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Multiple Criteria performance evaluation of YouTube mathematical educational videos by IS-TOPSIS

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Abstract

In this work, a TOPSIS-based approach is proposed based on the idea of ideal similarity. It considers the ideal solution not necessarily related to the optimum values of the decision criteria, but to any values between the minimum and maximum values of the criteria ranges. The proposed method allows the consideration of one or several decision makers; different types of data (single numerical values, intervals or linguistic variables); different normalization functions describing the importance given by the decision makers to the deviation of alternatives from the ideal solution and different weighting schemes. The procedure also allows the decision maker to decide how much information about the intervals he is willing to take into account (e.g. the expected value, the extremes of the interval or the entire set of values in the intervals).

In order to illustrate the practical applicability of the approach we include a real example consisting of the ranking of mathematical educational videos based on six didactical dimensions. The rating of educational videos is of great interest for educators due to their high popularity in Internet, especially in platforms as You Tube which has become one of the most used sources of information nowadays.

Keywords: MCDM, TOPSIS, normalization, ideal, educational videos, didactical performance.

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1. Introduction

Real world decision making problems are complex decision problems usually characterized by several conflicting criteria, different preferences and a great diversity of types of data with different degrees of uncertainty, ambiguity and imprecision. Multiple Criteria Decision Aid (MCDA) or Multiple Criteria Decision Making (MCDM) methods are concerned with structuring and solving decision problems involving multiple criteria with either continuous or discrete set of alternatives. In this paper, we are mainly concerned with the ranking of a set of discrete decision alternatives taking into account decision criteria of different nature: precise and imprecise and, a reference solution or benchmark which will be called the ideal solution. The decision matrix describing the valuation of alternatives with respect to each criterion will be formed by data of diverse nature: real numbers, intervals on the real line and/or linguistic or categorical variables handled by means of natural numbers.

One of the first steps in any ranking MCDM method consists of the normalization of the criteria. However, classical normalization procedures do not always take into account situations where the different nature of the data of the decision matrix (real numbers, intervals, linguistic variables) could make the ranking of the alternatives quite unstable. The proposed normalization method will be based on the similarity with the reference solution and will permit us to construct a new decision matrix composed of the similarity degrees of each alternative to the benchmark or ideal for each criterion. In this way, and thanks to this normalization procedure, the nature of the transformed normalized data will be homogeneous.

In order to illustrate our proposal, we will address the problem of the ranking of mathematical educational videos in You Tube based on their pedagogical characteristics. The Internet has become a strategic source of information worldwide. The use of educational videos in You Tube has dramatically increased over the last years. In fact, as stated by Azer et al. (2013) it "is the largest Internet video-sharing site and is a useful tool in social communication, business, advertising, and news as well as a promising learning resource for students and the general public". Therefore, the assessment and ranking of the performance of these videos in didactical terms is a crucial question for the educative community (from both, the teaching and learning perspectives). The ranking of instructional videos is clearly a multiple criteria decision making problem. The diverse nature of the decision making criteria makes this real problem especially adequate for the application of a new TOPSIS-based approach using the proposed normalization process, Ideal Similarity TOPSIS (IS-TOPSIS).

TOPSIS, Technique for Order Preference by Similarity to Ideal Solution (Hwang and Yoon, 1981), is one of the most widely used ranking methods due to its characteristics (it is simple, rational, comprehensible, efficient from a computational point of view and able to measure the relative performance for each alternative in a simple mathematical form).

TOPSIS attempts to choose alternatives that simultaneously have the shortest distance from the positive ideal solution (PIS) and the farther distance from the negative-ideal solution (NIS). In the classical approach the positive ideal solution maximizes criteria of the type "the more, the better" and minimizes criteria of the type "the less, the better",

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whereas the negative ideal solution maximizes "the more, the better" criteria and minimizes "the more, the better" criteria.

TOPSIS makes full use of the attribute information, provides a cardinal ranking of alternatives, and does not require the attribute preferences to be independent (Chen and Hwang, 1992; Yoon and Hwang, 1995). To apply this technique attribute values must be numeric, monotonically increasing or decreasing, and have commensurable units.

As far as the authors of this paper know, this is the first work in which the ranking of mathematical educational videos is addressed from a multiple criteria point of view taking into account six well-known epistemic dimensions of didactical mathematical knowledge (Godino et al., 2005, Pino-Fan et al., 2015). The utility of the didactical performance assessment of the mathematical educational videos available in You Tube goes beyond the simple control of their contents in mathematical terms. It takes into account the correctness and precision of the mathematical contents but also other important didactical features as the waste of time in the exposition, the empathy with the user, the capacity to attract the attention of the user or the degree of adaptation of the contents to the educational context. Therefore, the high number of possible uses of the videos assessed by the educators in these terms, makes them a very attractive educational tool.

In what follows, we will present a brief literature review concerning several methodological aspects related to the ranking method proposed in this paper, TOPSIS, highlighting the contribution of this paper with regards to the normalization procedure usually addressed in the first steps of the ranking method. In the next sections, we will present the TOPSIS-based proposed approach, Ideal Similarity TOPSIS (IS-TOPSIS). This new approach considers the ideal solution not necessarily related to the optimum values of the decision criteria, but to any values between the minimum and maximum values of the criteria ranges. The method makes use of this ideal solution with normalizing purposes avoiding in this way problems related to the simultaneous consideration of data of different nature (real numbers, intervals, linguistic variables...) generalizing the method proposed by Cables et al. (2016), Reference Ideal Method (RIM-TOPSIS) which only allows the consideration of decision matrices composed of real numbers. Finally, and in order to illustrate the applicability of the proposed method, we will present a real decision making application and the main conclusions of this work.

2. Literature review

Several MCDM methods have been proposed to assist decision makers in the process of ranking a set of discrete decision alternatives (Roy, 1985, Triantaphyllou, 2000). Among the most popular discrete MCDM methods for ranking alternatives we can highlight the Simple Additive Weighting (SAW) (Churchman and Ackoff, 1954), the Elimination et Choice Traduisant la Realité (ELECTRE) II and III methods (Roy, 1968), the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) (Hwang and Yoon, 1981), the Preference Ranking Organization Method for Enrichment Evaluations (PROMETHEE) II (Brans and Vincke, 1985), the compromise ranking method (VIKOR) (Orpocovic, 1998) and the Tomada de Decisão Interativa Multicritério (TODIM) (Gomes and Lima, 1992).

According to Zanakis et al. (1998) the decision maker must first face the task of selecting the most suitable method considering different dimensions, such as simplicity, reliability, robustness and quality. However, as these authors conclude, it is very difficult to answer questions such as "which method is the most appropriate for a specific type of problem and what are the advantages and disadvantages of using method rather than another?" (Zanakis et al. 1998, Triantaphyllou, 2000). In this work, as mentioned in the previous section and due to the previously described characteristics of the addressed real application, we have selected TOPSIS. This method has been widely used in a large number of real-world applications (see Behzadian et al. 2012 and Zyoud and Fuchs-Hanusch, 2017 for recent state of the art surveys showing its high applicability) which includes novel application areas as e-commerce (Arroyo-Cañada and Gil-Lafuente, 2017), new energy contexts (Thomaidis et al. 2008) or climate change or sustainability assessment (Bilbao-Terol et al. 2017) among others.

Another alternative to TOPSIS for the ranking of alternatives taking into account reference solutions is VIKOR. The method introduces a multiple criteria ranking index based on the particular measures of “closeness” to the ideal solution (Opricovic, 1998; Opricovic and Tzeng, 2004). The compromise solution is a feasible solution, which is the closest to the ideal solution.

Both methods are MCDM ranking approaches based on the distance to a reference point. However, in the case of TOPSIS both, distance to the positive ideal solution and distance to the negative ideal solution, are considered whereas in the VIKOR approach, the only goal is to minimize the distance to the positive ideal solution. This can lead to a situation where the best solution ranked by TOPSIS is not the closest to the ideal as the relative importance of the distances to the ideal and negative ideal are not considered (see Opricovic and Tzeng, 2004 for a further discussion on this topic). The selection of one method or another will depend on the real decision making problem to be solved. In the case of the ranking of instructional or educational videos the instructors usually aim at simultaneously taking into account maximum similarity with the best educational contents and avoidance of inappropriate educational contents. Therefore, in this work, the videos are ranked using TOPSIS instead of VIKOR although other approaches could also be considered depending on the didactical purposes of the instructors.

Decision criteria are usually measured in different units and therefore, do not necessarily have the same domains, nor do the same range (Cables et al., 2016). It is, therefore, necessary to normalize data in the decision matrix. Normalization can be carried out in different ways. The classical TOPSIS approach uses non-linear vector normalization although other types of normalization have been used in the TOPSIS literature (comparison about different normalization procedures and analysis of their impact on the decision results in the context of TOPSIS, have been widely discussed in the literature, e.g. See, Pavlicic, 2001, Zavadskas et al., 2003, Milani et al., 2005, Zavadskas et al., 2006, Chakraborty and Yeh, 2009, Çelen, 2014 or Cables et al., 2016).

Another controversial question regarding the classical TOPSIS procedure is related to the weighted scheme. In its classical formulation, criteria weights are the only subjective element and are directly determined by the decision maker. Many extensions of the classical TOPSIS method have been proposed combining TOPSIS with other methods such as AHP, PROMETHEE, ELECTRE or DEA in order to determine the criteria weights in those cases where they cannot be directly assigned by the decision maker (see Zavadskas et al., 2006 and more recently Yang et al. 2017 for an example of consensual weights).

Separation measurement of each alternative to the PIS and NIS involves, in the classical TOPSIS approach, consideration of a distance. Following the original proposal, most of the authors use the Euclidean distance to calculate the similarity of each alternative to the PIS and NIS. However, other distances have been also proposed to deal with this step of the TOPSIS procedure, as the Manhattan distance (see for example, Chang et al., 2010, Khademi-Zare et al., 2010, Tan, 2011 or Vega et al., 2014) or the Mahalanobis distance (Chang et al., 2010 or Vega et al., 2014) and comparison of the obtained results have been provided highlighting the advantages and disadvantages of each distance choice (Chang et al., 2010 and Vega et al., 2014).

One of the questions less discussed in the literature is the one concerning the considered ideal solution, the solution taken as a reference for the ranking of the alternatives. The classical TOPSIS approach, as well as most of its extensions, uses as references the positive and negative ideal solutions, PIS and NIS, respectively, which involves determining maximum and minimum solutions. That is, in the determination of the reference solutions there is an *optimizing philosophy*.

However, as pointed out by Cables et al. (2016), there may be decision contexts where the ideal solution is not necessarily one of the extreme values , but may be an intermediate value between the maximum and the minimum values. In this work we deal with those decision situations in which the ideal solution is not necessarily an optimum solution. Extending the results obtained by Cables et al. (2016) we propose a generalization of the method that allows the simultaneous use of data of different nature (simple numerical values, numerical intervals or linguistic sets of variables). Our proposal permits the consideration of more than one decision maker and proposes a very simple procedure to obtain consensual evaluations of the alternatives.

Based on the idea of ideal similarity we provide different normalizing functions and a support tool for the decision makers to facilitate the selection of the most adequate normalizing process. As we will see, this decision will depend on the particular decision contexts as it will take into account the importance given by the decision makers to the deviations from the ideal solutions (in terms of slack or surplus) and on the way they prefer to handle the information included in the numerical intervals.

3. Preliminaries on the proposed approach

The main steps of the TOPSIS procedure originally proposed by Hwang and Yoon (1981) comprise the forming of the decision matrix, followed by decision matrix normalization. In a next step, weights are assigned to the different criteria and the weighted normalized decision matrix is obtained. After that, the positive and negative ideal solutions are calculated and separation measures for each alternative are determined. Finally, in the last step relative closeness coefficients are obtained and used to rank the alternatives in descending order (see Table1).

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Table 1. Main steps of the classical TOPSIS approach

STEP 1. Determine the decision matrix X, where the number of alternatives is m and the number of criteria is n, $X = \left(x_{ij}\right)_{mxn}$, being x_{ij} real numbers.

STEP 2. Construct the normalized decision matrix,

$$r_{ij} = x_{ij} / \sqrt{\sum_{i=1}^{m} x_{ij}^2}, \quad 1 \le i \le m, \quad 1 \le j \le n.$$

STEP 3. Determine the weighted normalized decision matrix. Given, $w_j \in [0,1]$, with $w_1+w_2+\ldots+w_n=1$, we calculate

$$v_{ij} = w_j r_{ij}, \quad i = 1,\ldots,m, \quad j = 1,\ldots,n.$$

STEP 4. Determine the positive ideal A^+ (PIS) and negative ideal A^- solutions (NIS),

$$A^+ = \left\{ v_1^+,\ldots,v_n^+ \right\} = \left\{ \left(\max_i v_{ij}, j \in J\right)\left(\min_i v_{ij}, j \in J'\right) \right\} \quad i = 1,2,\ldots,m$$

$$A^- = \left\{ v_1^-,\ldots,v_n^- \right\} = \left\{ \left(\min_i v_{ij}, j \in J\right)\left(\max_i v_{ij}, j \in J'\right) \right\} \quad i = 1,2,\ldots,m$$

where J is associated with "the more, the better" criteria and J' is associated with "the less, the better" criteria.

STEP 5. Calculate the separation measures with respect to the PIS and NIS,

$$S_i^+ = \left(\sum_{j=1}^{n} \left(v_{ij} - v_j^+\right)^2 \right)^{1/2}, \quad S_i^- = \left(\sum_{j=1}^{n} \left(v_{ij} - v_j^-\right)^2 \right)^{1/2}, \quad 1 \le i \le m.$$

STEP 6. Calculate the relative proximity to the ideal solution using the relative index

$$R_i = \frac{S_i^-}{S_i^+ + S_i^-} \quad i = 1,\ldots,m.$$

STEP 7. Rank the best alternatives according to R_i in descending order.

The classical TOPSIS method solves problems in which both, data and decision processes are precisely represented by exact numerical values. However, most real-world problems, especially those involving human judgements, have a more complicated structure. As previously mentioned, human decision making processes are characterized by fuzziness, high complexity and uncertainty. In these situations Fuzzy Sets Theory becomes a useful tool and therefore, based on the original TOPSIS method, many other extensions have been proposed, providing support for fuzzy data or decision processes to model imprecision, uncertainty, lack of information or vagueness (see Dymova et al, 2013, Wang, 2014, Cables et al, 2016 for recent reviews on Fuzzy TOPSIS).

The main idea of Fuzzy Sets Theory is quite intuitive and natural: instead of determining the exact boundaries as in an ordinary set, a fuzzy set allows for no sharply

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defined boundaries because of the generalization of a characteristic function to a membership function. By letting X denote a universal set, a fuzzy set $\tilde{A}$ of X can be characterized as a set of ordered pairs of element x and the grade of membership of x in $\tilde{A}$, $\mu_{\tilde{A}}(x)$, and it is often written as

$$\tilde{A} = \left\{ \left(x, \mu_{\tilde{A}}(x) \right) / \, x \in X \right\} \tag{1}$$

Note that the membership function is an obvious extension of the idea of a characteristic function of an ordinary set because it takes values between 0 and 1, not only 0 and 1. A membership level equal to zero means no membership, a membership value equal to one means Boolean membership and intermediate numbers reflect intermediate membership degrees (see Kaufmann and Gupta, 1988 and Zimmermann, 1996).

A fuzzy number is one of the most common forms of fuzzy set application (Kaufmann and Gupta, 1988); it is defined as a fuzzy set defined on the real line with a convex, continuous and normalized membership function.

Dubois and Prade define a LR-fuzzy number $\tilde{M}$ as follows (Dubois and Prade, 1978):

$$\tilde{M} = \left(m^L, m^R, \delta^L, \delta^R \right)_{LR} \tag{2}$$

if its membership function has the following form:

$$\mu_{\tilde{M}}(x) = \begin{cases} L\left(\dfrac{m^L - x}{\delta^L} \right) & if \quad x \le m^L \\ 1 & if \quad m^L \le x \le m^R \\ R\left(\dfrac{x - m^R}{\delta^R} \right) & if \quad x \ge m^R \end{cases} \tag{3}$$

where $L, R : \left[0, +\infty \right[\rightarrow \left[0,1 \right]$ are strictly decreasing in $\text{supp}(\tilde{M}) = \left\{ x \in X / \mu_{\tilde{M}}(x) > 0 \right\}$ and upper semi-continuous functions such that $L(0) = R(0) = 1$.

If the support of $\tilde{M}$ is a bounded set, being $m^L - \delta^L$ the infimum and $m^R + \delta^R$ the supremum in that set, then functions L and R are defined on $\left[0,1 \right]$ and they satisfy that $L(1) = R(1) = 0$. When $L(z) = R(z) = \max\left\{ 0, 1 - z \right\}$, $\tilde{M}$ is said to be a fuzzy trapezoidal number with support $\left[m^L, m^R \right]$ and core $\left[M^L, M^R \right]$. If the core of the fuzzy number is a unique number, $\left[M^L = M^R = M \right]$, the fuzzy number is said to be triangular.

4. Ideal Similarity TOPSIS: IS-TOPSIS

In this section we will propose a new approach for the TOPSIS, which constitutes a generalization of the method proposed by Cables et al. (2016), the Reference Ideal Method, RIM. This method relies on the idea that the ideal solution, instead of being an optimal solution, can take any value between the minimum and maximum values of the range of the criteria. The proposal by Cables et al. (2016), the Reference Ideal Method based on TOPSIS (RIM-TOPSIS) does not present rank reversal given that, as we will see in what follows, the PIS solution will be the unitary vector and the NIS will be the null vector.

RIM-TOPSIS is based on the concepts of *range* (any interval, labels set or simple set of values belonging to a domain) and *reference ideal* (an interval, labels set or simple values representing the maximum importance or relevance in a given range). This procedure not only allows the simultaneous consideration of data of different nature known with different precision degrees but, it smooths the idea from TOPSIS requiring all the criteria to be monotonically increasing or decreasing. The important question is to attain the maximum similarity to the ideal, not to reach a maximum or minimum value for the criteria.

Cables et al. (2016) configure the range and ideal as intervals, with independence of the nature of the data. For example, if the range is {Very Good, Good, Fair, Bad, Very Bad}, they first transform the linguistic variables into numbers {5, 4, 3, 2, 1}, and then, the range is expressed as $[A, B]=[1, 5]$. In this context, an ideal $[C, D] = [3, 4]$, will mean that the ideal is {Fair, Good}.

With this notation, let us consider, $[A, B]$ the range of a decision criterion that belongs to a universe of discourse; $[C, D]$ representing the reference ideal for that criterion, with $\left[C,D\right]\subseteq\left[A,B\right]$ and $x\in[A,B]$ the valuation of an alternative with regards to the considered criterion. Cables et al. (2016) proposed a normalization process based on the distance to the reference ideal

$$d_{\min}\left(x,\left[C,D\right]\right)=\min\left(\left|x-C\right|,\left|x-D\right|\right) \tag{4}$$

The authors define a function, $f:[A,B]\rightarrow\left[0,1\right]$, that provides a value that belongs to the unitary interval (i.e. if it is equal to 1, then it coincides with the reference ideal and the more distant it is from 1, the more distant from the reference ideal)

$$f(x)=\begin{cases}1 & if \quad x\in\left[C,D\right]\\ 1-\dfrac{C-x}{C-A} & if \quad x\in\left[A,C\right]\wedge A\neq C\\ 1-\dfrac{x-D}{B-D} & if \quad x\in\left[D,B\right]\wedge D\neq B\end{cases} \tag{5}$$

Obviously, if $[A,B]$ and $[C,D]$ are intervals on the real line, (5) describes the membership function of a trapezoidal fuzzy number $T = (C, D, C\text{-}A, B\text{-}D)$ (see the graphical representation on the left in Figure 1).

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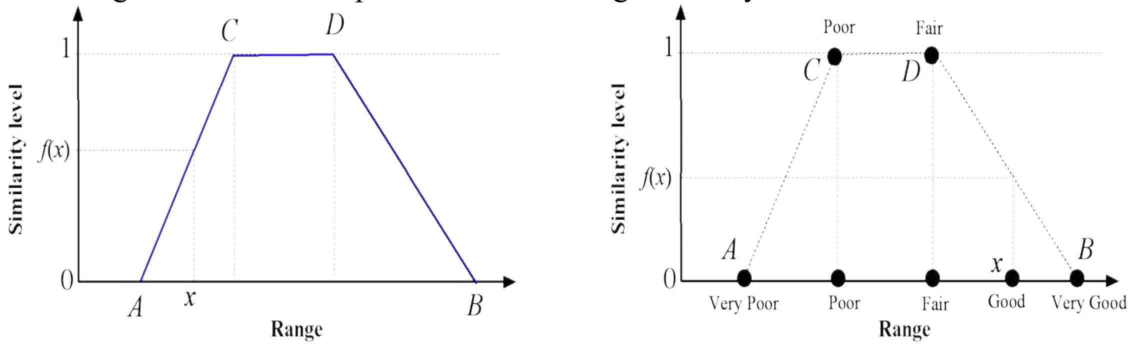
Figure 1. Membership functions reflecting similarity with the ideal solution



Bearing in mind this normalization procedure, Cables et al. (2016) proposed a replacement of step 2 in Table 1 by

$$r_{ij} = f(x_{ij}), \quad 1 \le i \le m, \quad 1 \le j \le n \tag{6}$$

By its own construction, (6) makes the positive ideal to be A^{+}=(1, 1, …, 1) and the negative ideal to be A^{-}=(0,0, …0). Notice that, regardless the initial optimization direction, with the new proposed normalization, the interest is to maximize all the criteria.

The function f given in (5) is designed to work with real numbers $x \in [A, B]$, because the original data are real numbers or because they have been transformed into real numbers. In what follows we will generalize the proposed procedure to the case in which valuations are given by intervals.

The first step consists of re-interpreting the meaning of *f(x),* obtained with (5), as the similarity of x with the nearest point of the ideal [C, D]. By construction, any point in the ideal interval will have the maximum similarity (that is, 1), whereas for any other point not belonging to the ideal interval, the similarity will be smaller the farther to the interval [C, D] (Zeng and Guo, 2008). In fact, this idea can be stated for two any fuzzy sets $\tilde{A}$, $\tilde{B}$. Given the normalized distance between these two sets, $d(\tilde{A}, \tilde{B})$ (Dubois and Prade, 1978), according to Zeng and Guo (2008), a similarity measure between $\tilde{A}$ and $\tilde{B}$ can be defined as

$$\mathrm{Sim}(\tilde{A}, \tilde{B}) = 1 - d(\tilde{A}, \tilde{B}) \in [0,1] \tag{7}$$

Let us consider a valuation of an alternative with respect to a given criterion described by an interval $[a,b] \subseteq [A, B]$. We will propose a normalization process based on the normalized distance between this valuation and the reference ideal,

$$d\left([a,b],[C,D]\right) \tag{8}$$

where d is any adequate distance between intervals.

From (7), we can construct a generalization of function (5) in the following way

$$F\left([x,y]\right) = \begin{cases} 1 & \text{if} \quad [x,y] \subseteq \left[C,D\right] \\ 1 - d\left([x,y],[C,D]\right) & \text{if} \quad [x,y] \not\subseteq \left[C,D\right] \end{cases} \tag{9}$$

Given a range [A, B], a reference ideal [C,D] $\subseteq$ [A,B] and valuation of an alternative [x,y] $\subseteq$ [A,B], this function provides a value that belongs to the unitary interval.

Using function F defined in (9) to normalize the data, we propose a TOPSIS approach, based on the concept of ideal similarity, IS-TOPSIS (see Table 2).

Table 2. Main steps of the IS-TOPSIS approach

STEP 1. Define the working context: type of data, number of decision makers, criteria range, reference ideal [C_j, D_j] and weights $w_j \in [0,1]$ with $w_1+\ldots+w_n=1$, associated to each criterion $1 \le j \le n$.
STEP 2. Obtain the valuation matrix $X = \left(\left[c_{ij}^L, c_{ij}^R\right]\right)_{mxn}$ being $\left[c_{ij}^L, c_{ij}^R\right]$ the valuation of alternative j with regards to criterion i expressed as an interval. In case of group decision making this matrix will be a consensual matrix based on the individual valuations of the decision makers.
STEP 3. Normalize the matrix X using function F given in (9), $Y = \left(y_{ij}\right)_{m\times n}$.
STEP 4. Calculate the weighted normalized matrix. Given $w_j \in [0,1]$ we calculate $Y' = \left(y'_{ij}\right)_{m\times n} = \left(w_j y_{ij}\right)_{m\times n}$.
STEP 5. Calculate the variation to the normalized reference ideal for each alternative i. $$I_i^+ = \left(\sum_{j=1}^n \left(y'_{ij} - y_j^+\right)^2\right)^{1/2}, I_i^- = \left(\sum_{j=1}^n \left(y'_{ij} - y_j^-\right)^2\right)^{1/2}$$ where $y_j^+ = \max_i y'_{ij}$, $y_j^- = \min_i y'_{ij}$, $i = 1,\ldots,m$
STEP 6. Calculate the relative index for each alternative i $$S_i = \frac{I_i^-}{I_i^+ + I_i^-} \quad where\ 0 < R_i < 1,\ i = 1,\ldots,m$$
STEP 7. Rank the alternatives according to R_i in descending order. The alternatives that are the top are the best solutions.

For the distance, we can use, as proposed by Canós et al. (2014), Hamming normalized distance for the range of data [A, B], that is,

$$d_H\left([a,b],[C,D]\right) = \frac{1}{2(B-A)}\left(\lvert a-C\rvert + \lvert b-D\rvert\right), \quad \forall [a,b] \subseteq [A,B] \tag{10}$$

Given the function f described in (5), any interval $[a,b] \subseteq [A,B]$ can be normalized either using the average of the membership degrees of the extremes of the interval

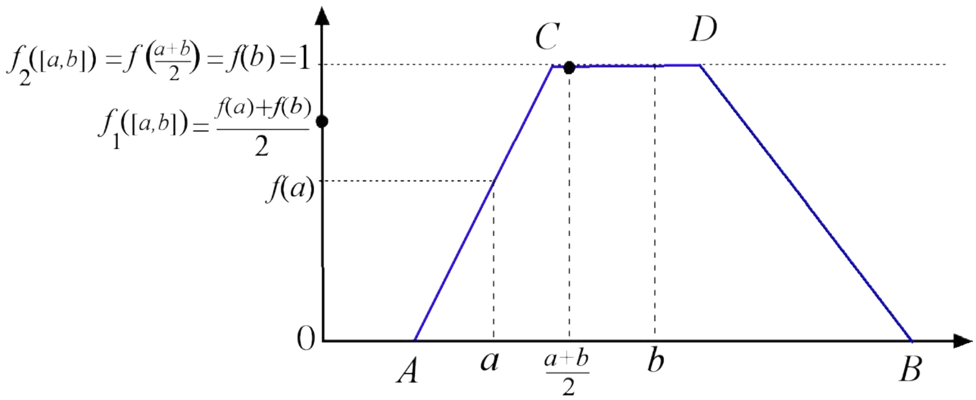
$$f_1([a,b]) := \frac{1}{2}\left(f(a) + f(b)\right) \tag{11}$$

or using the membership degree of the expected value of the interval (Heilpern, 1992)

$$f_2([a,b]) := f\left(\frac{a+b}{2}\right) \tag{12}$$

It is clear that the values $f_1([a,b])$ and $f_2([a,b])$ will not be always the same (see Figure 2). Let us see a numerical example.

Figure 2. Different normalization options



Example: Let us consider a range [A, B]=[0, 10] and an ideal [C, D]=[7, 8]. Let us also consider three decision alternatives A_1=[6, 9], A_2=[5.5, 9.5] and A_3=[5, 9]. If we normalize the alternatives using the previously proposed functions f_1, f_2 and F we obtain

$$f_1(A_1) = \frac{1}{2}\big(f(6) + f(9)\big) = 0.678571429,\ \ f_1(A_2) = \frac{1}{2}\big(f(5.5) + f(9.5)\big) = 0.589285714$$

$$f_1(A_3) = \frac{1}{2}\big(f(5) + f(9)\big) = 0.607142857.$$

$$f_2(A_1) = f\left(\frac{6+9}{2}\right) = 1,\ \ f_2(A_2) = f\left(\frac{5.5+9.5}{2}\right) = 1,\ \ f_2(A_3) = f\left(\frac{5+9}{2}\right) = 1.$$

$$F(A_1) = 1 - \frac{|6-7| + |9-8|}{20} = 0.9,\qquad F(A_2) = 1 - \frac{|5.5-7| + |9.5-8|}{20} = 0.85,$$

$$F(A_3) = 1 - \frac{|5-7| + |9-8|}{20} = 0.85.$$

Therefore, according to f_1, the ranking would be $A_1 > A_3 > A_2$, with f_2 it would be $A_1 = A_2 = A_3$ and with F the ranking would be $A_1 > A_3 = A_2$.

The selection of the normalizing function will depend therefore on the decision context. If the decision maker wants to equally penalize slack or surplus in the deviation of an alternative with respect to the ideal, then the suggested normalizing function would be F. On the contrary, if different importance is given to the deviations (for example, if the decision maker wants to penalize more the excess than the defect or *vice versa*) then normalizing functions f_1 or f_2 would be more suitable. The use of f_1 is recommended for those situations where the decision maker needs to take into account the extreme values of the intervals. Otherwise, if the decision maker is satisfied with the expected value representing the information in the interval, f_2 would be more adequate. Figure 3 displays the different scenarios and the different advises as for the use of the normalizing functions.

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Figure 3. Scenarios for the selection of the normalizing function

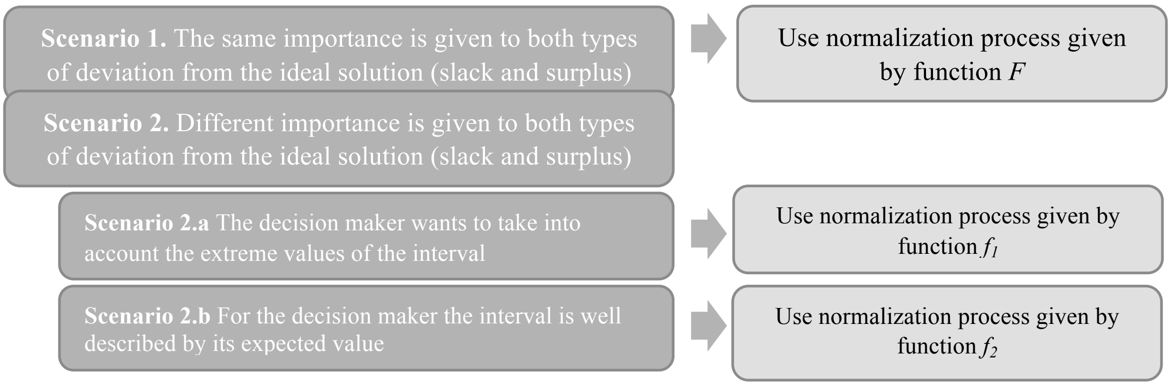
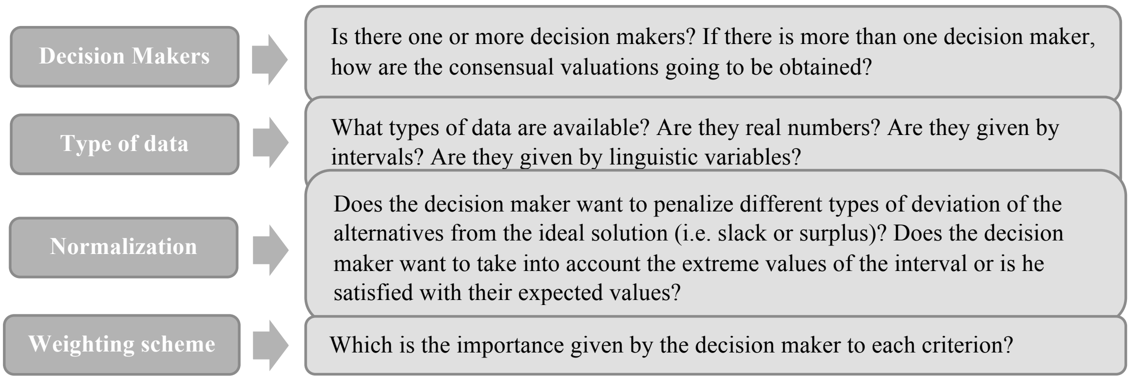


Figure 4 summarizes the key questions to be answered in the IS-TOPSIS approach. As we can observe the decisions derived from these questions will mainly depend on the number of decision makers, on the type of the data, on the decision context and on the decision maker preferences, all these factors being susceptible of change depending on the circumstances.

Figure 4. Key questions in the IS-TOPSIS approach



In the next section, we will illustrate the applicability of the proposed approach by means of a real example where several decision makers face the problem of ranking a set of mathematical educational videos published in You Tube based on didactical criteria given both in numerical and linguistic terms.

5. Application

Godino et al. (2007) introduce the notion of didactical suitability of an instructional process as the coherent and systemic articulation of six components (see Pino-Fan et al. 2015):

- *Epistemic facet*, refers to specialized knowledge of the mathematical dimension.
- *Cognitive facet*, refers to the knowledge about the students' cognitive aspects.
- *Affective facet*, refers to the knowledge about the students' affective, emotional and behavioural aspects.

- *Interactional facet*, refers to the knowledge of the interactions that occur within a classroom.
- *Mediational facet*, refers to the knowledge of resources and means which might foster the students' learning process.
- *Ecological facet*, refers to the knowledge of curricular, contextual, social, political and economic aspects that have an influence on the management of the students' learning.

A good performance in one of these dimensions might not correspond to a good performance in the other dimensions. Therefore, the assessment of the didactical suitability of an instructional process is clearly a multiple criteria decision making problem. Moreover, each of these dimensions is *a matter of degree* (Godino et al. 2007), thus being characterized by a high degree of ambiguity, uncertainty and imprecision and even, subjectivity based on the expertize level and experience of the instructors. In order to try to reflect this potential subjectivity and the ambiguity, uncertainty and imprecision of the assessment of the didactical suitability of the videos, we have asked three experts (mathematical instructors) from three higher education institutions from different countries to assess the didactical performance of 12 mathematical educational videos available in You Tube with respect to the didactical criteria. The three experts were given the same importance as we have considered their level of expertice and experience to be the same. However, other situations could be considered depending on the decisional context.

Table 3 describes the considered decision criteria and the questions used to obtain evaluations from the three experts. The questions were proposed by Santos-Mellado et al. (2017). Both positive and negative attitude statements have been used as a "positive" or "negative" set of statements might influence respondents' answers to the statements (i.e. a set of positive statements might produce higher agreement than the level of disagreement for a set of negative statements), (Gendall and Hoek, 1990).

We have chosen a mathematical concept, basis of a vector space, and we have conducted a search of You Tube videos in Google. We have searched educational videos introducing the concept of basis of a vector space. The selected videos are those appearing in the first 12 positions when using as key words "basis" and "vector space". Table 4 displays a description of our decision alternatives.

Table 3. Decision Criteria

Criteria	*Description*	*Question*
Z_1	Epistemic facet	To what extent are the treated mathematical concepts correct?
Z_2	Cognitive	To what extent does the author mention all the elements in a fluid way?
Z_3	Affective facet	To what extent do the contents of the video attract the attention of the user?
Z_4	Mediational facet	To what extent time and resources are wasted in the explanation?
Z_5	Ecological facet	To what extent is the video adapted to the concrete educational context?
Z_6	Interactional facet	To what extent is it difficult to understand the author?

Table 4. Decision alternatives

Video	*Title*	*Web References*
V_1	Basis for a set of vectors	[1]
V_2	Basis for a vector space	[2]

V_3	Basis and dimension of a vector space	[3]
V_4	Linear algebra - basis of a vector space	[4]
V_5	Basis of a vector space	[5]
V_6	Vector space, basis, dimension	[6]
V_7	Basis, vectors and coordinates	[7]
V_8	Basis and dimension	[8]
V_9	Linear combinations, span, and basis vectors	[9]
V_{10}	Basis and dimension	[10]
V_{11}	Linear algebra example problems	[11]
V_{12}	Concepts of basis and dimension	[12]

The experts were asked to score the videos with respect to each criterion using both, a numerical scale [0-10] and the following linguistic labels:

{Very Poor, Poor, Fair, Good, Very Good}

Once individual scores and linguistic labels were obtained for each video in each dimension, we obtained a consensual assessment for each dimension, *i*, and video, *j*, in the form of intervals. For the case of numerical scores we obtained the following intervals: $\left[z_{ij}^{L}, z_{ij}^{R}\right]$ where $z_{ij}^{L} = \min\left\{z_{ij}^{k}\right\}, k = 1,2,3$ and $z_{ij}^{R} = \max\left\{z_{ij}^{k}\right\}, k = 1,2,3$ being z_{ij}^{k} the score given by expert *k* to the video *i* with regards to criterion *j*.

Table 5 displays the obtained individual scores from each expert and the consensual intervals for the affective criterion. All the individual scores for all the considered criteria are displayed in Table 1A in the appendix. We can observe how for dimensions 3 and 6, the intervals have an amplitude equal or greater than 3 in more than 50% of the cases (values highlighted in Table 1A in the appendix).

Table 5. Example of the individual scores and consensual intervals

Video	*Expert 1*	*Expert 2*	*Expert 3*	$[z_{ij}^{L}, z_{ij}^{R}]$
V_1	8	9	7	[7, 9]
V_2	6	0	5	[0, 6]
V_3	6	9	6	[6, 9]
V_4	5	1	1	[1, 5]
V_5	5	0	1	[0, 5]
V_6	5	5	6	[5, 6]
V_7	7	3	2	[2, 7]
V_8	6	5	5	[5, 6]
V_9	9	10	10	[9, 10]
V_{10}	4	3	2	[2, 4]
V_{11}	7	5	8	[5, 8]
V_{12}	6	9	8	[6, 9]

For the linguistic case, the consensual assessment is expressed as the union set of the individual linguistic rates. Table 6 displays the obtained individual and consensual ratings for the affective criterion using linguistic variables (see Table 2A in the appendix for all the assessments).

Table 6. Example of the individual rates with linguistic labels and consensual intervals

Video	*Expert 1*	*Expert 2*	*Expert 3*	*Consensual*
V_1	G	VG	G	{G,VG}
V_2	F	VP	F	{VP,F}
V_3	F	G	F	{F,G}
V_4	F	VP	VP	{VP, F}

V_5	F	VP	VP	{P,F}
V_6	F	F	F	{F,F}
V_7	G	P	P	{VP,G}
V_8	F	F	F	{F,F}
V_9	VG	VG	VG	{VG,VG}
V_{10}	P	P	P	{P,P}
V_{11}	G	F	G	{F,G}
V_{12}	F	VG	VG	{G,VG}

From Table 2A, we can observe how from a linguistic perspective, those dimensions valued using negative statements are the ones presenting more difficulties (and therefore a greater disparity). However, the assessments obtained with linguistic labels are more similar than the ones obtained when the experts were asked to score the videos from 0 to 10.

Once the videos have been scored and rated in all the dimensions by the experts and consensual intervals have been obtained, a different expert is asked to provide the ideal scores and rates for each criterion (see Table 7). This expert belongs to the Ibero-American Laboratory for the Assessment of Education Processes (LABIPE, https://www.uv.es/liern/LABIPE).

Table 7. Ideal solutions for the numerical and linguistic cases

Criteria	*Types of statement*	*Numerical Ideal*	*Linguistic Ideal*
Z_1	Positive	[9, 10]	{VG}
Z_2	Positive	[8, 10]	{G,VG}
Z_3	Positive	[8, 9]	{G}
Z_4	Negative	[1 ,2]	{P}
Z_5	Positive	[7, 9]	{F,G}
Z_6	Negative	[0, 1]	{VP}

Based on the ideal solution and following the steps of the IS-TOPSIS described in the previous section (see Table 2), the rankings displayed in Table 8 were obtained. We first normalized the data (see STEP 2 in Table 2). For this, we considered Scenario 1 (see Figure 3) in which the same importance is given to both types of deviation from the ideal solution (slack and surplus). With regards to the weighting scheme, following the expert's advice, we considered equal weights for all the criteria (see Figure 4). Normalized weighted values were then calculated (see STEP 3 in Table 2).

In order to handle the linguistic rates and ideals, following the procedure proposed by Cables et al. (2016) the linguistic rates have been transformed into numerical values using the following scale (see Table 3A in the appendix):

$$\{VP=1,\ P=2,\ F=3,\ G=4,\ VG=5\}$$

Tables 8, 9 and 10 show the obtained rankings for three different situations regarding the type of data: numerical interval data, linguistic variables and a mixed case in which, those criteria for which the relative ranges were higher were handled by means of linguistic variables and those with more stability (lower relatives ranges) were handled using numerical intervals. As mentioned in the preceding section, the decision maker(s) will have to determine the type data to be collected depending on the characteristics of the available information (see Figure 4). In this example with illustration purposes, we

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have included the three possible situations: only numerical valuations, only linguistic valuations and mixed valuations depending on the characteristics of the data.

The first two columns in Table 8 display the video ranking and the relative index values corresponding to the numerical valuation (see STEP 5, STEP 6 and STEP 7 in Table 2). Next two columns show the ranking and relative index values for the linguistic case and, finally, the last two columns display the results for a mixed case in which criteria Z_1, Z_2, Z_4 and Z_5 are measured with numerical scores and criteria Z_3 and Z_6 are measured using linguistic terms.

Table 8. IS-TOPSIS: obtained rankings with equal weights

NUMERICAL		LINGUISTIC		MIXED	
Video Rank	R_i	*Video Rank*	R_i	*Video Rank*	R_i
9	0.93297	9	0.86954	1	0.92012
1	0.92266	1	0.84518	9	0.89099
3	0.87236	6	0.81452	3	0.83289
6	0.80047	3	0.81392	6	0.82087
11	0.74579	11	0.80481	11	0.77291
7	0.72070	8	0.76014	7	0.75221
10	0.71769	10	0.75000	10	0.74340
4	0.70973	7	0.74330	4	0.71641
12	0.70613	4	0.71059	12	0.70154
8	0.66261	12	0.70660	8	0.68399
5	0.64328	5	0.64446	5	0.66084
2	0.59383	2	0.62743	2	0.61132

When different importance is given to both types of deviation from the ideal solution (slack and surplus) the obtained rankings are the ones displayed in Tables 9 and 10. Concretely, Table 9 shows results for normalization in Scenario 2.a, that is, when the decision maker wants to take into account the extreme values of the interval. On the other hand, Table 10 shows results for normalization in Scenario 2.b, when for the decision maker the interval is well described by its expected value (see Figure 3).

Table 9. RIM-TOPSIS ranks with the average of the membership degrees

NUMERICAL		LINGUISTIC		MIXED	
Video Rank	R_i	*Video Rank*	R_i	*Video Rank*	R_i
1	0.95793	11	0.78757	3	0.82784
3	0.81895	3	0.75032	1	0.81800
9	0.81741	6	0.74739	11	0.78239
11	0.76260	8	0.70160	7	0.72471
4	0.71083	7	0.65808	10	0.71013
10	0.70819	10	0.65661	9	0.68745
7	0.70321	9	0.64837	4	0.68358
6	0.68174	12	0.63644	6	0.68225
12	0.65993	4	0.62098	8	0.66548
8	0.65639	1	0.60436	12	0.63825
2	0.58392	2	0.59917	2	0.58567

5	0.57065	5	0.57061	5	0.57403

Table 10. RIM-TOPSIS ranks with the membership degree of the expected value

NUMERICAL		LINGUISTIC		MIXED	
Video Rank	R_i	*Video Rank*	R_i	*Video Rank*	R_i
1	0.96636	8	0.75211	3	0.83698
3	0.89720	3	0.75032	1	0.81800
9	0.82087	6	0.74739	6	0.81150
6	0.81090	4	0.74138	4	0.78912
11	0.77189	11	0.72350	7	0.73847
7	0.71698	7	0.68096	9	0.69098
10	0.71651	2	0.67279	11	0.68902
4	0.70765	10	0.65469	8	0.66307
12	0.70587	9	0.64837	5	0.66254
8	0.65403	12	0.64471	12	0.65201
5	0.62988	1	0.60436	10	0.64649
2	0.58392	8	0.75211	3	0.83698

We can observe how for the RIM-TOPSIS approach, when linguistic valuations are used, video number 1 ranking the first in the numerical and mixed cases, ranks in one of the last positions. Video number 8 not performing well in the numerical and mixed cases performs quite well in the case of linguistic valuations. Video number 4 performing in the fifth position with numerical valuation, ranks in the ninth position in the case of linguistic valuations and in the seventh position in the case of mixed valuations. These variations in the rankings depending on the nature of the data are quite mitigated in the case of the IS-TOPSIS using the normalization approach proposed in this paper. As we can see the three rankings are quite similar regardless the type or nature of the data (see Table 8).

6. Conclusions

In this paper we have proposed a generalization of the TOPSIS approach based on the proposal by Cables et al. (2016), the so-called Reference Ideal Method based on TOPSIS (RIM-TOPSIS). Our method, based on the similarity of the alternatives with respect to the criteria with the ideal solution, Ideal Similarity TOPSIS (IS-TOPSIS) allows working with one or several decision makers and with different types of data simultaneously (numerical intervals, linguistic variables or mixed data). This last feature, the simultaneous different nature of the data, can lead to a situation where the classical normalization processes give rise to very unstable rankings. In order to solve this problem a new normalization process has been proposed which transforms the original data into new data reflecting the similarity of each alternative with respect to each decision criterion with an ideal solution taken as a reference. This ideal solution, as in the case of the approach proposed by Cables et al. (2016), does not need to be an optimum solution. On the contrary, it can be given by any interval in the range of the decision criteria, considered as satisfactory by the decision maker(s).

The proposed method has been completed with a decision aid tool intended to guide the decision maker(s) into the different steps of the ranking process. Several key decisions have to be taken regarding the attainment of consensual valuations in the case of several decision makers: the type of data given the available information; the type of normalization process given the characteristics of each decision context and the selection of a weighting scheme representing the preferences with regards to the different criteria.

A real decision example has been included to illustrate the potential of the IS-TOPSIS approach for the ranking of a set of discrete alternatives in different decision contexts. In particular, we have addressed the ranking of mathematical educational videos based on six didactical dimensions. A set of 12 videos from You Tube has been considered explaining the concept of basis of a vector space. The videos have been evaluated in those didactical criteria by three educational experts using numerical intervals and linguistic variables. An additional expert has fixed the ideal intervals for each criterion and different rankings have been attained depending on the type of data and the normalization process. The normalization process reflects, in the context of similarity with the ideal solution, the importance given by the decision maker to the deviations with respect to the ideal (slack or surplus) and his desire to take into account or not the extreme values of the intervals.

The ranking of educational videos based on multiple didactical criteria could be of great interest for practitioners in the education field. It is important to rate educational videos available in Internet platforms as You Tube because they could constitute a very useful and attractive tool for educators for its use both, in the classroom and outside it. Internet gives the students access to a high number of learning resources with different methodologies and procedures and with several degrees of success. The educational videos could give the students, academic support both in the short-term and the long-term, solving in-time particular questions or providing assistance in long-term learning processes. If the educational institutions have a wide repertory of mathematical educational videos rated from the previously described didactical criteria, then they would be able to build a historical memory of “best practices” in the teaching of specific mathematical contents.

One of the main features of the proposed ranking in this paper is that by construction the ranking is not sensible to changes in the positive ideal or negative ideal solutions. It has also very low sensitivity to changes on the data as ambiguity, imprecision and uncertainty have been taken into account from the first step of the method. The main fact potentially affecting the obtained ranking is the weighting scheme. In future works we will try to determine the criteria weights in an objective way directly from the available data. This will solve one of the main criticism of the TOPSIS ranking method. More dimensions will be also included in the decision problem in order to take into account the quality of the videos in terms of image, sound, interaction level with users and the authority principle. This would enrich the assessment of the free-online videos available in platforms as YouTube. The proposed normalization method will be also integrated into a VIKOR-based approach in order to rank videos following other didactical points of view, as commented previously.

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[6] https://www.youtube.com/watch?v=nOfY1ZATzIM

[7] https://www.youtube.com/watch?v=wYKAw5QanJY

[8] https://www.youtube.com/watch?v=AqXOYgpbMBM

[9] https://www.youtube.com/watch?v=k7RM-ot2NWY

[10] https://www.youtube.com/watch?v=lf5WacddAo4

[11] https://www.youtube.com/watch?v=3l3qfs2vINE

[12] https://www.youtube.com/watch?v=RO-XYVeaROw

Appendix

Table 1A. Individual scores and consensual intervals

	Scores from expert 1						*Scores from expert 2*						*Scores from expert 3*					
Video	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
V_1	9	7	8	2	8	2	10	7	9	1	8	1	10	8	7	2	9	2
V_2	7	6	6	3	6	4	5	3	0	4	2	7	8	7	5	3	4	8
V_3	9	6	6	4	7	4	10	10	9	2	9	2	10	10	6	3	8	4
V_4	8	7	5	4	6	3	10	7	1	7	9	5	10	6	1	2	9	5
V_5	9	6	5	3	7	3	10	4	0	8	9	7	10	6	1	4	10	8
V_6	9	7	5	3	7	2	10	8	5	5	9	5	10	8	6	5	9	4
V_7	8	9	7	3	6	3	10	3	3	8	7	6	9	3	2	1	8	4
V_8	6	8	6	4	7	2	9	2	5	7	5	8	9	9	5	3	2	2
V_9	9	9	9	2	8	1	10	10	10	2	6	1	10	10	10	2	7	1
V_{10}	8	7	4	5	6	4	9	9	3	5	9	5	9	9	2	2	8	6
V_{11}	7	7	7	4	7	3	7	4	5	2	6	7	8	9	8	5	6	6
V_{12}	5	7	6	7	6	3	3	10	9	7	2	0	4	10	8	4	6	0

Consensual intervals

	$\left[z_{i1}^L, z_{i1}^R\right]$	$\left[z_{i2}^L, z_{i2}^R\right]$	$\left[z_{i3}^L, z_{i3}^R\right]$	$\left[z_{i4}^L, z_{i4}^R\right]$	$\left[z_{i5}^L, z_{i5}^R\right]$	$\left[z_{i6}^L, z_{i6}^R\right]$
V_1	[9, 10]	[7, 8]	[7, 9]	[1, 2]	[8, 9]	[1, 2]
V_2	[5, 8]	[3, 7]	[0, 6]	[3, 4]	[2, 6]	[4, 8]
V_3	[9, 10]	[6, 10]	[6, 9]	[2, 4]	[7, 9]	[2, 4]
V_4	[8, 10]	[6, 7]	[1, 5]	[2, 7]	[6, 9]	[3, 5]
V_5	[9, 10]	[4, 6]	[0, 5]	[3, 8]	[7, 10]	[3, 8]
V_6	[9, 10]	[7, 8]	[5, 6]	[3, 5]	[7, 9]	[2, 5]
V_7	[8, 10]	[3, 9]	[2, 7]	[1, 8]	[6, 8]	[3, 6]
V_8	[6, 9]	[2, 9]	[5, 6]	[3, 7]	[2, 7]	[2, 8]
V_9	[9, 10]	[9, 10]	[9, 10]	[2, 2]	[6, 8]	[1, 1]
V_{10}	[8, 9]	[7, 9]	[2, 4]	[2, 5]	[6, 9]	[4, 6]
V_{11}	[7, 8]	[4, 9]	[5, 8]	[2, 5]	[6, 7]	[3, 7]
V_{12}	[3, 5]	[7, 10]	[6, 9]	[4, 7]	[2, 6]	[0, 3]

Table 2A. Individual linguistic rates and consensual rates

	Scores from expert 1						*Scores from expert 2*						*Scores from expert 3*					
Video	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
V_1	VG	G	G	VP	G	VP	VG	G	VG	VP	G	VP	VG	G	G	VP	VG	VP
V_2	G	F	F	P	F	P	F	P	VP	P	P	G	G	F	F	P	P	G
V_3	VG	F	F	P	G	P	VG	VG	G	P	VG	P	VG	VG	F	P	VG	F
V_4	G	G	F	P	F	P	VG	G	VP	G	VG	F	VG	F	VP	P	VG	F
V_5	VG	F	F	P	G	P	VG	P	VP	G	VG	G	VG	F	VP	P	VG	G
V_6	VG	G	F	P	G	VP	VG	G	F	F	VG	F	VG	G	F	F	VG	P
V_7	G	VG	G	P	G	P	VG	P	P	G	G	F	VG	P	P	P	VG	VP
V_8	G	G	F	P	G	VP	VG	P	F	G	F	G	VG	VG	F	P	P	VP
V_9	VG	VG	VG	VP	G	VP	VG	VG	VG	P	G	VP	VG	VG	VG	P	G	P
V_{10}	G	G	P	F	F	P	VG	VG	P	F	VG	F	VG	VG	P	P	VG	F
V_{11}	G	G	G	P	G	P	G	P	F	P	G	F	G	VG	G	F	G	F
V_{12}	F	G	F	G	F	P	P	G	VG	G	P	VP	F	VG	VG	F	F	VP

Table 2A. Individual linguistic rates and consensual rates (continuing)

	Consensual sets					
	{Z_1}	{Z_2}	{Z_3}	{Z_4}	{Z_5}	{Z_6}
V_1	{VG}	{G}	{G, VG}	{VP}	{G, VG}	{VP}
V_2	{F, G}	{P, F}	{VP, F}	{P}	{P, F}	{P, G}
V_3	{VG}	{F, VG}	{F, G}	{P}	{G, VG}	{P, F}
V_4	{G, VG}	{F, G}	{VP, F}	{P, G}	{F, VG}	{P, F}
V_5	{VG}	{P, F}	{VP, F}	{P, G}	{G, VG}	{P, G}
V_6	{VG}	{G}	{F}	{P, F}	{G, VG}	{VP, P, F}
V_7	{G, VG}	{P, VG}	{P, G}	{P, G}	{G, VG}	{P, F}
V_8	{G, VG}	{P, G, VG}	{F}	{P, G}	{P, F, G}	{VP, G}
V_9	{VG}	{VG}	{VG}	{VP, P}	{G}	{P,VP}
V_{10}	{G, VG}	{G, VG}	{P}	{P, F}	{F, VG}	{P, F}
V_{11}	{G}	{P, G, VG}	{F, G}	{P, F}	{G}	{P, F}
V_{12}	{P, F}	{G, VG}	{G, VG}	{F, G}	{P, F}	{VP, P}

Table 3A. Numerical transformation of linguistic labels

	Scores from expert 1						*Scores from expert 2*						*Scores from expert 3*					
Video	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
V_1	5	4	4	1	4	1	5	4	5	1	4	1	5	4	4	1	5	1
V_2	4	3	3	2	3	2	3	2	1	2	2	4	4	3	3	2	2	4
V_3	5	3	3	2	4	2	5	5	4	2	5	2	5	5	3	2	5	3
V_4	4	4	3	2	3	2	5	4	1	4	5	3	5	3	1	2	5	3
V_5	5	3	3	2	4	2	5	2	1	4	5	4	5	3	1	2	5	4
V_6	5	4	3	2	4	1	5	4	3	3	5	3	5	4	3	3	5	2
V_7	4	5	4	2	4	2	5	2	2	4	4	3	5	2	2	2	5	2
V_8	4	4	3	2	4	1	5	2	3	4	3	4	5	5	3	2	2	1
V_9	5	5	5	1	4	1	5	5	5	2	4	1	5	5	5	2	4	1
V_{10}	4	4	2	3	3	2	5	5	3	3	5	3	5	5	2	2	5	3
V_{11}	4	4	4	2	4	2	4	2	3	2	4	3	4	5	4	3	4	3
V_{12}	3	4	4	4	3	2	2	4	5	4	2	1	3	5	5	3	3	1

	Consensual sets					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
V_1	5	4	[4, 5]	1	[4, 5]	1
V_2	[3, 4]	[2, 3]	[1, 3]	2	[2, 3]	[2, 4]
V_3	5	[3, 5]	[3, 4]	2	[4, 5]	[2, 3]
V_4	[4, 5]	[3, 4]	[1, 3]	[2, 4]	[3, 5]	[2, 3]
V_5	5	[2, 3]	[1, 3]	[2, 4]	[4, 5]	[2, 4]
V_6	5	4	3	[2, 3]	[4, 5]	[1, 3]
V_7	[4, 5]	[2, 5]	[2, 4]	[2, 4]	[4, 5]	[2, 3]
V_8	[4, 5]	[2, 5]	3	[2, 4]	[2, 4]	[1, 4]
V_9	5	5	5	[1, 2]	4	1
V_{10}	[4, 5]	[4, 5]	2	[2, 3]	[3, 5]	[2, 3]
V_{11}	5	[2, 5]	[3, 4]	[2, 3]	4	[2, 3]
V_{12}	[2, 3]	[4, 5]	[4, 5]	[3, 4]	[2, 3]	[1, 2]