Bounding areas and lengths and Holditch's theorem

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Holditch's theorem is an old result on areas of planar closed curves related with moving chords. If a curve is traced out by a chosen point in a chord of constant length that go through another curve such that its endpoints always lie on that curve, then the difference between the areas of both curves is the same as the area of an ellipse with semiaxes the lengths in which the point divides the chord. Some generalizations of this result has been studied. The objective is to consider a 2-dimensional manifold with bounded curvature and to use Rauch comparison theorems for Jacobi fields to obtain some boundings on areas and lengths of curves generated by moving chords. As particular cases, Holditch's theorem, the Steiner's formulas for parallel or offset curves and the Barbier's theorem, all generalized for constant curvature surfaces are found.