

A VIKOR-based approach for the ranking of mathematical instructional videos

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Abstract

Purpose: In the last years, the use of free-online instructional videos has gained popularity among educators and students. Its success is mainly based on the provision of fast and inexpensive access to educational contents which can be consulted at the own convenience of students, all over the world. Free-online platforms as YouTube offer access to more than 10 million instructional videos. The objective of this paper is to assess and rank the educational quality of free-online instructional videos from a multidimensional perspective.

Design/methodology/approach: In this paper, we propose a MCDM approach based on a compromise ranking method, VIKOR. The approach integrates a normalization process which is especially suitable for situations where the nature of the different decision making criteria is such that it does not allow homogeneous aggregation.

Findings: With the proposed normalization approach, the initial valuations of the alternatives with respect to the criteria are transformed in order to reflect their similarity with a given reference point (ideal solution). The normalized data are then integrated in a VIKOR-based framework in order to obtain those mathematical videos closer to the ideal video from the instructors' perspective.

Originality: The ranking of instructional videos based on their quality from an educational multidimensional perspective is a good example of a real decision making problem where the nature of the criteria, qualitative and quantitative, implies heterogeneous data. The proposed IS-VIKOR approach overcomes some of the problems inherent to this real decision making problem.

Keywords: MCDM, VIKOR, TOPSIS, normalization, ideal, mathematical videos.

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1. Introduction

The United Nations Educational, Scientific and Cultural Organization (UNESCO) defines open educational resources (OER) as “teaching, learning and research materials in any medium, digital or otherwise, that reside in the public domain or have been released under an open license that permits no-cost access, use, adaptation and redistribution by others with no or limited restrictions” (UNESCO, 2002). The Universal Declaration of Human Rights states that everyone has the right to education and for any interested person, quality education must be made generally available and equally accessible with no barriers or limitations (UN, 1948).

Among all the open educational resources, instructional videos have an important role. They are excellent complementary resources for both, the instructor and the students, in those context where contact time with the students is a hard constraint allowing students to have fast and inexpensive access to educational contents at their own convenience (Lage et al., 2000; Acuña-Soto et al., 2018). These characteristics have promoted the proliferation of instructional videos and other open line resources in the last years. Online-free platforms as YouTube currently offers access to more than 10 million educational videos. In view of this situation and taking into account the easy predictable increasing tendency of instructional videos’ production, the assessment of the quality of these open line resources is a key question (Acuña-Soto et al., 2018). In the context of instructional online videos, quality is a multidimensional concept. Control of the quality of the contents from educational aspects, which is related to the authority principle, is essential. Authority reveals that the author of the video has the qualifications and knowledge to do so. In the context of free-online video platform where everybody can upload contents, identification of the author and description of his/her credentials are key questions. This question has been widely studied in the case of web pages (Kapoun, 1998). However, as far as the authors of this paper know, this discussion has not been enough conducted in the case of free online educational videos. As in the case of the publication of contents in web pages, accuracy is necessary with respect to the author identification in order to make contents reliable.

Introduced by Godino et al. (2007) and developed by other authors as Pino-Fan et al. (2015), the notion of didactical suitability of an instructional process is defined as the coherent and systemic articulation of six dimensions or facets which have been widely applied to the performance evaluation of mathematical instructional processes (see Godino et al., 2007): epistemic, cognitive, affective, interactional, mediational and ecological. The *epistemic facet*, refers to specialized knowledge of the mathematical dimension. The *cognitive facet*, refers to the knowledge about the students’ cognitive aspects. The *affective facet*, refers to the knowledge about the students’ affective, emotional and behavioural aspects. The *interactional facet*, refers to the knowledge of the interactions that occur within a classroom. The *mediational facet*, refers to the knowledge of resources and means which might foster the students’ learning process and, the *ecological facet*, refers to the knowledge of curricular, contextual, social political, economic... aspects that have an influence on the management of the students’ learning. We will based on these facets for the proposal of the didactical criteria in our multiple criteria ranking approach.

However, other quality aspects of the videos, interaction with users or accessibility are also important and need to be taken into account in the ranking of instructional online videos. Interaction with users is an important aspect in the educational performance assessment of instructional, especially educational, videos. This aspect refers to how

well the video is doing in terms of its audience (Acuña-Soto et al., 2018). This aspect is common to any video published in platforms as YouTube. The number of views, the number of “likes”, the number of “dislikes”, the number of comments and the number of shares reflect the degree of interaction with the users which is a highly desirable characteristic of any educational process (Guo et al., 2014, Vest, 2009; Laazer and Toloza, 2007).

It is also important to take into account the production quality of the videos which refers to the technical quality of the video in terms of image, sound and the technical support for the transmission of the educational contents: blackboard, specific software, direct recording in a classroom (Acuña-Soto et al., 2018). Technical high-quality is a key factor to retain the attention of the viewer and to facilitate the transmission of educational contents in contexts where the instructor is not present (see Koumi, 2006; Vest, 2009; Laazer and Toloza, 2007; Acuña-Soto et al., 2018).

Accessibility is another important aspect which describes how easy it is for the viewers to find the video. This is a crucial question in platforms as YouTube. Selection of appropriate keywords is essential as well as a precise and accurate title and description of the content of the video. If important keywords are missing or the title is imprecise the video will not get to the interested audience. A short and precise description of the contents and a clear and descriptive miniature of the video will avoid early drop-offs and dissatisfied viewers (see Laazer and Toloza, 2007; Acuña-Soto, et al., 2018).

All the previously described criteria are of very different nature, qualitative and quantitative and data are of heterogeneous character: real numbers, intervals, linguistic labels... This situation could lead to some problems related to the normalization of the criteria. Classical normalization procedures do not always take into account situations where the different nature of the data of the decision matrix (real numbers, intervals, sets linguistic variables...) could make the ranking of the alternatives quite unstable. In order to address this situation, in this paper we will use a normalization method based on the similarity with a given reference solution (ideal solution) that will allow the conversion of the original decision matrix into a transformed decision matrix composed of the similarity degrees of each alternative to the benchmark or ideal of each criterion. In this way, and thanks to this normalization procedure, the nature of the transformed normalized data will be homogeneous. The transformed normalized decision making matrix will be then integrated in a new VIKOR-based approach, Ideal Similarity VIKOR (IS-VIKOR). The objective of this paper is to propose a multiple criteria decision making approach based on this method, to assess and rank mathematical videos based on their educational quality taking into account the multiple points of views and dimensions of this concept).

The compromise ranking method (VIKOR) was first developed by Opricovic (1998). The method introduces the multiple criteria ranking index based on the particular measures of “closeness” to the ideal solution (Opricovic, 1998; Opricovic and Tzeng, 2004). The compromise solution is a feasible solution, which is the closest to the ideal solution.

In many real world applications, the nature of the data is very heterogeneous. This is the situation in this work. The quality assessment of the mathematical videos will rely on both, quantitative and qualitative criteria and will include continuous real variables, binary variables and linguistic variables and/or interval data. In order to avoid normalization problems associated to the different nature of the data, in this work, we

will transform the original data using a normalization process proposed by Acuña-Soto et al. (2018) that takes into account the degree of similarity of each observation with a reference (ideal) value. The transformed data will be then used in a new VIKOR based approach, ideal similarity VIKOR (IS-VIKOR), which relies on the idea that the reference solutions, instead of being optimal solutions can take any value between the minimum and maximum values of the range of the criteria (see Cables et al., 2016; Acuña-Soto et al., 2018).

The obtained results will be compared with those obtained by Acuña-Soto et al., (2018) using a TOPSIS-based approach which integrates the previously described normalization method, Ideal Similarity TOPSIS (IS-TOPSIS). The Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) (Hwang and Yoon 1981) ranks decision alternatives based on a set of decision criteria choosing firstly the alternatives that simultaneously have the shortest distance from the positive ideal solution (PIS) and the farther distance from the negative-ideal solution (NIS). The positive ideal solution maximizes criteria of the type “the more, the better” and minimizes criteria of the type “the less, the better”, whereas the negative ideal solution maximizes “the less, the better” criteria and minimizes “the more, the better” criteria. Our interest on this comparison relies on the interesting features of both MCDM methods, IS-TOPSIS and IS-VIKOR. They both incorporate a normalization procedure that overcomes the potential problem in ranking terms of a heterogeneous nature of the data in the decision matrix, based on the idea of similarity with the ideal solution. Moreover, they both allow to consider as ideal solution any reference point not necessarily being related to an optimizing philosophy. However, the selection of one MCDM method or another, a TOPSIS-based method or a VIKOR-based method, will depend on the educational purpose of the instructor. If in the search of online videos the instructor wants to encourage as much as possible the use of high quality videos from an educational point of view, then a VIKOR-based approach will be more adequate for the ranking of the videos. Nevertheless, if the instructor aims at finding videos as similar as possible to the positive ideal video (ideal reference) and simultaneously as different as possible to the negative ideal video, then, the best option would be a TOPSIS-based approach.

As we will see in the next sections, the selection of one ranking method or another may depend on the educational purpose of the instructor.

2. Two classical MCDM ranking methods: TOPSIS and VIKOR

In this section we will briefly review the main characteristics of the classical TOPSIS and VIKOR methods. Table 1 presents a comparative description of the main steps of these ranking methods. The main difference between TOPSIS and VIKOR is that VIKOR uses an aggregating function representing “closeness to the ideal”. TOPSIS, as described in the introduction, ranks decision alternatives based on a set of decision criteria choosing firstly the alternatives that simultaneously have the shortest distance from the positive ideal solution or ideal solution and the farther distance from the negative-ideal solution. This can lead to a situation where the best solution ranked by TOPSIS is not the closest to the ideal as the relative importance of the distances to the ideal and negative ideal are not considered (see Opricovic and Tzeng, 2004 for a further discussion on this topic).

The selection of one ranking method or another will be based on the importance given by the decision maker to the ideal solution. In the context of the ranking of educational mathematical videos, if the instructor prefers the students to use videos as much similar

as possible to the ideal or reference video then, a VIKOR-based approach will be more adequate. However, if the instructor also considers as important avoiding as much as possible the use of videos considered as anti-ideal, then a TOPSIS-based approach will be more suitable. In some situations, it is easier to properly explain a new concept (ideal) than to correct a wrong content (anti-ideal).

Table 1. Main steps in the classical TOPSIS and VIKOR approaches

| SIMILARITIES BETWEEN TOPSIS AND VIKOR | |
|---|--|
| <p>STEP 1. Determine the decision matrix $X = (x_{ij})_{m \times n}$ where the number of alternatives is m and the number of criteria is n, being x_{ij} real numbers representing the values of the ith criterion for the alternative j.</p> <p>STEP 2. Construct the normalized decision matrix, $(r_{ij})_{m \times n}$.</p> <p>STEP 3. Determine the weights associated to the different criteria, $w_j \in [0,1]$, $w_1 + w_2 + \dots + w_n = 1$.</p> <p>STEP 4. Determine the weighted normalized decision matrix. $v_{ij} = w_j r_{ij}$, $i = 1, \dots, m$, $j = 1, \dots, n$.</p> <p>STEP 5. Determine the positive ideal A^+ (PIS) and negative ideal A^- solutions (NIS), being F^+ associated with “the more, the better” criteria and F^- associated with “the less, the better” criteria</p> $I^+ = \{v_1^+, \dots, v_n^+\} = \left\{ \left(\max_i v_{ij}, j \in F^+ \right) \left(\min_i v_{ij}, j \in F^- \right) \right\} \quad i = 1, 2, \dots, m.$ $I^- = \{v_1^-, \dots, v_n^-\} = \left\{ \left(\min_i v_{ij}, j \in F^+ \right) \left(\max_i v_{ij}, j \in F^- \right) \right\} \quad i = 1, 2, \dots, m.$ | |
| DIFFERENCES BETWEEN TOPSIS AND VIKOR | |
| TOPSIS Method | VIKOR Method |
| <p>STEP 6. Calculate the separation measures, D_i^+ and D_i^-, distances to the PIS and NIS, respectively,</p> $D_i^+ = \left(\sum_{j=1}^n (v_{ij} - v_j^+)^2 \right)^{1/2}, \quad i = 1, \dots, m.$ $D_i^- = \left(\sum_{j=1}^n (v_{ij} - v_j^-)^2 \right)^{1/2}, \quad i = 1, \dots, m.$ <p>STEP 7. Calculate the relative proximity to the ideal solution using the relative index</p> $P_i = \frac{D_i^-}{D_i^+ + D_i^-}, \quad i = 1, \dots, m.$ <p>STEP 8. Rank the best alternatives according to P_i in descending order. The result is one ranking list.</p> | <p>STEP 6. Compute the values</p> $S_i = \sum_{j=1}^n w_j (x_j^+ - x_{ij}) / (x_j^+ - x_j^-) \quad i = 1, \dots, m.$ $R_i = \max_j [w_j (x_j^+ - x_{ij}) / (x_j^+ - x_j^-)] \quad i = 1, \dots, m.$ <p>STEP 7. Compute the ranking indexes</p> $Q_i = \lambda \frac{(S_i - S^+)}{(S^- - S^+)} + (1 - \lambda) \frac{(R_i - R^+)}{(R^- - R^+)},$ <p>where $S^+ = \min_i S_i$, $S^- = \max_i S_i$, $R^+ = \min_i R_i$, $R^- = \max_i R_i$.</p> <p>$\lambda \in [0,1]$ is introduced as weight of strategy of “the majority criteria” (or “the maximum group utility”).</p> <p>STEP 8. Rank the best alternatives according to the values S_i, R_i and Q_i in decreasing order. The results are three ranking lists.</p> |

Source: own elaboration based on Opricovic and Tzeng (2004).

In the TOPSIS approach alternative $i=1$, which is the best ranked by P (maximum), is better than alternative $i=2$ if the following condition is satisfied: $P_1 > P_2$. In the VIKOR

approach we propose as compromise solution the alternative $i=1$, which is the best ranked by Q (minimum) if the following two conditions are satisfied:

C1. *Acceptable advantage*: $Q_1 - Q_2 \geq DQ$, where $i=2$ is the second best ranked alternative in the ranking by Q and $DQ=1/(m-1)$.

C2. *Acceptable stability*: The alternative $i=1$ must also be the best ranked by R and/or S .

If one of the conditions is not satisfied, then a set of compromise solutions is proposed:

- Alternatives $i=1$ and $i=2$ if only condition C.2 is not satisfied, or
- Alternatives $i=1, 2, \dots, M$, if the condition C.1 is not satisfied, where M is determined by the relation $Q_M - Q_1 < DQ$, for maximum i .

Tables 2, 3 and 4 present a comparative description of the different approaches used to address four of the main decision to be made for each of the proposed ranking methods: normalization, weighting scheme and distance metric. A large number of approaches can be found in the literature dealing with the previous questions and giving rise to different TOPSIS-based and VIKOR-based approaches (see Behzadian et al., 2012; Zyoud and Fuchs-Hanusch, 2017 and Mardani et al., 2016).

Table 2. Commonly used normalization methods in TOPSIS and VIKOR

| Method | Formulae | Sample of References for TOPSIS | Sample of References for VIKOR |
|--|---|--|--|
| (i) Vector Normalization | $r_{ij} = \frac{x_{ij}}{\ x_j\ } = \frac{x_{ij}}{\sqrt{\sum_{k=1}^{\#X} x_{kj}^2}}$ $\text{if } j \in F^+$ $r_{ij} = \frac{1/x_{ij}}{\ 1/x_j\ } = \frac{1/x_{ij}}{\sqrt{\sum_{k=1}^{\#X} (1/x_{kj})^2}}$ $\text{if } j \in F^-$ $r_{ij} =$ | Isiklar and Li (2010), Secme et al. (2009), Shyur (2006), Wu et al. (2009), Wu et al. (2010), Yu et al. (2011), | Vahdani et al. (2012), Yazdani and Payam (2015), Rezaie et al. (2014), Chou et al. (2014) |
| (ii) Linear Scale Transformation (Max-Min) | $r_{ij} = \frac{x_{ij} - \min_{k=1, \dots, \#X} x_{kj}}{\max_{k=1, \dots, \#X} x_{kj} - \min_{k=1, \dots, \#X} x_{kj}}$ $\text{if } j \in F^+$ $r_{ij} = \frac{\max_{k=1, \dots, \#X} x_{kj} - x_{ij}}{\max_{k=1, \dots, \#X} x_{kj} - \min_{k=1, \dots, \#X} x_{kj}}$ $\text{if } j \in F^-$ | Lin et al. (2010), Bai et al. (2014) | Chatterjee et al. (2010), Zhu et al. (2015), Liu et al. (2015), Tzeng and Huang (2012), Peng (2015), Zavadskas and Antuchevičienė (2004) |
| (iii) Linear Scale Transformation (Max) | $r_{ij} = \frac{x_{ij}}{\max_{k=1, \dots, \#X} x_{kj}}$ $\text{if } j \in F^+$ $r_{ij} = 1 - \frac{x_{ij}}{\max_{k=1, \dots, \#X} x_{kj}}$ $\text{if } j \in F^-$ | if Ertugrul and Karakasglu (2008), KarimiAzari et al. (2011), Sun (2010), Sun and Lin (2009), Vahdani et al. (2012) if | Vinodh et al. (2014), Feng et al. (2013), Mousavi et al. (2013), Ebrahimnejad et al. (2012) |
| (iv) Linear Scale Transformation (Sum) | $r_{ij} = \frac{x_{ij}}{\sum_{k=1}^{\#X} x_{kj}} \quad \text{if } j \in F^+$ $r_{ij} = \frac{1/x_{ij}}{\sum_{k=1}^{\#X} (1/x_{kj})} \quad \text{if } j \in F^-$ | Huang and Peng (2012) | |
| (v) Other | | Acuña-Soto et al. (2018), Cables et al. (2016) | Acuña-Soto et al. (2018), Zeng et al. (2013), Jahan et al. (2011) |

Source: Own elaboration based on Ouenniche et al. (2017).

Table 3. Commonly used weighting schemes in TOPSIS and VIKOR

| | Description | Sample of references for TOPSIS | Sample of references for VIKOR |
|------------|--|---|---|
| Subjective | Direct assignment of weights | Benitez et al., (2007); Ertugrul and Karakasglu, (2009); Li, (2010) | Vinodh et al. (2014), Peng et al. (2015), Vučijak et al. (2015), Chatterjee et al. (2009). |
| | Analytical Hierarchy Process (AHP) based Methods | Khademi-Zare et al., (2010); Lin et al., (2010); Wu et al., (2010); Yu et al., (2011) | Chatterjee et al. (2010), Zhu et al. (2015), Liu et al. (2015), Tzeng and Huang (2012), , Wu et al. (2011), Chen and Chen (2010), |
| | PROMETHEE II | | Feng et al. (2013) |
| | SWARA (Step-wise Weight Assessment Ratio Analysis) | | Zolfani et al. (2013) |
| Objective | Equal weights | Chang et al. (2010) | Zeng et al. (2013) |
| | Entropy weight method | Chang et al. (2010) | Liu et al. (2015), Chatterjee et al. (2009), Jahan et al. (2011), Chauhan and Vaish (2012), |
| | Coefficient of Variation weight method | Chang et al. (2010) | Zavadskas and Antuchevičienė (2004) |
| | Data Envelopment Analysis | Chen et al., 2009 | Peng (2015), Lee and Pai (2015), Hsu (2014, 2015) |
| | Regression techniques | Olson (2004); Wu and Olson (2006) | |

Source: Own elaboration based on Ouenniche et al. (2017).

Table 4. Commonly used distance metrics in TOPSIS and VIKOR

| Distance | Formulae | Sample of references for TOPSIS | Sample of references for VIKOR |
|---|---|---|--|
| Euclidean distance (Minkowski with p=2) | $d_{i,k} = \sqrt{\sum_{j=1}^m (x_{i,j} - x_{k,j})^2}$ | Chang et al. (2010), Huang and Peng (2012), Li (2010), Yu et al. (2011) | |
| Manhattan/Cityblock distance (Minkowski with p=1) | $d_{i,k} = \sum_{j=1}^m x_{i,j} - x_{k,j} $ | Chang et al. (2010) | Chatterjee et al. (2010), Zhu et al. (2015), Tzeng and Huang (2012), Peng (2015) |
| Chebishev distance (Minkowski with p=∞) | $d_{i,k} = \max x_{i,j} - x_{k,j} $ | | Chatterjee et al. (2010), Zhu et al. (2015), Peng (2015), Vučijak et al. (2015) |
| Mahalanobis distance | $d_{i,k} = \sqrt{(x_i - x_k)^t \Sigma^{-1} (x_i - x_k)}$ Σ = variance-covariance matrix | Chang et al. (2010), Vega et al. (2014) | |

Source: Own elaboration based on Ouenniche et al. (2017).

The VIKOR-based approaches use in most of the cases a Linear Scale Transformation (Max-Min) in order to normalize data. However, this classical normalization approach could give rise to ranking problems in presence of data of heterogeneous nature. In the

following section we will describe a normalization process that will allow us to handle data of different nature.

3. Normalization process

In what follows we describe an alternative to traditional normalization in VIKOR-based approaches, the linear scale transformation (Max-Min), that takes into account the degree of similarity of each criterion with its ideal. Aggregation is addressed for criteria that have been transformed into homogeneous as they all reflect the degree of similarity with the ideal valuation (see Acuña-Soto et al., 2018).

Let us consider a criterion c_j with range $[A_j, B_j]$, for instance, $[A_j, B_j] = [\min_i x_{ij}, \max_i x_{ij}]$. We can interpret that any of the normalizations in Table 2 reflects, with a value in $[0, 1]$, the similarity of x_{ij} with B_j (when the objective is to maximize the criterion) or with A_j (when the objective is to minimize the criterion). This idea can be generalized when an ideal exists for criterion c_j given by an interval $[a_j, b_j] \subseteq [A_j, B_j]$.

Let us consider a valuation of an alternative with respect to a given criterion j described by an interval $[x_{ij}^L, x_{ij}^R] \subseteq [A_j, B_j]$. We will propose a normalization process based on the similarity between $[x_{ij}^L, x_{ij}^R]$ and $[a_j, b_j]$, i.e.

$$\text{Sim}([x_{ij}^L, x_{ij}^R], [a_j, b_j]) = 1 - d_H([x_{ij}^L, x_{ij}^R], [a_j, b_j]), \quad \forall [x_{ij}^L, x_{ij}^R] \subseteq [A_j, B_j], \quad (3)$$

where $d_H([x_{ij}^L, x_{ij}^R], [a_j, b_j])$ is the Hamming normalized distance (Canós et al., 2014), for the range of data $[A_j, B_j]$, that is,

$$d_H([x_{ij}^L, x_{ij}^R], [a_j, b_j]) = \frac{|x_{ij}^L - a_j| + |x_{ij}^R - b_j|}{2(B_j - A_j)}, \quad \forall [x_{ij}^L, x_{ij}^R] \subseteq [A_j, B_j]. \quad (4)$$

It is important to notice that although the previously introduced Hamming distance has very suitable properties, other normalized distances could be also used (see for instance, Zeng and Guo, 2008).

Then, the normalization is given by

$$N([x_{ij}^L, x_{ij}^R]) = \begin{cases} 1, & [x_{ij}^L, x_{ij}^R] \subseteq [a_j, b_j] \\ 1 - \frac{|x_{ij}^L - a_j| + |x_{ij}^R - b_j|}{2(B_j - A_j)}, & [x_{ij}^L, x_{ij}^R] \not\subseteq [a_j, b_j], [x_{ij}^L, x_{ij}^R] \subseteq [A_j, B_j] \end{cases} \quad (5)$$

Other cases can be considered giving rise to other normalization functions (see Acuña-Soto et al., 2018 for more details). Based on (5), in what follows we will propose a VIKOR-based approach, IS-VIKOR, comparing it with the IS-TOPSIS approach proposed by Acuña-Soto et al. (2018).

4. Ideal Similarity VIKOR (IS-VIKOR)

Let us consider, $[A_j, B_j]$ the range of a decision criterion j that belongs to a universe of discourse; $[a_j, b_j]$ the reference ideal for that criterion, with $[a_j, b_j] \subseteq [A_j, B_j]$ and

$x_{ij} \in [A_j, B_j]$ the valuation of an alternative i with regards to the considered criterion j . Table 5 describes the main steps of the IS-VIKOR proposed approach. We have also included a summary of the IS-TOPSIS steps proposed by Acuña-Soto et al. (2018) in order to compare both approaches. Main differences, as in the case of traditional TOPSIS and VIKOR, are referred to the simultaneous consideration of maximization of the distance to the anti-ideal or negative ideal solution in the aggregation step and the type of distance function used for that aggregation. In Step 6 in the VIKOR approach, λ is introduced as weight of the strategy of ‘the majority of attributes’ (or ‘the maximum group utility’). The selection of this value plays an important role as determines the type of compromise solution. A value of λ greater than 0.5 implies a “voting by majority” compromise solution whereas a value of λ smaller than 0.5 implies a “veto” compromise. If the decision maker is looking for a “consensus” solution between the “majority” and the “veto” solutions, then the value of λ would be equal to 0.5. The value of $\lambda \in [0,1]$ is usually taken equal to 0.5 (Chatterjee et al. (2009).

Table 5. Main steps in the IS-TOPSIS and IS-VIKOR approaches

| Similarities between IS-TOPSIS and IS-VIKOR | |
|--|--|
| STEP 1. Define the working context: type of data, number of decision makers, criteria range, reference ideal for each criterion $[a_j, b_j]$ and weights $w_j \in [0,1]$ with $w_1 + \dots + w_n = 1$, associated to each criterion $1 \leq j \leq n$. | |
| STEP 2. Obtain the valuation matrix $X = \left([x_{ij}^L, x_{ij}^R] \right)_{m \times n}$ being $[x_{ij}^L, x_{ij}^R]$ the valuation of alternative i with regards to criterion j expressed as an interval. In case of group decision making this matrix will be a consensual matrix based on the individual valuations of the decision makers. | |
| STEP 3. Normalize the matrix X using function in (5), $Y = (y_{ij})_{m \times n}$. | |
| STEP 4. Given, $w_j \in [0,1]$, we calculate the weighted normalized matrix, $Y' = (y'_{ij})_{m \times n} = (w_j y_{ij})_{m \times n}$. | |
| Differences between IS-TOPSIS and IS-VIKOR | |
| IS-TOPSIS Method | IS-VIKOR Method |
| STEP 5. Calculate the variation to the normalized reference ideal for each alternative i . Let us notice that the vector representing the reference ideal will be $(1,1,\dots,1)$ and the vector representing the reference anti-ideal will be $(0,0,\dots,0)$ therefore the weighted reference ideal will coincide with the weights vector (w_1, w_2, \dots, w_n) | STEP 5. Calculate the variation to the normalized reference ideal for each alternative i . Let us notice that the vector representing the reference ideal will be $(1,1,\dots,1)$ and the vector representing the reference anti-ideal will be $(0,0,\dots,0)$ therefore the weighted reference ideal will coincide with the weights vector (w_1, w_2, \dots, w_n) |
| $I_i^+ = \sqrt{\sum_{j=1}^n (y'_{ij} - w_j)^2}, I_i^- = \sqrt{\sum_{j=1}^n (y'_{ij})^2}, i = 1, \dots, m.$ | $I_i^{p=1} = \sum_{j=1}^n (w_j - y'_{ij}) \quad i = 1, \dots, m.$ $I_i^{p=\infty} = \max_i [(w_j - y'_{ij})] \quad i = 1, \dots, m.$ |
| STEP 6. Calculate the relative proximity to the ideal solution using the relative index | STEP 6. Compute the ranking indexes |
| $P_i = \frac{I_i^-}{I_i^+ + I_i^-}, \quad 1 \leq i \leq m.$ | $Q_i = \lambda \frac{(I_i^{p=1} - S^+)}{(S^- - S^+)} + (1 - \lambda) \frac{(I_i^{p=\infty} - R^+)}{(R^- - R^+)},$ where $S^+ = \min_i I_i^{p=1}, \quad S^- = \max_i I_i^{p=1},$ $R^+ = \min_i I_i^{p=\infty}, \quad R^- = \max_i I_i^{p=\infty}.$ |
| | $\lambda \in [0,1]$ is introduced as weight of strategy of “the majority criteria” (or “the maximum group utility”). |

STEP 7. Rank the best alternatives according to P_i in descending order. The result is one ranking list.

STEP 7. Rank the best alternatives according to the values $I_i^{p=1}$, $I_i^{p=\infty}$ and Q_i in decreasing order. The results are three ranking lists.

Source: Own elaboration.

The first four steps are common to both ranking methods and include normalization proposed in the previous section which transforms all the criteria in order to reflect the similarity with the ideal of each alternative valued in each criterion. Let us notice that using normalization N given by (5) we transform the original criteria in such a way that now they are all of the type “the more, the better” in the sense of maximizing their similarity with the ideal solution. In what follows we will try following example, we highlight two of the main advantages of the proposed normalization method for those situations in which the nature of the data is highly heterogeneous. On the one hand, we obtain a higher stability of the solution. On the other hand, our proposal allows alternatives to be ranked even in the case of an ideal solution occupying an intermediate position in the range. In what follows we will briefly illustrate these two features.

Example: Let us consider 5 alternatives valued with respect to 2 criteria (columns 2 and 3 in Table 3). The first criterion is given by linguistic terms and the second one by real number between 0 and 1. We will consider two different situations. In the first one, the reference ideal will be form by the optimum values of the decision criteria. In the second one, we will set the reference ideal as a set of intermediate values for the decision criteria belonging to the range of each criteria.

a) Ideal solution composed of the optima (maximum values) of all the criteria (bold values in Table 6).

Let us consider the following weights $w_1=0.4$ and $w_2=0.6$. Let us model criterion C1 in two different ways: using a numerical scale as in (6) (see column 4 in Table 6) and using intervals on the real line (column 7 in Table 6). The “consensual” compromise solution, Q_i for $\lambda=0.5$, is displayed in columns 6 and 9. The obtained rankings for the different modeling of the valuations are displayed in the last row in Table 6.

Table 6. Decision Criteria

| Criteria | Description | Question |
|------------------------|---------------------|---|
| Didactic | | |
| Z_1 | Epistemic facet | To what extent are the treated mathematical concepts correct? |
| Z_2 | Cognitive | To what extent does the author mention all the elements in a fluid way? |
| Z_3 | Affective facet | To what extent do the contents of the video attract the attention of the user? |
| Z_4 | Mediational facet | To what extent time and resources are wasted in the explanation? |
| Z_5 | Ecological facet | To what extent is the video adapted to the concrete educational context? |
| Z_6 | Interactional facet | To what extent is it difficult to understand the author? |
| Interaction with users | | |
| Z_7 | Views | Number of visualizations without taking into account if the video has been watched entirely |
| Z_8 | Likes | Number of "likes" taking into account publication date |
| Z_9 | Dislikes | Number of "dislikes" taking into account publication date |
| Z_{10} | Comments | Number of comments without taking into account author's responses |
| Z_{11} | Shares | Number of times the video has been shared taking into account |

| | | publication date |
|---------------------------|------------------------------|---|
| Production quality | | |
| Z ₁₂ | Quality of image | Quality of the image of the video (current/optimal resolution) |
| Z ₁₃ | Quality of sound | Quality of the sound of the video (volume/normalized) content loudness (dB) |
| Z ₁₄ | Quality of technical support | Quality of the technical support of the video: To what extent is the medium used to transmit the contents clear? e.g. blackboard capture, normal video, notebook capture, hand write, use of computer software... |
| Accessibility | | |
| Z ₁₅ | Title | Precision of the title: To what extent does the title describe precisely the contents of the video? |
| Z ₁₆ | Keywords | Precision of keywords: To what extent do the keywords describe precisely the contents of the video? |
| Z ₁₇ | Description | Quality of the video description: To what extent does the description of the video precisely indicate the contents of the video? |
| Z ₁₈ | Miniature | Quality of the video miniature: To what extent does the miniature of the video describe precisely the contents of the video? |
| Authority | | |
| Z ₁₉ | Identification | The author is clearly identified (Yes/No) |
| Z ₂₀ | Biography | Quality of the short biography description |
| Z ₂₁ | Professionalism | Degree in which the author is an adequate professional from the educational sector |
| Z ₂₂ | Authorship | The author is the person uploading the video (Yes/No) |
| Z ₂₃ | Subscribers | Number of subscribers which follow the author or person who uploads the video |
| Z ₂₄ | Officiality | The video is published in the context of an official institution or body (Yes/No) |

Source: Acuña-Soto et al. (2018)

As we can observe the selected modeling for the linguistic variables determines the obtained ranking. Positions 2 and 3 are different. Alternative 5 ranks second when the use numerical valuations for the qualitative criterion and ranks third when interval valuations are used to value this criterion.

In order to check whether the modeling done with intervals is more stable than the first one we will replace value 0.246 for alternative A₂ in criterion C₂ by value 0.249. In the last row in Table 7 we can observe how now the ranking obtained with numbers varies whereas the ranking obtained using intervals remains the same.

Table 7. Decision alternatives

| <i>Video</i> | <i>Title</i> | <i>URL</i> |
|-----------------|---|---|
| V ₁ | Basis for a set of vectors | https://www.youtube.com/watch?v=rUJ5B-swc9Y |
| V ₂ | Basis for a vector Space | https://www.youtube.com/watch?v=XeU6ixsv1IE |
| V ₃ | Basis and dimension of a vector space | https://www.youtube.com/watch?v=-42bA6CKRnU |
| V ₄ | Linear Algebra - basis of a vector space | https://www.youtube.com/watch?v=XErZLJYwhcE |
| V ₅ | Basis of a vector space | https://www.youtube.com/watch?v=dOIVxQCHT0k |
| V ₆ | Vector space, basis, dimension | https://www.youtube.com/watch?v=nOfY1ZATzIM |
| V ₇ | Basis, vectors and coordinates | https://www.youtube.com/watch?v=wYKAw5QanJY |
| V ₈ | Basis and dimension | https://www.youtube.com/watch?v=AqXOYgpbMBM |
| V ₉ | Linear combinations, span, and basis vectors | https://www.youtube.com/watch?v=k7RM-ot2NWY |
| V ₁₀ | Basis and dimension | https://www.youtube.com/watch?v=lf5WacddAo4 |
| V ₁₁ | Linear algebra example problems - vector space basis example #1 | https://www.youtube.com/watch?v=3l3qfs2vINE |

Source: Acuña-Soto et al. (2018)

b) Let us suppose that for criterion C_1 the ideal is considered to be an intermediate value {Fair} (bold values in Table 8). In this case, although the classical VIKOR approach allows modeling using linguistic variables, it could not be used as the ideal is not an optimal solution.

Table 8. Individual and consensual rates with linguistic labels for Z_{14}

| <i>Video</i> | <i>Expert 1</i> | <i>Expert 2</i> | <i>Expert 3</i> | <i>Consensual</i> |
|-----------------|-----------------|-----------------|-----------------|-------------------|
| V ₁ | G | G | F | {F,G} |
| V ₂ | VP | P | VP | {VP,P} |
| V ₃ | F | P | P | {P,F} |
| V ₄ | G | G | G | G |
| V ₅ | P | P | F | {P,F} |
| V ₆ | VG | VG | G | {G,VG} |
| V ₇ | P | P | F | {P,F} |
| V ₈ | G | VG | P | {P,VG} |
| V ₉ | VG | VG | G | {G,VG} |
| V ₁₀ | VG | VG | P | {P,VG} |
| V ₁₁ | VG | G | G | {G,VG} |
| V ₁₂ | VP | VP | VP | VP |

Source: Acuña-Soto et al. (2018)

However, with the proposed normalization ideal solutions different than the optimal can be considered and do not present any difficulties. In Table 8 we have displayed the obtained ranking for weights $w_1=0.4$ and $w_2=0.6$, considering a different ideal solution.

In the next section, we will illustrate and compare both ranking approaches using a real multiple criteria decision making problem.

5. Ranking of mathematical instructional videos

In order to illustrate the suitability of the proposed methodological approach to the resolution of real decision making problems we will apply IS-VIKOR to the ranking of mathematical educational videos in You Tube. Obtained results will be compared with those obtained by Acuña-Soto et al. (2018) using IS-TOPSIS. We have considered 24 decision criteria organized in 5 fundamental dimensions: didactic, interaction with users, production quality, accessibility and authority (see Tables 9 and 10).

Table 9. Criteria description, ideals and weights

| <i>Criteria</i> | <i>Description</i> | <i>Statement</i> | <i>Ideal</i> | <i>Weight</i> |
|------------------------|---------------------|------------------|--------------|---------------|
| Didactic | | | | 0.329555219 |
| Z ₁ | Epistemic facet | Positive | {VG} | 0.128531242 |
| Z ₂ | Cognitive | Positive | {G,VG} | 0.070062226 |
| Z ₃ | Affective facet | Positive | {G} | 0.017176916 |
| Z ₄ | Mediational facet | Negative | {P} | 0.029547704 |
| Z ₅ | Ecological facet | Positive | {F,G} | 0.015375083 |
| Z ₆ | Interactional facet | Negative | {VP} | 0.068862048 |
| Interaction with users | | | | 0.169464716 |
| Z ₇ | Views | Positive | 812.7005 | 0.019420127 |
| Z ₈ | Likes | Positive | 13.5695 | 0.048848884 |

| | | | | |
|--------------------|------------------------------|----------|----------|-------------|
| Z ₉ | Dislikes | Negative | 0.0000 | 0.007171995 |
| Z ₁₀ | Comments | Positive | 1.1765 | 0.082625834 |
| Z ₁₁ | Shares | Positive | 2.0588 | 0.011397876 |
| Production quality | | | | 0.10090001 |
| Z ₁₂ | Quality of image | Positive | {F,G} | 0.067987493 |
| Z ₁₃ | Quality of sound | Positive | {F,G} | 0.022756534 |
| Z ₁₄ | Quality of technical support | Positive | G | 0.010155983 |
| Accessibility | | | | 0.066259587 |
| Z ₁₅ | Title | Positive | {VG} | 0.038568971 |
| Z ₁₆ | Keywords | Positive | {F,G} | 0.017478961 |
| Z ₁₇ | Description | Positive | {F,G} | 0.00756904 |
| Z ₁₈ | Miniature | Positive | {F,G} | 0.002642615 |
| Authority | | | | 0.333820468 |
| Z ₁₉ | Identification | Positive | Yes | 0.085088368 |
| Z ₂₀ | Biography | Positive | {F,G} | 0.050240591 |
| Z ₂₁ | Professionalism | Positive | G | 0.140542773 |
| Z ₂₂ | Authorship | Positive | Yes | 0.011039751 |
| Z ₂₃ | Subscribers | Positive | 885.0267 | 0.029495463 |
| Z ₂₄ | Officiality | Positive | Yes | 0.017413523 |

Source: Acuña-Soto et al. (2018).

We have asked educational experts to assess the performance of the videos in the decision criteria. As this assessment is in most of the cases imprecise, ambiguous and uncertain we have chosen three different higher education institutions from two different countries to assess the educational performance of a small sample of 12 mathematical videos published in You Tube. These videos were the ones displayed first by Google search engine for the used keywords. The selected number of videos have only illustration purposes and the proposed ranking method could be easily applied to any set of videos.

Table 10 describes the considered decision criteria and the questions used to obtain evaluations from the three experts. As an example, we have chosen a mathematical concept, basis of a vector space, which is concrete and can be easily and shortly explained, and we have conducted a search of You Tube videos in Google. However, any other mathematical concept could be also selected. Table 10 displays a description of our decision alternatives.

Table 10. Obtained rankings with IS-TOPSIS and IS-VIKOR

| Video | IS-TOPSIS Rank | P_i | IS- VIKOR Consensual Rank | Q_i | IS- VIKOR Efficiency Rank | $I_i^{p=1}$ | IS- VIKOR Equity Rank | $I_i^{p=\infty}$ |
|-----------------|----------------|---------|------------------------------------|---------|------------------------------------|-------------|--------------------------------|------------------|
| V ₁ | 9 | 0.72822 | 9 | 0.75209 | 9 | 0.36484 | 9 | 0.08508 |
| V ₂ | 8 | 0.66235 | 8 | 0.77473 | 8 | 0.42780 | 8 | 0.0819 |
| V ₃ | 5 | 0.65108 | 10 | 0.78951 | 10 | 0.3853 | 11 | 0.08508 |
| V ₄ | 10 | 0.65020 | 5 | 0.88627 | 5 | 0.43818 | 2 | 0.08508 |
| V ₅ | 7 | 0.63799 | 11 | 0.65170 | 1 | 0.3493 | 6 | 0.08262 |
| V ₆ | 6 | 0.62940 | 7 | 0.71870 | 7 | 0.39075 | 7 | 0.08232 |
| V ₇ | 11 | 0.62278 | 6 | 0.69561 | 11 | 0.37581 | 5 | 0.08247 |
| V ₈ | 1 | 0.62153 | 1 | 0.56168 | 3 | 0.33733 | 10 | 0.08029 |
| V ₉ | 3 | 0.60807 | 2 | 0.00000 | 6 | 0.22707 | 1 | 0.06798 |
| V ₁₀ | 2 | 0.58999 | 3 | 0.63091 | 2 | 0.33797 | 3 | 0.08262 |
| V ₁₁ | 4 | 0.58180 | 4 | 0.68150 | 4 | 0.38264 | 4 | 0.08156 |

| | | | | | | | | |
|----------|----|---------|----|---------|----|---------|----|---------|
| V_{12} | 12 | 0.51206 | 12 | 1.00000 | 12 | 0.50033 | 12 | 0.08508 |
|----------|----|---------|----|---------|----|---------|----|---------|

Source: Own elaboration and Acuña-Soto et al. (2018).

We have measured the performance of each video in each quantitative criterion taking into account the number of days the video had been available in YouTube until the date of collection of the data (20 of August 2017) to make data comparable (see Table 11 and Table A in the appendix).

Table 11. Individual and consensual rates with linguistic labels for Z_{14}

| <i>Video</i> | <i>Expert 1</i> | <i>Expert 2</i> | <i>Expert 3</i> | <i>Consensual</i> |
|--------------|-----------------|-----------------|-----------------|-------------------|
| V_1 | G | G | F | {F,G} |
| V_2 | VP | P | VP | {VP,P} |
| V_3 | F | P | P | {P,F} |
| V_4 | G | G | G | G |
| V_5 | P | P | F | {P,F} |
| V_6 | VG | VG | G | {G,VG} |
| V_7 | P | P | F | {P,F} |
| V_8 | G | VG | P | {P,VG} |
| V_9 | VG | VG | G | {G,VG} |
| V_{10} | VG | VG | P | {P,VG} |
| V_{11} | VG | G | G | {G,VG} |
| V_{12} | VP | VP | VP | VP |

Source: Acuña-Soto et al. (2018)

The three experts scored the performance of the videos with respect to each qualitative criterion using following linguistic labels:

{Very Poor, Poor, Fair, Good, Very Good}

Once linguistic labels were obtained for each video in each dimension in each group, we obtained a consensual assessment for each dimension, i , and video, j , in the form of intervals. The three experts were given the same importance as we have considered their level of expertise and experience to be the same. However, other situations could be considered depending on the decisional context.

The consensual assessment is expressed as the union set of the individual linguistic rates. Although the use of other aggregation operators is possible (minimum, maximum, intersection...) in this paper the union operator between sets of linguistic valuations has been selected in an attempt to reflect in this way the potential disparity in the assessments due to the inherent ambiguity and subjectivity of some of the criteria (see Gil-Aluja, 1999 for more details). Table 11 displays the obtained individual and consensual ratings for the quality of technical support criterion using linguistic variables (the assessments of all qualitative and quantitative criteria can be found in the appendix). In these tables we can observe that, although there is a high degree of similarity in the valuations from the three experts concerning some of the criteria, there are other criteria for which the valuations are quite different. This disparity intends to be captured in a suitable way by the consensual intervals obtained applying the union operator to the sets composed of the linguistic valuations from the three experts.

Once the videos have been scored and rated in all the dimensions by the experts and consensual intervals have been obtained, a different expert is asked to provide the ideal (reference) scores and rates for each criterion (see Table 12). This expert belongs to the Ibero-American Laboratory for the Assessment of Education Processes (LABIPE, <https://www.uv.es/liern/LABIPE>).

Table 12. Criteria description, ideals and weights

| Criteria | Description | Statement | Ideal | Weight |
|-------------------------------|------------------------------|-----------|----------|-------------|
| Didactic | | | | 0.329555219 |
| Z ₁ | Epistemic facet | Positive | {VG} | 0.128531242 |
| Z ₂ | Cognitive | Positive | {G,VG} | 0.070062226 |
| Z ₃ | Affective facet | Positive | {G} | 0.017176916 |
| Z ₄ | Mediational facet | Negative | {P} | 0.029547704 |
| Z ₅ | Ecological facet | Positive | {F,G} | 0.015375083 |
| Z ₆ | Interactional facet | Negative | {VP} | 0.068862048 |
| Interaction with users | | | | 0.169464716 |
| Z ₇ | Views | Positive | 812.7005 | 0.019420127 |
| Z ₈ | Likes | Positive | 13.5695 | 0.048848884 |
| Z ₉ | Dislikes | Negative | 0.0000 | 0.007171995 |
| Z ₁₀ | Comments | Positive | 1.1765 | 0.082625834 |
| Z ₁₁ | Shares | Positive | 2.0588 | 0.011397876 |
| Production quality | | | | 0.10090001 |
| Z ₁₂ | Quality of image | Positive | {F,G} | 0.067987493 |
| Z ₁₃ | Quality of sound | Positive | {F,G} | 0.022756534 |
| Z ₁₄ | Quality of technical support | Positive | G | 0.010155983 |
| Accessibility | | | | 0.066259587 |
| Z ₁₅ | Title | Positive | {VG} | 0.038568971 |
| Z ₁₆ | Keywords | Positive | {F,G} | 0.017478961 |
| Z ₁₇ | Description | Positive | {F,G} | 0.00756904 |
| Z ₁₈ | Miniature | Positive | {F,G} | 0.002642615 |
| Authority | | | | 0.333820468 |
| Z ₁₉ | Identification | Positive | Yes | 0.085088368 |
| Z ₂₀ | Biography | Positive | {F,G} | 0.050240591 |
| Z ₂₁ | Professionalism | Positive | G | 0.140542773 |
| Z ₂₂ | Authorship | Positive | Yes | 0.011039751 |
| Z ₂₃ | Subscribers | Positive | 885.0267 | 0.029495463 |
| Z ₂₄ | Officiality | Positive | Yes | 0.017413523 |

Source: Acuña-Soto et al. (2018).

In order to handle the linguistic rates and ideals, following the procedure proposed by Cables et al. (2016) the linguistic rates have been transformed into numerical values using the following scale:

$$\{VP=1, P=2, F=3, G=4, VG=5\} \quad (6)$$

Based on the ideal solution and following the steps of the IS-TOPSIS and IS-VIKOR approaches described in the previous section, the rankings displayed in Table 13 were obtained considering subjective weights from the expert (see Acuña-Soto et al., 2018 for more details). Opricovic (1998) proposed a method that determines the weight stability intervals. Based on his method, the compromise solution obtained with the initial weights will be replaced if the value of a weight is not within the stability interval (Opricovic and Tzeng, 2004). This stability analysis could be useful for the decision maker in those situations in which the importance of the assigned weights is not prefix in advance by an expert (e.g. the expert is willing to accept small changes in the weighting scheme for the criteria).

Second and third columns in Table 13 display the video ranking and the relative index values obtained with IS-TOPSIS (obtained by Acuña-Soto et al., 2018) whereas columns fourth to ninth show the ranking obtained using IS-VIKOR for Q_i , $I_i^{p=1}$ and $I_i^{p=\infty}$.

Table 13. Obtained rankings with IS-TOPSIS and IS-VIKOR

| <i>Video</i> | <i>IS-TOPSIS Rank</i> | P_i | <i>IS-VIKOR Consensual Rank</i> | Q_i | <i>IS- VIKOR Efficiency Rank</i> | $I_i^{p=1}$ | <i>IS- VIKOR Equity Rank</i> | $I_i^{p=\infty}$ |
|-----------------|-----------------------|---------|---|---------|--|-------------|--|------------------|
| V ₁ | 9 | 0.72822 | 9 | 0.00000 | 9 | 0.3648 | 9 | 0.06799 |
| V ₂ | 8 | 0.66235 | 8 | 0.56168 | 8 | 0.4278 | 8 | 0.08030 |
| V ₃ | 5 | 0.65108 | 10 | 0.63091 | 10 | 0.3853 | 11 | 0.08156 |
| V ₄ | 10 | 0.65020 | 5 | 0.65170 | 5 | 0.4382 | 2 | 0.08192 |
| V ₅ | 7 | 0.63799 | 11 | 0.68150 | 1 | 0.3493 | 6 | 0.08232 |
| V ₆ | 6 | 0.62940 | 7 | 0.69561 | 7 | 0.3908 | 7 | 0.08247 |
| V ₇ | 11 | 0.62278 | 6 | 0.71870 | 11 | 0.3758 | 5 | 0.08263 |
| V ₈ | 1 | 0.62153 | 1 | 0.75209 | 3 | 0.3373 | 10 | 0.08263 |
| V ₉ | 3 | 0.60807 | 2 | 0.77473 | 6 | 0.2271 | 1 | 0.08509 |
| V ₁₀ | 2 | 0.58999 | 3 | 0.78951 | 2 | 0.3380 | 3 | 0.08509 |
| V ₁₁ | 4 | 0.58180 | 4 | 0.88627 | 4 | 0.3826 | 4 | 0.08509 |
| V ₁₂ | 12 | 0.51206 | 12 | 1.00000 | 12 | 0.5003 | 12 | 0.08509 |

Source: Own elaboration and Acuña-Soto et al. (2018)

If we compare the obtained results, we can observe how best ranked videos depend on the applied MCDM. When both, the ideal and anti-ideal solutions are taken into account (i.e. when we choose alternatives which are simultaneously close to the positive ideal and far from the negative ideal) the best ranked video is V₈ (see column 3 in Table 13 displaying the obtained results using the IS-TOPSIS results). However, if we choose the maximum efficient ranking (compromise solution L₁) obtained using IS-VIKOR and taking into account only the distance to the ideal solution, the best ranked video is V₅. If the decision maker wants a maximum equilibrium solution, maximum equity solution (solution where the maximum regret is minimized), then the best video would be V₉. This solution is the one obtained using the IS-VIKOR approach for the metric $p=\infty$. Finally, when we are only interested in approaching the ideal solution, a consensual compromise solution can be also provided as an intermediate solution between compromise solution L₁ and compromise solution L _{∞} , taking $\lambda=0.5$ in STEP 6 in the IS-VIKOR approach. As we can observe comparing results displayed in Table 13, the obtained ranking in this last case, is the most similar to the IS-TOPSIS ranking.

The order of search on Google's search-results pages is based on a priority rank called a "PageRank" that helps rank web pages that match a given search string. The "PageRank algorithm analyzes human-generated links assuming that web pages linked from many important pages are themselves likely to be important. The algorithm computes a recursive score for pages, based on the weighted sum of the PageRanks of the pages linking to them" (https://en.wikipedia.org/wiki/Google_Search). The obtained first twelve results are the videos in the order displayed in Table 10. We can observe how the obtained order is highly different than the rankings obtained based on educational criteria. This highlights the importance of the design of future search engines taking into account several educational dimensions.

6. Conclusions

The use of free-online instructional videos is gaining popularity among instructors and students. Popular free-online video platforms as YouTube are acknowledging the great

potential of this open online resources and have created a large number of important educational channels where authors upload videos with instructional purposes.

The quality of free-online instructional videos is a multidimensional concept which needs to take into account several qualitative and quantitative criteria of highly heterogeneous nature. In the specific case of educational videos, their quality assessment needs to take into account not only the quality of the contents from an academic perspective but also, interaction with users, production quality, accessibility and authority. This last aspect is a critical issue within the education context. Authority reveals to what extent the author of the video has the qualifications and knowledge to produce an educational tool which, especially in the case of free-online platforms as YouTube, will be available all around the world. In the context of popular free video platforms, where everybody can upload contents, identification of the author and description of his/her credentials are key questions. However, as far as the authors of this paper know, although briefly discussed in the context of web pages, this discussion has not been conducted in the case of free online educational videos. As in the case of the publication of contents in web pages, accuracy is necessary with respect to the author identification in order to make contents reliable. With this work, we hope to try to catch the attention of free-online video platforms as YouTube on the importance of providing users with some quality guarantees from an educational point of view. The searching algorithm used by these platforms should be different than the one used for the search of other video contents, as entertainment videos.

In this paper, we have proposed a framework for the multidimensional assessment of the quality of instructional free-online videos. The proposed approach is based in a popular ranking technique, VIKOR, which allows the ranking of the videos based on their similarity with an ideal video which is considered as a reference for the instructor. The idea of similarity with the ideal solution is also used in a first phase of the decision making problem with normalization purposes. The necessity of the aggregation of homogeneously normalized alternatives with respect to different criteria of different nature is especially relevant in the case of the quality assessment of instructional videos from educational aspects. Decision making criteria are both, qualitative and quantitative and are measured using continuous, binary and linguistic variables. Normalization of such heterogeneous criteria can lead to highly decompensated valuation of the alternatives. The transformation of these valuations in order to measure their similarity with the ideal solution can constitute a good alternative in order to avoid the heterogeneity problems in the nature of the data.

The application of the proposed IS-VIKOR approach to the ranking of instructional mathematical videos allows us to provide the instructor with different rankings. The final selection of one ranking or another will depend on the didactical perspective of the educator. On the other hand, the proposed approach aims at contributing to illustrate which dimensions and variables should be taken into account and to propose a suitable procedure for the ranking of mathematical educational videos in terms of their quality, useful for free-online platforms as YouTube. The use of these type of rankings in the display of educational videos by searching engines as the one designed by YouTube or the one implemented by Google could be of great help for the users, especially for the students, which in some cases are not completely able to discern among online contents of different educational quality.

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Appendix

Table A. Individual decision matrices for the qualitative criteria

| <i>Video</i> | <i>Z₁</i> | <i>Z₂</i> | <i>Z₃</i> | <i>Z₄</i> | <i>Z₅</i> | <i>Z₆</i> | <i>Z₁₂</i> | <i>Z₁₃</i> | <i>Z₁₄</i> | <i>Z₁₅</i> | <i>Z₁₆</i> | <i>Z₁₇</i> | <i>Z₁₈</i> | <i>Z₂₀</i> | <i>Z₂₁</i> |
|-----------------|----------------------|----------------------|----------------------|----------------------|----------------------|----------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|
| Expert 1 | | | | | | | | | | | | | | | |
| V ₁ | VG | G | G | VP | G | VP | F | F | G | VG | VP | G | G | VG | VG |
| V ₂ | G | F | F | P | F | P | P | G | VP | VG | VP | VP | G | P | VG |
| V ₃ | VG | F | F | P | G | P | G | G | F | VG | VP | G | G | F | G |
| V ₄ | G | G | F | P | F | P | F | P | G | VG | VP | VP | F | VP | VG |
| V ₅ | VG | F | F | P | G | P | G | F | P | VG | VP | VG | G | VG | VG |
| V ₆ | VG | G | F | P | G | VP | VG | VG | VG | VG | VP | VP | VG | VP | VG |
| V ₇ | G | VG | G | P | G | P | P | F | P | VG | VP | VG | VP | VG | VG |
| V ₈ | G | G | F | P | G | VP | G | VG | G | VG | VP | VP | VG | VP | VG |
| V ₉ | VG | VG | VG | VP | G | VP | VG | VG | VG | VG | VP | P | F | VP | P |
| V ₁₀ | G | G | P | F | F | P | G | G | VG | VG | VP | VP | G | VP | VP |
| V ₁₁ | G | G | G | P | G | P | VG | G | VG | F | VP | F | VG | VG | VG |
| V ₁₂ | F | G | F | G | F | P | G | G | VP | G | VP | VP | VP | VP | VP |
| Expert 2 | | | | | | | | | | | | | | | |
| V ₁ | VG | G | VG | VP | G | VP | F | F | G | VG | VP | F | F | VG | VG |
| V ₂ | F | P | VP | P | P | G | P | G | P | VG | VP | VP | F | P | VG |
| V ₃ | VG | VG | G | P | VG | P | G | G | P | VG | VP | F | F | F | G |
| V ₄ | VG | G | VP | G | VG | F | F | P | G | VG | VP | P | F | P | G |
| V ₅ | VG | P | VP | G | VG | G | G | F | P | VG | VP | VG | G | VG | VG |
| V ₆ | VG | G | F | F | VG | F | VG | VG | VG | VG | VP | | VG | F | VG |
| V ₇ | VG | P | P | G | G | F | P | F | P | G | VP | VG | VP | VG | VG |
| V ₈ | VG | P | F | G | F | G | G | VG | VG | VG | VP | | VG | VG | VG |
| V ₉ | VG | VG | VG | P | G | VP | VG | VG | VG | VG | VP | F | F | P | P |
| V ₁₀ | VG | VG | P | F | VG | F | G | G | G | VG | VP | | G | F | P |
| V ₁₁ | G | P | F | P | G | F | VG | G | VG | P | VP | F | VG | VG | VG |
| V ₁₂ | P | G | VG | G | P | VP | G | G | VP | VG | VP | VP | P | F | P |
| Expert 3 | | | | | | | | | | | | | | | |
| V ₁ | VG | G | G | VP | VG | VP | F | F | F | F | VP | G | G | VG | G |
| V ₂ | G | F | F | P | P | G | P | G | VP | F | VP | F | G | P | G |
| V ₃ | VG | VG | F | P | VG | F | G | G | P | G | VP | G | F | F | G |
| V ₄ | VG | F | VP | P | VG | F | F | P | G | F | VP | G | G | VP | G |
| V ₅ | VG | F | VP | P | VG | G | G | F | F | F | VP | F | F | VG | G |
| V ₆ | VG | G | F | F | VG | P | VG | VG | G | P | VP | F | G | F | G |
| V ₇ | VG | P | P | P | VG | VP | P | F | F | P | VP | G | P | VG | G |
| V ₈ | VG | VG | F | P | P | VP | G | VG | P | P | VP | P | F | F | G |
| V ₉ | VG | VG | VG | P | G | P | VG | VG | G | P | VP | F | G | P | G |
| V ₁₀ | VG | VG | P | P | VG | F | G | G | P | P | VP | F | F | F | G |
| V ₁₁ | G | VG | G | F | G | F | VG | G | G | G | VP | G | F | VG | G |
| V ₁₂ | F | VG | VG | F | F | VP | G | G | VP | VP | VP | P | F | F | G |