean membership and intermediate numbers reflect intermediate membership degrees (see Kaufmann and Gupta 1988 and Zimmermann 1996).

In this context, it is natural trying to establish the concept of similarity between two fuzzy sets $\tilde{A}$, $\tilde{B}$. Given the normalized distance between two sets, $d(\tilde{A},\tilde{B})$, a similarity measure between $\tilde{A}$ and can be defined as follows (Dubois and Prade 1978, Zeng and Guo 2008):

$$\mathrm{Sim}(\tilde{A},\tilde{B}) = 1 - d(\tilde{A},\tilde{B}) \in [0,1] \tag{2}$$

Given this idea of similarity, we will propose a normalizing function based on the similarity of each data with its corresponding ideal value. With this objective, we will also need to take into account the different nature of the data.

CASE 1. If the data is a real number $x_{ij} \in [A_j, B_j]$, we need to distinguish two situations: an interval expressing an ideal does not exist for the criterion (Case 1.1) or an ideal does exist expressed by an interval (Case 1.2).

Case 1.1. If an ideal solution is not available for the criterion we will use the normalization process given in (ii) in Table 1, that is,

$$N_1(x_{ij}) = \frac{x_{ij} - A_j}{B_j - A_j}, \qquad x_{ij} \in [A_j, B_j]. \tag{3}$$

Case 1.2. According to Cables et al. (2016) and Acuña-Soto et al. (2017), if the ideal of the criterion c_j is $[a_j, b_j]$, the authors propose a normalization process that can be expresssed as a measure of the similarity of x with $[a_j, b_j]$, $Sim(x_{ij}, [a_j, b_j]) = 1 - d_{\min}(x_{ij}, [a_j, b_j])$, where $d_{\min}$ is the distance

$$d_{\min}\left(x_{ij}, \left[a_j, b_j \right] \right) = \min\left(\left| x_{ij} - a_j \right|, \left| x_{ij} - b_j \right| \right). \tag{4}$$

Thus, in the Case 1.2, the normalization N_2 given by

$$N_2(x_{ij}) = \begin{cases} \dfrac{x_{ij} - A_j}{a_j - A_j}, & A_j \le x_{ij} < a_j, \\ 1, & a_j \le x_{ij} \le b_j, \\ \dfrac{B_j - x_{ij}}{B_j - b_j}, & b_j < x_{ij} \le B_j, \end{cases} \tag{5}$$

is the more adequate for the problem. In Figure 2, we show the difference between N_1 and N_2. On one hand, it can be proven that the existence of an ideal solution can reorder

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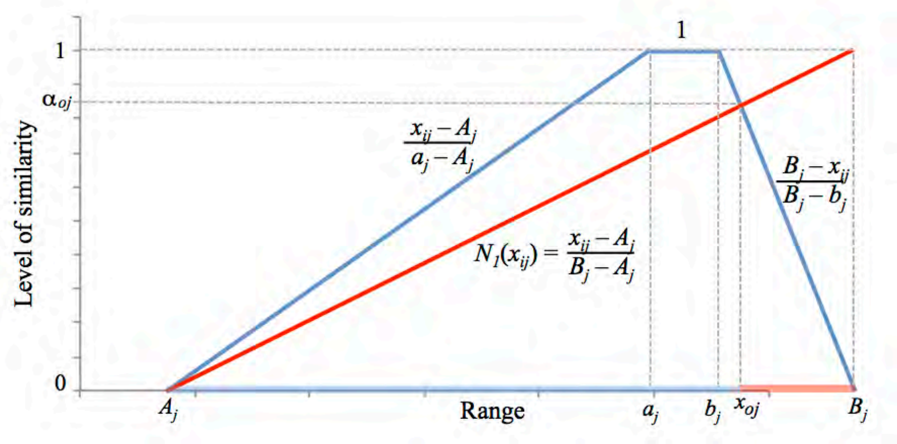
the data: with normalization N_1, if $x_{ij}<y_{ij}$, $N_1(x_{ij})< N_2(y_{ij})$. However, this does not happen with N_2. For example, the normalized value of $x_{ij} = B_j$ is $N_2(B_j) = 0$, although B_j is the greatest value that x_{ij} can take.

Figure 2. Comparison between normalizations N_1 and N_2



Source: own elaboration.

On the other hand, in Theorem 1 it is proven that a value x_{0j} exists such that for any $x_{ij} \in [A_j, x_{0j}[$, it is verified that $N_1(x_{ij})>N_2(x_{ij})$, while the contrary happens if $x_{ij} \notin [A_j, x_{0j}[$.

Theorem 1. *With the previous notation, given* $x_{0j} = \dfrac{B_j^2 - A_j b_j}{2B_j - A_j - b_j}$, *it is verified:*

(i) $x_{0j} \in [b_j, B_j]$ *and* $N_1(x_{0j}) = N_2(x_{0j})$.

(ii) $N_2(x_{ij}) > N_1(x_{ij})$ *for any* $x_{ij} \in \left[A_j, x_{0j}\right[$.

(iii) $N_2(x_{ij}) < N_1(x_{ij})$ *for any* $x_{ij} \in \left]x_{0j}, B_j\right]$.

PROOF. By construction, B_j-A_j and B_j-b_j are non-negative numbers, then (i) is verified:

$$\frac{B_j^2 - A_j b_j}{2B_j - A_j - b_j} \geq b_j \leftrightarrow B_j^2 - A_j b_j \geq 2B_j b_j - A_j b_j - b_j^2 \leftrightarrow (B_j - b_j)^2 \geq 0.$$

$$\frac{B_j^2 - A_j b_j}{2B_j - A_j - b_j} \leq B_j \leftrightarrow B_j^2 - A_j b_j \leq 2B_j^2 - A_j B_j - b_j B_j \leftrightarrow 0 \leq (B_j - A_j)(B_j - b_j).$$

In addition, $N_1(x_{0j}) = \dfrac{B_j^2 - A_j b_j}{2B_j - A_j - b_j} = N_2(x_{0j})$.

To prove (ii) and (iii), it is enough to take into account that

$$\frac{B_j - x_{ij}}{B_j - b_j} > \frac{x_{ij} - A_j}{B_j - A_j} \leftrightarrow x_{ij} < \frac{B_j^2 - A_j b_j}{2B_j - A_j - b_j}.$$

□

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CASE 2: Let us consider a valuation of an alternative with respect to a given criterion j described by an interval $[x_{ij}^L, x_{ij}^R] \subseteq [A_j, B_j]$. We will propose a normalization process based on the similarity between $[x_{ij}^L, x_{ij}^R]$ and $[a_j, b_j]$, i.e.

$$\mathrm{Sim}\left([x_{ij}^L, x_{ij}^R],[a_j,b_j]\right) = d_H\left([x_{ij}^L, x_{ij}^R],[a_j,b_j]\right), \quad \forall [x_{ij}^L, x_{ij}^R] \subseteq [A_j, B_j], \tag{6}$$

where $d_H\left([x_{ij}^L, x_{ij}^R],[a_j,b_j]\right)$ is the Hamming normalized distance (Canós et al. 2014) for the range of data $[A_j, B_j]$, that is,

$$d_H\left([x_{ij}^L, x_{ij}^R],[a_j,b_j]\right) = \frac{|x_{ij}^L - a_j| + |x_{ij}^R - b_j|}{2(B_j - A_j)}, \quad \forall [x_{ij}^L, x_{ij}^R] \subseteq [A_j, B_j]. \tag{7}$$

Then, in this case, normalization is given by

$$N_3\left([x_{ij}^L, x_{ij}^R]\right) = \begin{cases} 1, & [x_{ij}^L, x_{ij}^R] \subseteq [a_j, b_j] \\ 1 - \dfrac{|x_{ij}^L - a_j| + |x_{ij}^R - b_j|}{2(B_j - A_j)}, & [x_{ij}^L, x_{ij}^R] \not\subseteq [a_j, b_j],\ [x_{ij}^L, x_{ij}^R] \subseteq [A_j, B_j] \end{cases} \tag{8}$$

The following result shows some relations among the three types of normalization.

Theorem 2. *With the previous notation, if the ideal is* $[a_j, b_j] = [B_j, B_j]$, *then*

$$N_3\left([x_{ij}^L, x_{ij}^R]\right) = N_2\left(\frac{x_{ij}^L + x_{ij}^R}{2}\right) = N_1\left(\frac{x_{ij}^L + x_{ij}^R}{2}\right), \quad \forall [x_{ij}^L, x_{ij}^R] \subseteq [A_j, B_j].$$

PROOF.

By construction, if $[a_j, b_j] = [B_j, B_j]$ we have $N_2(x_{ij}) = N_1(x_{ij}), \ \forall x_{ij} \in [A_j, B_j]$.

Let us call $z_{ij} = \dfrac{x_{ij}^L + x_{ij}^R}{2}$. If $z_{ij} \in [a_j, b_j]$, by hypothesis $[a_j, b_j] = \{B_j\}$, therefore, $x_{ij}^L = x_{ij}^R = B_j$. Thus,

$$N_3\left([x_{ij}^L, x_{ij}^R]\right) = N_2\left(\frac{x_{ij}^L + x_{ij}^R}{2}\right) = N_1\left(\frac{x_{ij}^L + x_{ij}^R}{2}\right) = 1.$$

Let us suppose that $z_j \neq B_j$,

$$N_3\left([x_{ij}^L, x_{ij}^R]\right) = 1 - \frac{|x_{ij}^L - B_j| + |x_{ij}^R - B_j|}{2(B_j - A_j)} = 1 - \frac{B_j - x_{ij}^L + B_j - x_{ij}^R}{2(B_j - A_j)}$$

$$= 1 - \frac{B_j - \frac{x_{ij}^L + x_{ij}^R}{2}}{B_j - A_j} = \frac{\frac{x_{ij}^L + x_{ij}^R}{2} - A_j}{B_j - A_j} = N_2\left(z_j\right) = N_1\left(z_j\right). \qquad \square$$

4. Ideal Similarity TOPSIS: IS-TOPSIS

In this section we will present a new approach for the ranking of a set of discrete alternatives based on their similarity with an ideal alternative, IS-TOPSIS.

Let us consider, $[A_j, B_j]$ the range of a decision criterion j that belongs to a universe of discourse; $[a_j, b_j]$ representing the reference ideal for that criterion, with $\left[a_j, b_j\right] \subseteq \left[A_j, B_j\right]$ and $x_{ij} \in [A_j, B_j]$ the valuation of an alternative I with regards to the considered criterion j.

Taking into account the results presented in the previous section to normalize the data, we propose a TOPSIS approach, based on the concept of ideal similarity, IS-TOPSIS. It is important to notice that with the proposed normalization all the criteria are now considered to be maximized as now the objective is to reach the highest similarity degree with the ideal value. The main steps of the method are the following:

STEP 1. Define the working context: type of data, number of decision makers, criteria range, reference ideal for each criterion $[a_j, b_j]$ and weights $w_j \in [0,1]$ with $w_1 + \ldots + w_n = 1$, associated to each criterion $1 \le j \le n$.

STEP 2. Obtain the valuation matrix $X = \left(\left[x_{ij}^L, x_{ij}^R\right]\right)_{mxn}$ being $\left[x_{ij}^L, x_{ij}^R\right]$ the valuation of alternative i with regards to criterion j expressed as an interval. In case of group decision making this matrix will be a consensual matrix based on the individual valuations of the decision makers.

STEP 3. Normalize the matrix X using functions in (3), (5) or (8), $Y = \left(y_{ij}\right)_{m \times n}$.

STEP 4. Calculate the weighted normalized matrix. Given, $w_j \in [0,1]$, we calculate $Y' = \left(y'_{ij}\right)_{m \times n} = \left(w_j y_{ij}\right)_{m \times n}$.

STEP 5. Calculate the variation to the normalized reference ideal for each alternative i. Let us notice that the vector representing the reference ideal will be (1,1,…,1) and the vector representing the reference anti-ideal will be (0,0,…,0) therefore the weighted reference ideal will coincide with the weights vector (w_1, w_2,…,w_n)

$$I_i^+ = \sqrt{\sum_{j=1}^{n} \left(y'_{ij} - w_j\right)^2}, \quad I_i^- = \sqrt{\sum_{j=1}^{n} \left(y'_{ij}\right)^2}, i = 1, 2, \ldots, m$$

STEP 6. Calculate the relative index for each alternative i

$$R_i = \frac{I_i^-}{I_i^+ + I_i^-}, \qquad 1 \le i \le m.$$

STEP 7. Rank the alternatives according to R_i in descending order. The alternatives that are the top are the best solutions.

Let us notice that within this approach and similarly to what happens in the RIM-TOPSIS approach, the rank reversal problem is avoided.

In the classical TOPSIS formulation, criteria weights are the only subjective element and are directly determined by the decision maker. Many extensions of the traditional TOPSIS method have been proposed combining TOPSIS with other methods such as AHP, PROMETHEE, ELECTRE or DEA which aid the decision maker in determining criteria weights in those cases where they cannot be directly assigned (see Zavadskas et al. 2006, Zyoud and Fuchs-Hanusch 2017). Although any of the previous methods can be applied to derive the criteria weights, in this work given the hierarchical character of the decision criteria in the real problem addressed in the next section, we will apply one of the most popular methods for the derivation of priorities, the eigenvalue method (EM).

Table 2. Converting ''verbal judgements'' into ''numbers''

Verbal expressions	*Corresponding numbers*
Equal	1
Equal to moderate	2
Moderate	3
Moderate to strong	4
Strong	5
Strong to very strong	6
Very strong	7
Very strong to extreme	8
Extreme	9

[a] In Saaty (1996, 2005) the verbal expressions "equal to moderate", "moderate to strong", "strong to very strong" and "very strong to extreme" are replaced by "weak", "moderate plus", "strong plus" and "very, very strong", respectively.

The EM is used in the Analytic Hierarchy Process (AHP). The AHP proposed by Saaty is a measurement theory of intangible criteria (Saaty 1980). AHP is based on the fact that the inherent complexity of a multiple criteria decision making problem can be solved through the construction of hierarchic structures consisting of a goal, criteria and alternatives. In each hierarchical level paired comparisons are made with judgments using numerical values taken from the AHP absolute fundamental scale of 1-9 (see Table 2). These comparisons lead to dominance matrices from which ratio scales are derived in the form of principal eigenvectors. These matrices are positive and reciprocal ($a_{ij} = 1/a_{ji}$).

The method is one of the most extended multiple criteria decision making techniques (see Emrouznejad and Marra 2017 and Ishizaka and Labib 2011 for a recent review on main features of the AHP approach), adapts very well to the hierarchy of criteria proposed in the context of educational video quality assessment and also has the additional advantage of being easy to explain to the experts that have to assess the different criteria in a simple and systematic way. However, it has also received some criticisms (see Bana e Costa and Vansnick 2008 and Belton and Steward 2002). Cardinal consistency is rarely observed in practice. When matrices are inconsistent, the ratio between two priorities may differ from our direct estimations. Some solutions have been proposed to deal with this potential problem (see Ishizaka and Lusti 2004 and Wang et al. 2009). However, as indirect estimations could contain important information, we think that their influence on the final priorities must be in the context of the real application solved in this paper, taken into account.

In the following example, we highlight two of the main advantages of our method:

a) The use of intervals in the modeling of the problem increases the stability of the solution.
b) Alternatives can be ranked even in the case of an ideal solution occupying an intermediate position in the range.

Example: Let us consider 5 alternatives valued with respect to 2 criteria (columns 2 and 3 in Table 3). The first criterion is given by linguistic terms and the second one by real number between 0 and 1.

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a) Let us suppose that the most desirable option is to reach the maximum value for the two criteria.

a.1. Stability respect to modifications in the decision matrix.

We consider both criteria to have the same importance, $w_1=w_2=0.5$. Let us model criterion C1 using a numerical scale as in (9) (see column 4 in Table 3) and using intervals (column 7 in Table 3).

Table 3. Normalization with valuations of different nature

	Original valuations		Numerical valuations			Interval valuations		
Alternatives	C_1	C_2	C_1	C_2	Similarity	C_1	C_2	Similarity
A_1	Very Good	0.115	5	0.115	0.53795	[4.5, 5.0]	0.115	0.66928
A_2	Fair	0.246	3	0.246	0.45872	[2.5, 3.5]	0.246	0.50745
A_3	Good	0.421	4	0.421	0.81167	[3.5, 4.5]	0.421	0.82051
A_4	Poor	0.223	2	0.223	0.28620	[1.5, 2.5]	0.223	0.28007
A_5	Very Poor	0.441	1	0.441	0.46205	[1.0, 1.5]	0.441	0.33072
Ranking			$A_3 > A_1 > A_5 > A_2 > A_4$			$A_3 > A_1 > A_2 > A_5 > A_4$		

As we can observe in the last row in Table 3, depending on the selected modeling done for the linguistic variables, rankings vary for positions 3 and 4. In order to check that the modeling done with intervals is more stable than the first one we will replace value 0.246 for alternative A_2 in criterion C_2 by value 0.249. In the last row in Table 4 we can observe how now the ranking obtained with numbers varies whereas the ranking obtained using intervals remains the same.

Table 4. Normalization with small changes on valuations

	Original valuations		*Numerical valuations*	*Interval valuations*
Alternative	C_1	C_2	*Similarity*	*Similarity*
A_1	Very Good	0.115	0.53832	0.66961
A_2	Fair	0.249	0.46259	0.50930
A_3	Good	0.421	0.81156	0.82046
A_4	Poor	0.223	0.28614	0.28004
A_5	Very Poor	0.441	0.46168	0.33039
Ranking			$A_3 > A_1 > A_2 > A_5 > A_4$	$A_3 > A_1 > A_2 > A_5 > A_4$

a.2. Stability with respect to modifications in the weights.

As an example, for the data provided in Table 3, let us consider the weights $w_1=0.6$ and $w_2=0.4$. For both modeling options, the numerical and the one based on intervals, we obtain the ranking $A_3 > A_1 > A_2 > A_5 > A_4$. If we observe the rankings displayed in Table 3, we can see how the order coincides with the one obtained for the modeling using intervals with weights $w_1=w_2=0.5$, whereas this order has changed compared with the one using a modeling based on numerical valuations and equal weights.

b) Let us suppose that for criterion C_1 the ideal is considered to be an intermediate value {Fair}. In this case, although the classical TOPSIS approach allow modeling using linguistic variables, it could not be used as the ideal is not an optimal solution.

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Table 5. Normalization and ranking with a non-optimal ideal solution

	Original valuations		*Numerical valuations*
Alternative	C_1	C_2	*Similarity*
A_1	Very Good	0.115	0.08932
A_2	Fair	0.249	0.65816
A_3	Good	0.421	0.50051
A_4	Poor	0.223	0.42426
A_5	Very Poor	0.441	0.48093
Ranking			$A_2 > A_3 > A_5 > A_4 > A_1$

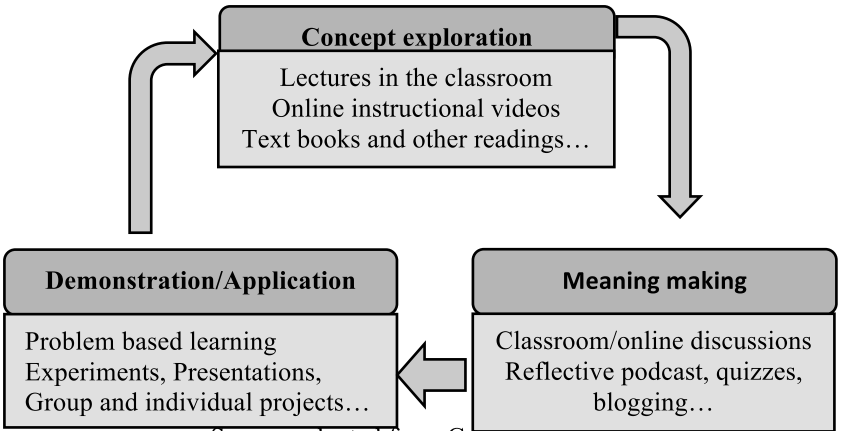
However, with the proposed normalization ideal solutions different than the optimal can be considered and do not present any difficulties. In Table 5 we have displayed the obtained ranking for equal weights w_1=w_2=0.5.

In what follows we will apply the proposed method to a real decision making problem.

5. Case study: ranking of mathematical videos from an educational point of view

In order to illustrate the suitability of the proposed methodological approach to the resolution of real decision making problems we will apply IS-TOPSIS to the ranking of mathematical educational videos in You Tube. Educational videos can be excellent complementary resources for instructors and students, in a context where contact time with the students is a hard constraint. They allow students to access contents at their own convenience and to suit their pace of learning, permitting as many repetitions of the contents as required by the students. The instructor can then focus on certain contents and conduct deeper analysis during the classes promoting discussions with and among the students (see Figure 3).

Figure 3. Learning opportunities with online videos and other resources



Source: adapted from Gerstein (2011)

The transition from "traditional" lectures to the "flipped classroom" is not the only potential advantage of the use of online videos, especially in the case of free-video platforms as YouTube. The use of free-online educational videos has the potential of creating a positive social impact (Liern and Pérez-Gladish 2018) as can provide fast and inexpensive access to educational contents to a broad community of students from less developed countries, rural or marginal areas (see Zeng et al. 2015, Laaser 1999 for the advantages of the use of new technologies for developing countries).

Recognising the growing influence of video-on-demand in education (Hansch et al. 2015; Bates 2015) YouTube has created "YouTube EDU" where educational content is aggregated into playlists and categories. More than 300 universities are collaborating in this initiative to provide more than 65,000 free lectures, news items and snippets of campus life (https://theconversation.com).

In this situation, and with more than 10 million videos tagged as educational in YouTube, the evaluation and ranking of educational videos in terms of their quality is crucial. However, in this context, quality is a multidimensional concept which needs to consider important dimensions related not only to the quality of the contents from an educational point of view but to other aspects related to the quality of the instructional process, the quality of the production of the videos or the authority of the authors of the videos. Although there has been a dramatic increase of educational videos uploads in free online platforms as YouTube, a recent report, the Kaltura report, shows how "the active use of video by students is still in its infancy" (Kaltura 2015). The great potential of this educational tool deserves then attention from decision makers in the educational process, particularly in the case of free-online-video platforms where the quality of the videos should be evaluated in educational terms from a multidimensional perspective.

In an attempt to proposed a global approach for the assessment of mathematical videos published in free-video platforms as YouTube, in this work, we have considered 5 fundamental points of view and 24 decision criteria or dimensions based on educational criteria (see Tables 6 and 7 and Figure 4).

Table 6. Fundamental points of view in the educational performance assessment

Point of view	*Description and justification*
Didactic	Introduced by Godino et al.. (2007) and developed by other authors as Pino-Fan et al.. (2015), the notion of didactical suitability of an instructional process is defined as the coherent and systemic articulation of six dimensions or facets: *Epistemic facet*, refers to specialized knowledge of the mathematical dimension. *Cognitive facet*, refers to the knowledge about the students' cognitive aspects. *Affective facet*, refers to the knowledge about the students' affective, emotional and behavioural aspects. *Interactional facet*, refers to the knowledge of the interactions that occur within a classroom. *Mediational facet*, refers to the knowledge of resources and means which might foster the students' learning process. *Ecological facet*, refers to the knowledge of curricular, contextual, social political, economic… aspects that have an influence on the management of the students' learning. These dimensions have been widely applied to the performance evaluation of mathematical instructional processes (see Godino et al. 2007).
Interaction with users	This point of view refers to how well the video is doing in terms of its audience. This point of view is common to any video published in platforms as YouTube. We have decided to take into account the number of views, the number of "likes", the number of "dislikes", the number of comments and the number of shares because, in the case of educational videos, in particular mathematical videos, these criteria reflect the degree of interaction with the users which is a highly desirable characteristic of any educational process. YouTube provides a tool, YouTube Analytics which provides few key statistics on these and other interesting criteria. However, not all the statistics are available to the public. This is the case of data on what time of the video. Knowing statistics about the drop-off point of the video could be of great interest in order to measure the performance of the video in terms of interaction with users (Guo et al. 2014). Nevertheless, these statistics are only available for the creators of the videos and therefore this criterion has

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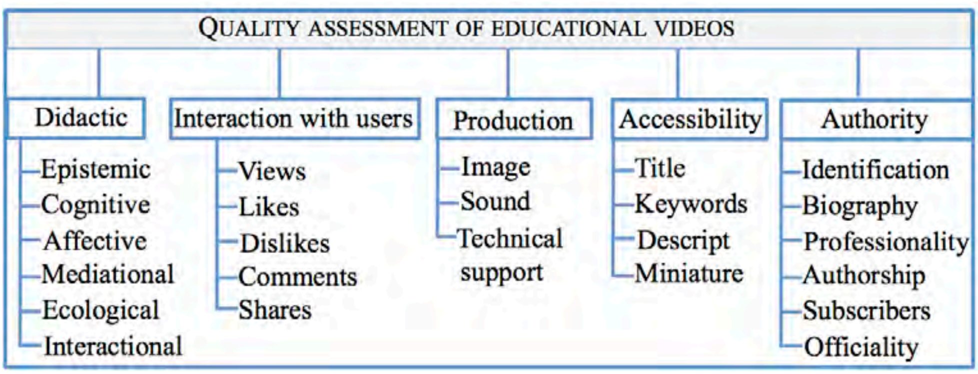
	not been included in our analysis of the videos (see https://creatoracademy.youtube.com, Vest 2009, Laazer and Toloza 2007).
Production quality	Production quality refers to the technical quality of the video in terms of image, sound and the technical support for the transmission of the educational contents: blackboard, specific software, direct recording in a classroom… Technical high-quality is a key factor to retain the attention of the viewer and to facilitate the transmission of educational contents in contexts where the instructor is not present (see Koumi 2003, 2006; Vest 2009, Laazer and Toloza 2007)
Accessibility	This point of view answers the question: How are viewers finding the video? This is a crucial question in platforms as YouTube. Selection of appropriate keywords is essential as well as a precise and accurate title and description of the content of the video. If important keywords are missing or the title is imprecise the video will not get to the interested audience. A short and precise description of the contents and a clear and descriptive miniature of the video will avoid early drop-offs and dissatisfied viewers (see Laazer and Toloza 2007)
Authority	Authority reveals that the author of the video has the qualifications and knowledge to do so. Evaluating a video for authority requires taking into account: authorship (it should be clear who creates the video content); contact information should be clearly provided; credentials (the author should state qualifications, credentials, or personal background that gives him/her authority to present the contents) and support by an official organization. In the context of free-online video platform where everybody can upload contents, identification of the author and description of his/her credentials are key questions. This question has been widely studied in the case of web pages (Kapoun 1998). However, as far as the authors of this paper know, this discussion has not been enough conducted in the case of free online educational videos. As in the case of the publication of contents in web pages, accuracy is necessary with respect to the author identification in order to make contents reliable. Therefore, we have included as criteria the quality of the short biography of the author and whether or not he/she is a professional from the educational sector. As anybody can upload educational contents in platforms like YouTube it is also important to take into account if the authorship of the video is acknowledge by the person who uploads the video (as in some occasions they are not necessary the same). The number of subscribers of the author can be also considered as a signal guaranteeing the quality of the contents as well as what we have called the officiality of the video which aims to inform about the existence of a formal educational institution behind the video.

Source: Own elaboration based on the experts' knowledge.

A good performance in one of the considered dimensions might not correspond to a good performance in the other dimensions. Therefore, the assessment of the educational quality of any instructional process based on videos is clearly a multiple criteria decision making problem.

Figure 4. Criteria hierarchy



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The 24 considered dimensions have been used as decision criteria by 3 educational experts to assess the educational performance of 12 mathematical videos published in You Tube. Table 4 describes the considered decision criteria and the questions used to obtain evaluations from the three experts. The questions for the didactic point of view have been adapted from Santos-Mellado et al. (2017). Both positive and negative attitude statements have been used as a "positive" or "negative" set of statements might influence respondents' answers to the statements (i.e. a set of positive statements might produce higher agreement than the level of disagreement for a set of negative statements), (Gendall and Hoek 1990).

We have chosen a mathematical concept, basis of a vector space, and we have conducted a search of You Tube videos in Google. We searched educational videos introducing the concept of basis of a vector space. The selected videos are those appearing in the first 12 positions when using as key words "basis" and "vector space". Table 8 displays a description of our decision alternatives.

We have distinguished between qualitative and quantitative criteria. Quantitative criteria refer to objective and precise data: number of views, number of likes, number of dislikes, number of comments, number of shares and number of subscribers and to data represented by binary variables: identification, authorship and officiality.

Table 7. Decision Criteria

Criteria	*Description*	*Question*
		Didactic
Z_1	Epistemic facet	To what extent are the treated mathematical concepts correct?
Z_2	Cognitive	To what extent does the author mention all the elements in a fluid way?
Z_3	Affective facet	To what extent do the contents of the video attract the attention of the user?
Z_4	Mediational facet	To what extent time and resources are wasted in the explanation?
Z_5	Ecological facet	To what extent is the video adapted to the concrete educational context?
Z_6	Interactional facet	To what extent is it difficult to understand the author?
Interaction with users		
Z_7	Views	Number of visualizations without taking into account if the video has been watched entirely
Z_8	Likes	Number of "likes" taking into account publication date
Z_9	Dislikes	Number of "dislikes" taking into account publication date
Z_{10}	Comments	Number of comments without taking into account author's responses
Z_{11}	Shares	Number of times the video has been shared taking into account publication date
Production quality		
Z_{12}	Quality of image	Quality of the image of the video (current/optimal resolution)
Z_{13}	Quality of sound	Quality of the sound of the video (volume/normalized) content loudness (dB)
Z_{14}	Quality of technical support	Quality of the technical support of the video: To what extent is the medium used to transmit the contents clear? e.g. blackboard capture, normal video, notebook capture, hand write, use of computer software…
Accessibility		
Z_{15}	Title	Precision of the title: To what extent does the title describe precisely the contents of the video?
Z_{16}	Keywords	Precision of keywords: To what extent do the keywords describe precisely the contents of the video?
Z_{17}	Description	Quality of the video description: To what extent does the description of the video precisely indicate the contents of the video?
Z_{18}	Miniature	Quality of the video miniature: To what extent does the miniature of the video describe precisely the contents of the video?
Authority		
Z_{19}	Identification	The author is clearly identified (Yes/No)

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Z_{20}	Biography	Quality of the short biography description
Z_{21}	Professionality	Degree in which the author is an adequate professional from the educational sector
Z_{22}	Authorship	The author is the person uploading the video (Yes/No)
Z_{23}	Subscribers	Number of subscribers which follow the author or person who uploads the video
Z_{24}	Officiality	The video is published in the context of an official institution or body (Yes/No)

Qualitative criteria refer to those aspects that are subjective, ambiguous and imprecise and highly depend on the opinion of experts: didactic facets, quality of image, sound and technical support, precision of title, keywords, description and miniature of the videos and professionality and quality of the biography of the author.

We have measured the performance of each video in each quantitative criterion taking into account the number of days the video had been available in YouTube until the date of collection of the data (20 of August 2017) to make data comparable.

Table 8. Decision alternatives

Video	*Title*	*URL*
V_1	Basis for a set of vectors	https://www.youtube.com/watch?v=rUJ5B-swc9Y
V_2	Basis for a vector Space	https://www.youtube.com/watch?v=XeU6ixsv1lE
V_3	Basis and dimension of a vector space	https://www.youtube.com/watch?v=-42bA6CKRnU
V_4	Linear Algebra - basis of a vector space	https://www.youtube.com/watch?v=XErZLJYwhcE
V_5	Basis of a vector space	https://www.youtube.com/watch?v=dOlVxQCHT0k
V_6	Vector space, basis, dimension	https://www.youtube.com/watch?v=nOfY1ZATzIM
V_7	Basis, vectors and coordinates	https://www.youtube.com/watch?v=wYKAw5QanJY
V_8	Basis and dimension	https://www.youtube.com/watch?v=AqXOYgpbMBM
V_9	Linear combinations, span, and basis vectors	https://www.youtube.com/watch?v=k7RM-ot2NWY
V_{10}	Basis and dimension	https://www.youtube.com/watch?v=lf5WacddAo4
V_{11}	Linear algebra example problems - vector space basis example #1	https://www.youtube.com/watch?v=3l3qfs2vINE
V_{12}	Concepts of basis and dimension	https://www.youtube.com/watch?v=RO-XYVeaROw

The three experts (educators experts on mathematics from three different higher education institutions from different countries) were then asked to score the performance of the videos with respect to each qualitative criterion using following linguistic labels:

{Very Poor, Poor, Fair, Good, Very Good}

The three experts were given the same importance as we have considered their level of expertise and experience to be the same. However, other situations could be considered depending on the decisional context.

Once linguistic labels were obtained for each video in each dimension in each group, we obtained a consensual assessment for each dimension, i, and video, j, in the form of intervals. The consensual assessment is expressed as the union set of the individual linguistic rates. Table 9 displays the obtained individual and consensual ratings for the quality of technical support criterion using linguistic variables (see Tables 1A, 2A and 3A in the appendix for the assessments of all qualitative and quantitative criteria).

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Table 9. Individual and consensual rates with linguistic labels for Z_{14}

Video	*Expert 1*	*Expert 2*	*Expert 3*	*Consensual*
V_1	G	G	F	{F,G}
V_2	VP	P	VP	{VP,P}
V_3	F	P	P	{P,F}
V_4	G	G	G	G
V_5	P	P	F	{P,F}
V_6	VG	VG	G	{G,VG}
V_7	P	P	F	{P,F}
V_8	G	VG	P	{P,VG}
V_9	VG	VG	G	{G,VG}
V_{10}	VG	VG	P	{P,VG}
V_{11}	VG	G	G	{G,VG}
V_{12}	VP	VP	VP	VP

Once the videos have been scored and rated in all the dimensions by the experts and consensual intervals have been obtained, a different expert is asked to provide the ideal scores and rates for each criterion (see Table 10). This expert belongs to the Ibero-American Laboratory for the Assessment of Education Processes (LABIPE, https://www.uv.es/liern/LABIPE).

Table 10. Criteria description, ideals and weights

Criteria	*Description*	*Statement*	*Ideal*	*Weight*
Didactic				0.329555219
Z_1	Epistemic facet	Positive	{VG}	0.128531242
Z_2	Cognitive	Positive	{G,VG}	0.070062226
Z_3	Affective facet	Positive	{G}	0.017176916
Z_4	Mediational facet	Negative	{P}	0.029547704
Z_5	Ecological facet	Positive	{F,G}	0.015375083
Z_6	Interactional facet	Negative	{VP}	0.068862048
Interaction with users				0.169464716
Z_7	Views	Positive	812.7005	0.019420127
Z_8	Likes	Positive	13.5695	0.048848884
Z_9	Dislikes	Negative	0.0000	0.007171995
Z_{10}	Comments	Positive	1.1765	0.082625834
Z_{11}	Shares	Positive	2.0588	0.011397876
Production quality				0.10090001
Z_{12}	Quality of image	Positive	{F,G}	0.067987493
Z_{13}	Quality of sound	Positive	{F,G}	0.022756534
Z_{14}	Quality of technical support	Positive	G	0.010155983
Accessibility				0.066259587
Z_{15}	Title	Positive	{VG}	0.038568971
Z_{16}	Keywords	Positive	{F,G}	0.017478961
Z_{17}	Description	Positive	{F,G}	0.00756904
Z_{18}	Miniature	Positive	{F,G}	0.002642615
Authority				0.333820468
Z_{19}	Identification	Positive	Yes	0.085088368
Z_{20}	Biography	Positive	{F,G}	0.050240591
Z_{21}	Professionality	Positive	G	0.140542773
Z_{22}	Authorship	Positive	Yes	0.011039751
Z_{23}	Subscribers	Positive	885.0267	0.029495463

Z_{24}	Officiality	Positive	Yes	0.017413523

Based on the ideal solution and following the steps of the IS-TOPSIS described in the previous section, the rankings displayed in Table 11 were obtained considering the weights elicited using the process described in the AHP process (see Table 10).

We can observe how two points of view have a weight greater than 30%, the didactic and authority points of view. These two points of view and their correspondent criteria are considered highly non-compensatory by the experts. The proposed method respects this important characteristic of many real decision making applications.

In order to handle the linguistic rates and ideals, following the procedure proposed by Cables et al. (2016) the linguistic rates have been transformed into numerical values using the following scale:

$$\{VP=1, P=2, F=3, G=4, VG=5\} \tag{9}$$

The normalized matrix and the weighted normalized matrix are displayed in the appendix in Tables 4A and 5A, respectively. The last two columns in Table 8 display the video ranking and the relative index values obtained with IS-TOPSIS whereas the first column shows the order in which Google searches results when a search is conducted using the keywords basis, vector space, videos and YouTube.

The order of search on Google's search-results pages is based on a priority rank called a "PageRank" that helps rank web pages that match a given search string. The "PageRank algorithm analyzes human-generated links assuming that web pages linked from many important pages are themselves likely to be important. The algorithm computes a recursive score for pages, based on the weighted sum of the PageRanks of the pages linking to them" (https://en.wikipedia.org/wiki/Google_Search).

Table 11. IS-TOPSIS: obtained rankings with weighs obtained with AHP based method

Video	*Google Rank*	*IS-TOPSIS Video Rank*	R_i
V_1	1	9	0.04776747
V_2	2	5	0.04457899
V_3	3	6	0.04438585
V_4	4	8	0.04413395
V_5	5	1	0.04406220
V_6	6	11	0.04344854
V_7	7	10	0.04343819
V_8	8	3	0.04339437
V_9	9	7	0.04217911
V_{10}	10	4	0.04008244
V_{11}	11	2	0.03830633
V_{12}	12	12	0.03485353

As we can observe, there is only one coincidence in the two rankings. This is due to the fact that Google's rank of the video results is based on a unique criterion which is not directly related with educational aspects. Definitively, these aspects should be taken into account in any search of educational contents in free-online platforms such us YouTube.

6. Conclusions

The proposed method, based on the similarity of the alternatives with respect to the criteria with the ideal solution, Ideal Similarity TOPSIS (IS-TOPSIS), allows working with one or several decision makers and with different types of data simultaneously (numerical intervals, linguistic variables or mixed data). This last feature, the simultaneous different nature of the data, can lead to a situation where the classical normalization processes give rise to very unstable rankings. In order to solve this problem a new normalization process has been proposed which transforms the original data into new data reflecting the similarity of each alternative with respect to each decision criterion with an ideal solution taken as a reference. The proposed normalization approach can be easily incorporated into any MCDM method based on reference solutions, as the ideal solution.

Discussion on the ideal solution is also addressed and we show how the idea of ideal solution can be generalized and can take any value between the minimum and maximum values of the range of the criteria. The new approach, based on the similarity with this ideal solution, Ideal Similarity TOPSIS, is applied to the ranking of free-online mathematical instructional videos based on their quality from an educational point of view. With the increasing success of free-online video platforms as YouTube and the observed change in some educational patters which encourages the use of free-online videos in the instructional process, the ranking of videos based on their quality is a crucial question.

The quality of free-online educational videos is a multidimensional concept which needs to take into account several qualitative and quantitative criteria of highly heterogeneous nature. Educational assessment needs to consider the quality of the contents, interaction with users, production quality, accessibility and authority. This last aspect is essential. Authority reveals that the author of the video has the qualifications and knowledge to do so. In the context of free-online video platform where everybody can upload contents, identification of the author and description of his/her credentials are key questions. However, as far as the authors of this paper know, although highly discussed in the context of web pages, this discussion has not been conducted in the case of free online educational videos. As in the case of the publication of contents in web pages, accuracy is necessary with respect to the author identification in order to make contents reliable. With this work, we hope to try to catch the attention of educators on the increasing importance of this educational instrument and the necessity of reliable global assessment of free-online educational contents.

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Appendix

Table 1A. Individual decision matrices for the qualitative criteria

Video	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6	Z_{12}	Z_{13}	Z_{14}	Z_{15}	Z_{16}	Z_{17}	Z_{18}	Z_{20}	Z_{21}
Expert 1															
V_1	VG	G	G	VP	G	VP	F	F	G	VG	VP	G	G	VG	VG
V_2	G	F	F	P	F	P	P	G	VP	VG	VP	VP	G	P	VG
V_3	VG	F	F	P	G	P	G	G	F	VG	VP	G	G	F	G
V_4	G	G	F	P	F	P	F	P	G	VG	VP	VP	F	VP	VG
V_5	VG	F	F	P	G	P	G	F	P	VG	VP	VG	G	VG	VG
V_6	VG	G	F	P	G	VP	VG	VG	VG	VG	VP	VP	VG	VP	VG
V_7	G	VG	G	P	G	P	P	F	P	VG	VP	VG	VP	VG	VG
V_8	G	G	F	P	G	VP	G	VG	G	VG	VP	VP	VG	VP	VG
V_9	VG	VG	VG	VP	G	VP	VG	VG	VG	VG	VP	P	F	VP	P
V_{10}	G	G	P	F	F	P	G	G	VG	VG	VP	VP	G	VP	VP
V_{11}	G	G	G	P	G	P	VG	G	VG	F	VP	F	VG	VG	VG
V_{12}	F	G	F	G	F	P	G	G	VP	G	VP	VP	VP	VP	VP
Expert 2															
V_1	VG	G	VG	VP	G	VP	F	F	G	VG	VP	F	F	VG	VG
V_2	F	P	VP	P	P	G	P	G	P	VG	VP	VP	F	P	VG
V_3	VG	VG	G	P	VG	P	G	G	P	VG	VP	F	F	F	G
V_4	VG	G	VP	G	VG	F	F	P	G	VG	VP	P	F	P	G
V_5	VG	P	VP	G	VG	G	G	F	P	VG	VP	VG	G	VG	VG
V_6	VG	G	F	F	VG	F	VG	VG	VG	VG	VP		VG	F	VG
V_7	VG	P	P	G	G	F	P	F	P	G	VP	VG	VP	VG	VG
V_8	VG	P	F	G	F	G	G	VG	VG	VG	VP		VG	VG	VG
V_9	VG	VG	VG	P	G	VP	VG	VG	VG	VG	VP	F	F	P	P
V_{10}	VG	VG	P	F	VG	F	G	G	G	VG	VP		G	F	P
V_{11}	G	P	F	P	G	F	VG	G	VG	P	VP	F	VG	VG	VG
V_{12}	P	G	VG	G	P	VP	G	G	VP	VG	VP	VP	P	F	P
Expert 3															
V_1	VG	G	G	VP	VG	VP	F	F	F	F	VP	G	G	VG	G
V_2	G	F	F	P	P	G	P	G	VP	F	VP	F	G	P	G
V_3	VG	VG	F	P	VG	F	G	G	P	G	VP	G	F	F	G
V_4	VG	F	VP	P	VG	F	F	P	G	F	VP	G	G	VP	G
V_5	VG	F	VP	P	VG	G	G	F	F	F	VP	F	F	VG	G
V_6	VG	G	F	F	VG	P	VG	VG	G	P	VP	F	G	F	G
V_7	VG	P	P	P	VG	VP	P	F	F	P	VP	G	P	VG	G
V_8	VG	VG	F	P	P	VP	G	VG	P	P	VP	P	F	F	G
V_9	VG	VG	VG	P	G	P	VG	VG	G	P	VP	F	G	P	G
V_{10}	VG	VG	P	P	VG	F	G	G	P	P	VP	F	F	F	G
V_{11}	G	VG	G	F	G	F	VG	G	G	G	VP	G	F	VG	G
V_{12}	F	VG	VG	F	F	VP	G	G	VP	VP	VP	P	F	F	G

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Table 2A. Consensual decision matrix for the qualitative criteria

Video	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6	Z_{12}	Z_{13}	Z_{14}	Z_{15}	Z_{16}	Z_{17}	Z_{18}	Z_{20}	Z_{21}
V_2	{F, G}	{P, F}	{VP, F}	P	{P, F}	{P, G}	P	G	{VP, P}	{F, VG}	VP	{VP, F}	{F, G}	{P, P}	{G, VG}
V_3	VG	{F, VG}	{F, G}	P	{G, VG}	{P, F}	G	G	{P, F}	{G, VG}	VP	{F, G}	{F, G}	{F, F}	{G, G}
V_4	{G, VG}	{F, G}	{VP, F}	{P, G}	{F, VG}	{P, F}	F	P	G	{F, VG}	VP	{VP, F}	{F, G}	{VP, P}	{G, VG}
V_5	VG	{P, F}	{VP, F}	{P, G}	{G, VG}	{P, G}	G	F	{P, F}	{F, VG}	VP	{F, VG}	{F, G}	VG	{G, VG}
V_6	VG	G	F	{P, F}	{G, VG}	{VP, F}	VG	VG	{G, VG}	{P, VG}	VP	{F, F}	{G, VG}	F	{G, VG}
V_7	{G, VG}	{P, VG}	{P, G}	{P, G}	{G, VG}	{P, F}	P	F	{P, F}	{P, VG}	VP	{G, VG}	{VP, P}	VG	{G, VG}
V_8	{G, VG}	{P, VG}	F	{P, G}	{P, G}	{VP, G}	G	VG	{P, VG}	{F, VG}	VP	{P, F}	{F, VG}	F	{G, VG}
V_9	VG	VG	VG	{VP, P}	G	VP	VG	VG	{G, VG}	{F, VG}	VP	{P, F}	{F, G}	{VP, P}	{P, G}
V_{10}	{G, VG}	{G, VG}	P	{P, F}	{F, VG}	{P, F}	G	G	{P, VG}	{F, VG}	VP	{F, F}	{F, G}	F	{P, G}
V_{11}	VG	{P, VG}	{F, G}	{P, F}	G	{P, F}	VG	G	{G, VG}	{P, G}	VP	{F, G}	{F, VG}	VG	{G, VG}
V_{12}	{P, F}	{G, VG}	{G, VG}	{F, G}	{P, F}	{VP, P}	G	G	VP	{VP, F}	VP	{VP, P}	{VP, F}	F	{P, G}

Table 3A. Decision matrix for the quantitative criteria

Video	Z_7	Z_8	Z_9	Z_{10}	Z_{11}	Z_{19}	Z_{22}	Z_{23}	Z_{24}
V_1	14.0612	0.5726	0.0154	0.0579	0.1228	0.0000	1.0000	332.0870	0.0000
V_2	13.4879	0.0324	0.0033	0.0100	0.0000	1.0000	1.0000	2.4290	1.0000
V_3	30.7037	0.0543	0.0150	0.0031	0.0175	0.0000	1.0000	1.8714	0.0000
V_4	1.7194	0.0172	0.0017	0.0017	0.0069	0.0000	1.0000	0.2048	1.0000
V_5	0.4282	0.0088	0.0000	0.0000	0.0029	1.0000	1.0000	0.1554	1.0000
V_6	7.8509	0.0257	0.0111	0.0043	0.0077	1.0000	0.0000	0.1859	1.0000
V_7	2.6344	0.0096	0.0000	0.0022	0.0015	1.0000	1.0000	1.4771	1.0000
V_8	64.2969	0.3852	0.0102	0.0332	0.1156	1.0000	1.0000	0.5850	1.0000
V_9	812.7005	13.5695	0.0615	1.1765	2.0588	1.0000	1.0000	885.0260	0.0000
V_{10}	1.5310	0.0078	0.0000	0.0000	0.0000	1.0000	1.0000	17.8294	0.0000
V_{11}	28.2656	0.1184	0.0015	0.0152	0.0804	1.0000	1.0000	18.2094	1.0000
V_{12}	1.8928	0.0173	0.0009	0.0009	0.0022	0.0000	1.0000	4.3149	0.0000

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Table 4A. Normalized decision matrix

Video	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6	Z_7	Z_8	Z_9	Z_{10}	Z_{11}	Z_{12}
V_1	1.0000	0.8750	0.8750	0.7500	0.7500	1.0000	0.0168	0.0417	0.7494	0.0492	0.0597	1.0000
V_2	0.6250	0.5000	0.5000	1.0000	0.7500	0.5000	0.0161	0.0018	0.9458	0.0085	0.0000	0.5000
V_3	1.0000	0.8750	0.8750	1.0000	0.7500	0.6250	0.0373	0.0034	0.7565	0.0027	0.0085	1.0000
V_4	0.8750	0.7500	0.5000	0.7500	0.8750	0.6250	0.0016	0.0007	0.9720	0.0015	0.0033	1.0000
V_5	1.0000	0.5000	0.5000	0.7500	0.7500	0.5000	0.0000	0.0001	1.0000	0.0000	0.0014	1.0000
V_6	1.0000	0.8750	0.7500	0.8750	0.7500	0.7500	0.0091	0.0013	0.8189	0.0036	0.0037	0.0000
V_7	0.8750	0.7500	0.7500	0.7500	0.7500	0.6250	0.0027	0.0001	1.0000	0.0019	0.0007	0.5000
V_8	0.8750	0.7500	0.7500	0.7500	0.8750	0.6250	0.0786	0.0278	0.8335	0.0282	0.0561	1.0000
V_9	1.0000	0.8750	0.7500	0.8750	0.8750	1.0000	1.0000	1.0000	0.0000	1.0000	1.0000	0.0000
V_{10}	0.8750	1.0000	0.5000	0.8750	0.8750	0.6250	0.0014	0.0000	1.0000	0.0000	0.0000	1.0000
V_{11}	1.0000	0.7500	0.8750	0.8750	0.8750	0.6250	0.0343	0.0082	0.9753	0.0129	0.0391	0.0000
V_{12}	0.3750	1.0000	0.8750	0.6250	0.7500	0.8750	0.0018	0.0007	0.9860	0.0007	0.0010	1.0000

Video	Z_{13}	Z_{14}	Z_{15}	Z_{16}	Z_{17}	Z_{18}	Z_{19}	Z_{20}	Z_{21}	Z_{22}	Z_{23}	Z_{24}
V_1	1.0000	0.8750	0.7500	0.0000	1.0000	1.0000	0.0000	0.6250	0.8750	1.0000	0.3751	0.0000
V_2	1.0000	0.3750	0.7500	0.0000	0.6250	1.0000	1.0000	0.6250	0.8750	1.0000	0.0026	1.0000
V_3	1.0000	0.6250	0.8750	0.0000	1.0000	1.0000	0.0000	0.8750	0.8750	1.0000	0.0019	0.0000
V_4	0.5000	1.0000	0.7500	0.0000	0.6250	1.0000	0.0000	0.5000	0.8750	1.0000	0.0001	1.0000
V_5	1.0000	0.6250	0.7500	0.0000	0.8750	1.0000	1.0000	0.6250	0.8750	1.0000	0.0000	1.0000
V_6	0.0000	0.8750	0.6250	0.0000	0.8750	0.7500	1.0000	0.8750	0.8750	0.0000	0.0000	1.0000
V_7	1.0000	0.6250	0.6250	0.0000	0.7500	0.5000	1.0000	0.6250	0.8750	1.0000	0.0015	1.0000
V_8	0.0000	0.8750	0.7500	0.0000	0.7500	0.8750	1.0000	0.8750	0.8750	1.0000	0.0005	1.0000
V_9	0.0000	0.6250	0.7500	0.0000	0.7500	1.0000	1.0000	0.5000	0.7500	1.0000	1.0000	0.0000
V_{10}	1.0000	0.6250	0.7500	0.0000	0.8750	1.0000	1.0000	0.8750	0.7500	1.0000	0.0200	0.0000
V_{11}	1.0000	0.8750	0.5000	0.0000	1.0000	0.8750	1.0000	0.6250	0.8750	1.0000	0.0204	1.0000
V_{12}	1.0000	0.2500	0.2500	0.0000	0.5000	0.6250	0.0000	0.8750	0.7500	1.0000	0.0047	0.0000

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Table 5A. Weighted decision matrix

Video	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6	Z_7	Z_8	Z_9	Z_{10}	Z_{11}	Z_{12}
V_1	0.1285	0.0613	0.0150	0.0222	0.0115	0.0689	0.0003	0.0020	0.0054	0.0041	0.0007	0.0680
V_2	0.0803	0.0350	0.0086	0.0295	0.0115	0.0344	0.0003	0.0001	0.0068	0.0007	0.0000	0.0340
V_3	0.1285	0.0613	0.0150	0.0295	0.0115	0.0430	0.0007	0.0002	0.0054	0.0002	0.0001	0.0680
V_4	0.1125	0.0525	0.0086	0.0222	0.0135	0.0430	0.0000	0.0000	0.0070	0.0001	0.0000	0.0680
V_5	0.1285	0.0350	0.0086	0.0222	0.0115	0.0344	0.0000	0.0000	0.0072	0.0000	0.0000	0.0680
V_6	0.1285	0.0613	0.0129	0.0259	0.0115	0.0516	0.0002	0.0001	0.0059	0.0003	0.0000	0.0000
V_7	0.1125	0.0525	0.0129	0.0222	0.0115	0.0430	0.0001	0.0000	0.0072	0.0002	0.0000	0.0340
V_8	0.1125	0.0525	0.0129	0.0222	0.0135	0.0430	0.0015	0.0014	0.0060	0.0023	0.0006	0.0680
V_9	0.1285	0.0613	0.0129	0.0259	0.0135	0.0689	0.0194	0.0488	0.0000	0.0826	0.0114	0.0000
V_{10}	0.1125	0.0701	0.0086	0.0259	0.0135	0.0430	0.0000	0.0000	0.0072	0.0000	0.0000	0.0680
V_{11}	0.1285	0.0525	0.0150	0.0259	0.0135	0.0430	0.0007	0.0004	0.0070	0.0011	0.0004	0.0000
V_{12}	0.0482	0.0701	0.0150	0.0185	0.0115	0.0603	0.0000	0.0000	0.0071	0.0001	0.0000	0.0680

Video	Z_{13}	Z_{14}	Z_{15}	Z_{16}	Z_{17}	Z_{18}	Z_{19}	Z_{20}	Z_{21}	Z_{22}	Z_{23}	Z_{24}
V_1	0.0228	0.0089	0.0289	0.0000	0.0076	0.0026	0.0000	0.0314	0.1230	0.0110	0.0111	0.0000
V_2	0.0228	0.0038	0.0289	0.0000	0.0047	0.0026	0.0851	0.0314	0.1230	0.0110	0.0001	0.0174
V_3	0.0228	0.0063	0.0337	0.0000	0.0076	0.0026	0.0000	0.0440	0.1230	0.0110	0.0001	0.0000
V_4	0.0114	0.0102	0.0289	0.0000	0.0047	0.0026	0.0000	0.0251	0.1230	0.0110	0.0000	0.0174
V_5	0.0228	0.0063	0.0289	0.0000	0.0066	0.0026	0.0851	0.0314	0.1230	0.0110	0.0000	0.0174
V_6	0.0000	0.0089	0.0241	0.0000	0.0066	0.0020	0.0851	0.0440	0.1230	0.0000	0.0000	0.0174
V_7	0.0228	0.0063	0.0241	0.0000	0.0057	0.0013	0.0851	0.0314	0.1230	0.0110	0.0000	0.0174
V_8	0.0000	0.0089	0.0289	0.0000	0.0057	0.0023	0.0851	0.0440	0.1230	0.0110	0.0000	0.0174
V_9	0.0000	0.0063	0.0289	0.0000	0.0057	0.0026	0.0851	0.0251	0.1054	0.0110	0.0295	0.0000
V_{10}	0.0228	0.0063	0.0289	0.0000	0.0066	0.0026	0.0851	0.0440	0.1054	0.0110	0.0006	0.0000
V_{11}	0.0228	0.0089	0.0193	0.0000	0.0076	0.0023	0.0851	0.0314	0.1230	0.0110	0.0006	0.0174
V_{12}	0.0228	0.0025	0.0096	0.0000	0.0038	0.0017	0.0000	0.0440	0.1054	0.0110	0.0001	0.0000