

SESIÓN ESPECIAL 18 (9 HORAS)

## OPTIMIZACIÓN DINÁMICA Y CONTROL. APLICACIONES A LA ECONOMÍA

Coordinadores: David Martín de Diego (CSIC)  
Juan Pablo Rincón Zapatero (UC3M)

### Conferenciantes

**Emilio Cerdá–Tena** (Universidad Complutense de Madrid): Optimal Control in Environmental and Resource Economics.

**Gregorio Díaz** (Universidad Complutense de Madrid): On an Optimal Investment–Consumption Model with a Wealth Constraint.

**Alberto Ibort** (Universidad Carlos III de Madrid): A Numerical Analysis Perspective of Control Theory.

**Ricardo Josa–Fombellida** (Universidad de Valladolid): A New Perspective to Stochastic Control Problems: Application to Consumption–Investment Models.

**Manuel de León** (CSIC): Symmetry and Reduction in Optimal Control Problems.

**Jesús Marín–Solano** (Universitat de Barcelona): Inverse Problems in Dynamic Optimization.

**David Martín de Diego** (CSIC): Geometric Integrators and Optimal Control Problems.

**Pascal Mossay** (Universidad de Alicante): Local Markets and Expectations.

**Jorge Navas** (Universitat de Barcelona): Investment Grants and the User Cost of Capital.

**Juan Pablo Rincón–Zapatero** (Universidad Carlos III de Madrid): Brief Review of Some Tools for Solving Stochastic Control Problems.

**Rosario Romera** (Universidad Carlos III de Madrid): Some Recent Results on Markovian Decision Models.

**Santiago J. Rubio** (Universidad de Valencia): Tariffs Agreements and International Non-renewable Resources Monopolies: Prices versus Quantities.

**Angels Xabadia** (Universidad de Zaragoza): The Economics of Competiton Between Individuals in Biological Populations.

### Horario

Lunes 31 de enero	Aula A8 <b>Plenarias</b>
11:30–12:40	Emilio Cerdá Tena
12:50–13:30	Gregorio Díaz
18:00–20:00	Alberto Ibort (Minicurso)
Martes 1 de febrero	Aula A8 <b>Comunicaciones</b>
11:30–12:00	David Martín de Diego
12:00–12:30	Manuel de León
12:30–13:00	Jesús Marín Solano
13:00–13:30	Jorge Navas
15:30–16:00	Santiago J. Rubio
16:00–16:30	Angels Xabadia
16:30–17:00	Pascal Mossay
17:00–17:30	Rosario Romera
18:00–18:30	Ricardo Josa Fombellida
18:30–19:00	Juan Pablo Rincón Zapatero

# Optimal Control in Environmental and Resource Economics

Emilio Cerdá Tena

*Departamento de Fundamentos del Análisis Económico I*

*Universidad Complutense de Madrid*

**Abstract.** In this lecture we present a panoramic view of different problems arising in Environmental and Resource Economics, which are solved by using Optimal Control. We study problems of pollution control and optimal management of both renewable and non-renewable natural resources. We present deterministic and stochastic models, in discrete and continuous time, with finite and infinite time horizon. We study conditions and paths that lead to steady states. We comment on and interpret from the economic point of view the main results obtained. We comment on specific problems on which the author has been working.

**Keywords:** Optimal Control, Pontryagin Maximum Principle, Dynamic Programming, Stochastic Control, Renewable Resources, Non-renewable Resources, Pollution.

# On an Optimal Investment–Consumption Model with a Wealth Constraint

Gregorio Díaz

*Departamento de Matemática Aplicada*

*Universidad Complutense de Madrid*

**Abstract.** Here we revisit an optimal investment–consumption model proposed by Merton (1971). Our contribution deals with the treatment by mean of the Dynamics Programming Principle in order to characterize the optimal value function by a Hamilton-Jacobi-Bellman equation solved in the viscosity solution sense. We consider a wealth constraint: in some sense, it is a kind of risk aversion term of a single investor. Relative to the involved boundary value problem, we extend some results for Dirichlet conditions to a more general boundary conditions.

# The Economics of Competition Between Individuals in Biological Populations

Renan U. Goetz

*Departament de Economia  
Universitat de Girona*

Angels Xabadia

*Departamento de Análisis Económico  
Universidad de Zaragoza*

**Abstract.** Describing biological growth where the changes in time and in the size of individuals of a biological population together with the fact that individuals compete for scarce resources are considered give rise to a partial integrodifferential equation. With respect to the determination of the optimal forest management regime this set up leads to a distributed optimal control problem. The paper presents a numerical technique that allows transforming the original distributed control problem into an ordinary control problem. In contrast to the previous literature this procedure does not require to program complex numerical algorithms but can be implemented by standard optimization software. To demonstrate the applicability of our approach, the empirical part of the paper determines the optimal selective-logging regime of a forest that consists of Scots Pine (*pinus sylvestris*) from a private perspective.

# A Numerical Analysis Perspective of Control Theory

Alberto Iborr

*Departamento de Matemáticas*  
*Universidad Carlos III de Madrid*

**Abstract.** Control theory has provided a variety of numerical problems and is at the origin of important developments in computation. In this talk we would like to look back at control theory from the perspective of modern numerical analysis and discuss some fundamental numerical concepts like stability, well-posedness, conditioning, etc. of control theory problems and how a look to these questions could help in the development of new algorithms and numerical methods.

# A New Perspective to Stochastic Control Problems: Application to Consumption–Investment Models

Ricardo Josa Fombellida

*Dpto. Estadística e Investigación Operativa  
Universidad de Valladolid*

Juan Pablo Rincón Zapatero

*Dpto. de Economía  
Universidad Carlos III de Madrid*

**Abstract.** We establish in this paper a new perspective in the solution of stochastic control problems with one state variable and two or more control variables. The approach is based in the characterization of the optimal control as solution of a second order quasilinear system of PDEs, instead of resorting to the Hamilton–Jacobi–Bellman equation, which for these types of problems turns to be non-linear. We find necessary and sufficient optimality conditions and also the connection of our approach with that of dynamic programming. We apply the method to some problems as the linear-quadratic and the consumption–investment model.

**Keywords:** Stochastic control, Hamilton-Jacobi-Bellman equation, consumption-investment model.

# Symmetry and Reduction in Optimal Control Problems

Manuel de León  
*CSIC*

**Abstract.** Symmetries are a valuable tool for integrating the equations of motion of a dynamical system. Indeed, by Noether's theorem, each symmetry gives rise to a first integral of the equations of motion for a variational problem, which allows us to reduce the total number of differential equations to be integrated. This viewpoint allows us to reduce the number of equations associated with optimal control problems with symmetry and compare the solutions of the original system with the solutions of the reduced one. The reconstruction of the optimal controls starting from the reduced problem will be also explored. A simple example from optimal control in economy will be considered.



# Inverse Problems in Dynamic Optimization

Jesús Marín Solano

*Departament de Matemàtica Econòmica, Financera i Actuarial  
Universitat de Barcelona*

**Abstract.** The integrability problem has a long tradition in economic theory. The questions to be solved are: given a demand function, can we find preferences which rationalize the demand function?; and, in the affirmative case, how can we find those preferences? In this talk we will analyze different approaches that can be found in the literature concerning the dynamic version of the problem: given the dynamics of a system, is it possible to find utility functionals such that their optimal solutions are equivalent to the dynamics of the system? More concretely, we will study the relationships between the inverse problem of variational calculus (given a system of differential equations, are those differential equations the critical solutions for a variational problem?) and different versions of the inverse problem which can be found in optimal control theory and, in particular, in the economic literature.

# Geometric Integrators and Optimal Control Problems

David Martín de Diego  
*CSIC*

**Abstract.** We propose a derivation of numerical integrators for optimal control problems. It is based in the classical technique of generating functions adapted to the special features of optimal control problems. We will also analyze some possible applications to economic problems.

# Local Markets and Expectations

Pascal Mossay

*Departamento de Fundamentos del Análisis Económico*

*Universidad de Alicante*

**Abstract.** Our primary objective is to develop a framework to think about the implications of people's rational decisions concerning migration in a continuous spatial economy. Local markets are assumed perfectly competitive. While adjusting consumption is costless, migration is costly. So as to capture this distinction, prices on markets will form instantaneously as in the Walrasian tradition and migration is modelled as an adjustment process. First we show that the consumer's problem may be split in a pure consumption problem and a pure migration problem. The pure migration problem is a problem in the calculus of variations. Existence of a solution to this problem is proved under mild conditions. As a special case, when the spatial distribution of prices is time-invariant, there is a one-to-one relationship between mobility and utility: high migration costs are associated with low utility levels. Then the interaction of these individual decisions concerning consumption and migration through the market mechanism is examined. We study whether prices will converge across locations over time. It is shown that spatial convergence is obtained in a linear sense provided that local goods are gross substitutes.

# Investment Grants and the User Cost of Capital

Jorge Navas and Jesús Marín-Solano

*Departament de Matemàtica Econòmica, Financera i Actuarial*  
*Universitat de Barcelona*

**Abstract.** Since Jorgenson (1963), the cost of capital concept has become a useful tool for the study of the investment demand and its sensitivity to government actions. Among the set of government policies to stimulate capital accumulation, fiscal policies are being used with the aim of reducing the user cost of capital, and several works in the tradition of the neoclassical theory of investment (e.g., Ruane, 1982, and Van Loon, 1983) have studied the effects of financial incentives on the investment policy of firms in the form of grants related to assets. In this paper we address some non-standard situations that will affect the user cost of capital, derive the corresponding expression for the cost of capital, and discuss the associated policy implications.

# Brief Review of Some Tools for Solving Stochastic Control Problems

Juan Pablo Rincón–Zapatero  
*Departamento de Economía*  
*Universidad Carlos III de Madrid*

**Abstract.** In this talk we attempt to give an overview to the most well-known techniques in the study of stochastic control problems: the dynamic programming principle, the stochastic maximum principle and the martingale approach.

## Some Recent Results in Markovian Decision Models

Rosario Romera

*Departamento de Estadística*

*Universidad Carlos III de Madrid*

**Abstract.** Markovian decision processes (MDPs) have been traditionally used in Control Engineering and Economics. However, a growing interest in areas such as Finance and Insurance is observed. In this talk we review the main features of MDP and how these models are suitable to study real problems arising in the aforementioned disciplines. Furthermore, some recent results on the existence of optimal strategies in discrete time MDPs with unbounded costs are presented.

# Tariffs Agreements and International Non-renewable Resources Monopolies: Prices versus Quantities

Santiago J. Rubio

*Departamento de Análisis Económico  
Universidad de Valencia*

**Abstract.** In this paper the scope of Bergstrom's (1982) results is studied. Moreover, his analysis is extended assuming that extraction cost is directly related to accumulated extractions. For the case of a competitive market it is found that the optimal policy is a constant tariff if extraction is costless. However, with depletion effects, the optimal tariff must ultimately be decreasing. For the case of a monopolistic market the results depend crucially on the kind of strategies the importing country governments can play and on whether the monopolist chooses the price or extraction rate. For a price-setting monopolist it is shown that the importing countries cannot use a tariff to capture monopoly rents if they are constrained to use open-loop strategies, even if the governments sign a tariff agreement. This result is drastically modified if the importing countries in the tariff agreement use Markov (feedback) strategies. For a quantity-setting monopolist the nature of the game changes and the importing country governments find it advantageous to set a tariff on resource importations. Moreover, in this case the importing countries in a tariff agreement enjoy a strategic advantage which allows them to behave as a leader.

**Keywords.** Tariffs, tariff agreements, nonrenewable resources, depletion effects, price-setting monopolist, quantity-setting monopolist, differential games, open-loop strategies, linear strategies, Markov-perfect Nash equilibrium, Markov-perfect Stackelberg equilibrium.