ERE–2009, Bilbao

CREATABLE UNIVERSES:

A NEW APPROACH

R. Lapiedra and J. A. Morales-Lladosa

Departament d'Astronomia i Astrofísica

Universitat de València

- One often uses to say that if the Universe arose out of a quantum vacuum fluctuation, one could expect it to have

  zero energy, zero 3-momenta, and also

  zero angular 4-momenta

- So, we will consider both:

  linear 4-momentum, $P^\alpha = (P^0, P^i)$, and

  angular 4-momentum, $J^{\alpha\beta} = (J^{0i}, J^{ij})$.

- Only those universes with

  $P^\alpha = 0$, and $J^{\alpha\beta} = 0$ (with respect to any event)

  could have arisen from a quantum vacuum fluctuation.

  Only those ones would be then 'creatable universes'.

- The big problem of the Universe energy and momenta definition is the lack of uniqueness:

  they are coordinate dependent quantities

  (whatever the energy-momentum complex we take).

- In asymptotically flat space-times, there is no problem:

  P^α and $J^{\alpha\beta}$ are uniquely defined

  (in asymptotic Minkowskian coordinate systems).

- But, since we are interested in the energy and momenta of the Universe, we need to go beyond asymptotically flat space-times.

- So, our first goal will be to reduce as much as possible the number of acceptable coordinate systems to properly define the Universe energy and momenta.

- We expect any possible Universe to have well defined energy and momenta, i.e., P^α and $J^{\alpha\beta}$ are finite and conserved in time. Even better, we expect them to be zero:

We expect the actual Universe to be 'creatable'

- Which coordinate time, then, to begin with? A physical universal time. That is, in order to properly define P^α and $J^{\alpha\beta}$, we need Gauss coordinates:

$$ds^2 = -dt^2 + dl^2 \,, \qquad dl^2 = g_{ij} dx^i dx^j \,, \quad i, j = 1, 2, 3.$$

- Since we treat all momenta at the same level, we need a symmetric energy-momentum complex. We will take the Weinberg one. Then, one obtains in Gauss coordinates:

- Linear 4-momentum:

$$P^0 = \frac{1}{16\pi G}\int(\partial_j g_{ij} - \partial_i g)d\Sigma_{2i}\,, \quad P^i = \frac{1}{16\pi G}\int(\dot{g}\delta_{ij} - \dot{g}_{ij})d\Sigma_{2j}$$

- Angular 4-momentum:

$$\begin{aligned} J^{jk} &= \frac{1}{16\pi G}\int(x_k\dot{g}_{ij} - x_j\dot{g}_{ki})d\Sigma_{2i}\,, \\ J^{0i} &= P^i t - \frac{1}{16\pi G}\int[(\partial_k g_{kj} - \partial_j g)x_i + g\delta_{ij} - g_{ij}]d\Sigma_{2j} \end{aligned}$$

- Notation:

$g \equiv \delta^{ij} g_{ij}\,, \qquad \dot{g}_{ij} \equiv \partial_t g_{ij};$

Σ_2 is the 2-surface boundary of Σ_3;

$d\Sigma_{2i}$ is the corresponding surface element.

Indices are raised and lowered with the Minkowski tensor.

- Since P^α and $J^{\alpha\beta}$ are supposed to be conserved, we only need this metric in the neighborhood of $\Sigma_3 \quad (t=0)$.

- Since P^α and $J^{\alpha\beta}$ are written as surface integrals on Σ_2, or as a limit of a sequence of surface integrals, all we need is a Gauss coordinate system on Σ_2 and its immediate neighborhood.

- In a Gauss coordinate system $\{x'^i\}$ on Σ_3, with base on Σ_2, one can write

$$\Sigma_2 : \quad x'^3 = 0$$

and from the above expressions for the momenta we obtain:

- Linear 4-momentum:

$$P^0 = -\frac{1}{2\kappa}\int \partial_3 g_{aa}\; dx^1 dx^2\,, \quad P^3 = \frac{1}{2\kappa}\int \partial_0 g_{aa}\; dx^1 dx^2$$

$$P^a = -\frac{1}{2\kappa}\int \partial_0 g_{3a}\; dx^1 dx^2 \qquad (a = 1, 2)$$

where $\kappa \equiv 8\pi G$ and $g_{aa} = g_{11} + g_{22}$.

- Angular 4-momentum:

$$\begin{aligned} J^{ij} &= \frac{1}{2\kappa}\int (x_j \partial_0 g_{3i} - x_i \partial_0 g_{3j})\; dx^1 dx^2\,, \\ J^{0a} &= \frac{1}{2\kappa}\int x^a \partial_3 g_{bb}\; dx^1 dx^2\,, \\ J^{03} &= -\frac{1}{2\kappa}\int g_{aa}\; dx^1 dx^2\,. \end{aligned}$$

- Let us consider a *universe*: a space-time where both 4-momenta

$$P^{\alpha} \qquad \text{and} \qquad J^{\alpha\beta} \ ,$$

  have been well defined, and then, conserved quantities.

- Then, the question is: for any *universe*,

  there exist Gauss coordinate systems such that both 3-momenta,

  P^i and J^{ij} irrespective of the momentum origin vanish ?

- The answer is YES, and consequently, the energy P^0 and momenta J^{0i} have an "intrinsic" meaning in these coordinate systems.

More precisely, the following statement occurs:

- For any *universe*, a 3-surface Σ_3 exists such that

$$P^i = 0, \qquad J^{ij} = 0$$

$$g_{ab}|_{\Sigma_2} = f\delta_{ab}, \qquad g_{33}|_{\Sigma_2} = 1, \qquad g_{3a}|_{\Sigma_2} = 0$$

in a Gauss coordinate system $\{x^\alpha\}$ based on Σ_3 $(x^0 = 0)$ and its boundary Σ_2 $(x^0 = x^3 = 0)$.

Such a coordinate system is unique up to a dilation and a rotation on Σ_2 at each point of Σ_2.

ABOUT THE CREATIVENESS OF THE *UNIVERSES*

Is any *universe* creatable?, that is,

does a *universe* satisfies that $P^\alpha = 0$ and $J^{\alpha\beta} = 0$ for any Σ_3?

- Generically, the answer is NO.
- Consequently, creativeness seems to be a non intrinsic property for *universes*.

- In fact, under a change from Σ_3 to Σ_3' that preserves the boundary of Σ_3, the energy density e and the linear momentum density p_3 of the P^3 component transform according as a generalized boost:

$$
\begin{aligned}
e' &= \frac{f'}{f}\,(e\cosh\psi - p_3\sinh\psi) \\
p_3' &= \frac{f'}{f}\,(e\sinh\psi - p_3\cosh\psi)
\end{aligned}
$$

where $\psi = \psi(x^a)$, $g_{ab} = f\delta_{ab}$, and $g'_{ab} = f'\delta_{ab}$.

- Then, it could seem that *universe* creativeness would be preserved only if

$$\frac{f'}{f} \quad \text{and} \quad \psi \quad \text{were} \quad \mathrm{CONSTANT}$$

which would be a very restrictive condition.

A 'GORDIAN DECISION'

The 'impasse' is bypassed if we assume that the notion of a creatable *universe* obeys to one of the following alternatives:

(A1) the area of Σ_2 vanishes, and the metric components have a sufficiently regular behavior in the vicinity of Σ_2,

(A2) Σ_3 does not contain any point of Σ_2, and furthermore

- the energy density, $e \propto \partial_3 g_{aa}$
- the lineal momentum density $p_3 \propto \partial_0 g_{aa}$, and
- the angular momentum densities $j^{03} \propto g_{aa}$ and $j^{0b} \propto x^b \partial_0 g_{aa}$

approach to zero when we approach to Σ_2 over Σ_3, OR

(A3) The components

$$P^0, \quad P^3 \quad \text{and} \quad J^{0i}$$

are zero because:

1) the corresponding integrals provides the necessary compensation, and

2) this compensation remains preserved under the above generalized boost, when we demand that metric symmetry becomes conveniently respected.

It can be seen that if $P^\alpha = 0$, $J^{\alpha\beta} = 0$ for some Σ_3, then this vanishing is preserved under any change of Σ_3: the *universe is "creatable"*.

WORK IN PROGRESS

- It remains to apply this scheme to particular situations (FLRW).
- Stephani and Bianchi models, deserves an special attention.